Minimum Spanning Tree Application in the Currency Market

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Our paper has been inspired by the minimum spanning tree (MST) methodology originally used in the field of stock returns. The objective of our contribution is to apply the approach in the foreign exchange markets. The methodology enables us to visualise complex and difficult market relations in a very obvious manner. We identified mutual positions of the relevant European and world currencies. Our findings are confirmed and complemented by the Sammon method as well. The European currencies are characterized by comparatively high diversity. The central position of China-related economies such as Hong-Kong and Singapore has been proved. Polish currency seems to be crucial in the Central European region.

1. Introduction

Undoubtedly, the financial market is a complex system with many complex features and various types of relations within it. Among other issues raised in the research, there is a relevant question of the suitability of scientific representation of the relations and interactions between the market structures. Recently, a lot of authors have focused on graph-theoretical representation and analysis of the financial market. One of the proponents of this approach is Mantegna (1999) famous for his seminal paper *Hierarchical structures in Financial Markets*. He suggested mapping of the stocks into the graph nodes, while the relations between stocks are projected into the graph edges. In such construction the edges are simply weighted by the Pearson's correlations between the pairs of stock returns. The classical correlation does not fulfil axioms of Euclidean distance. Therefore correlations between the pairs of stocks are transformed into the Euclidean distances. The concept of the *minimum spanning tree* (MST) as a minimal graph that covers all nodes without loops enables the complexity reduction view. It should be noted that MST obtained is close but not identical to the concept of the nearest neighbour single linkage clustering. The approach of Mantegna (1999) inspired other researchers (Bonanno, 2003; Onnela, 2003; Micciche, 2003) to apply the aforementioned methodology with several modifications on the stock markets. In our paper the MST representations of the world most important and regional currencies are compared. Since the concept of the MST in regards to the edges does not correctly visualise data similarities and differences, we suggest the use of the combination of MST with the Sammon planar projection methodology (Sammon, 1969).

2. Data and Methodology

We investigate a set of 18 currencies denominated in euro covering time period January 4, 1999 – December 12, 2012[1]. List of the currencies is given in Table 1.

Our research is inspired by the methodology of Mantegna (1999). We explore interactions (correlations) between n currency returns. From the logarithmic returns $r_i^t = \ln P_i^t - \ln P_i^{t-1}$ of the ith currency exchange rate P_i^t on day t we compute Pearson cross correlation coefficients between $n \times n$ pairs of currencies

$$\rho_{ij} = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{(\langle r_i^2 \rangle - \langle r_i \rangle^2)(\langle r_j^2 \rangle - \langle r_j \rangle^2)}}, \qquad (1)$$

where $\langle \ldots \rangle$ stands for the average over studied period. The coefficients ρ_{ij} constitute the square variance-covariance matrix.

Then, the correlation matrix is transformed into the distance matrix involving the elements

$$d_{ij} = \sqrt{2(1-\rho_{ij})}. \qquad (2)$$

The matrix is symmetric $d_{ij} = d_{ji}$ with zero diagonal values. It serves as the issue to derive the "minimum spanning tree" – connected distance weighted graph consisting of n nodes and $n - 1$ edges and containing no loops. The "minimum" attribute means that the sum of all weights of edges is minimal among all trees defined on the distance matrix. The Kruskal's and Prim's algorithms are most frequently used algorithms to identify MST. It should be also noted that the first algorithm was proposed by the outstanding Czech mathematician Otakar Borůvka.

3. Results – comparing MST approach and Sammon mapping

The graphical form of the correlation matrix with assigned MST edges is given in Figure 1, while the diagram of the MST is presented in Figure 2.

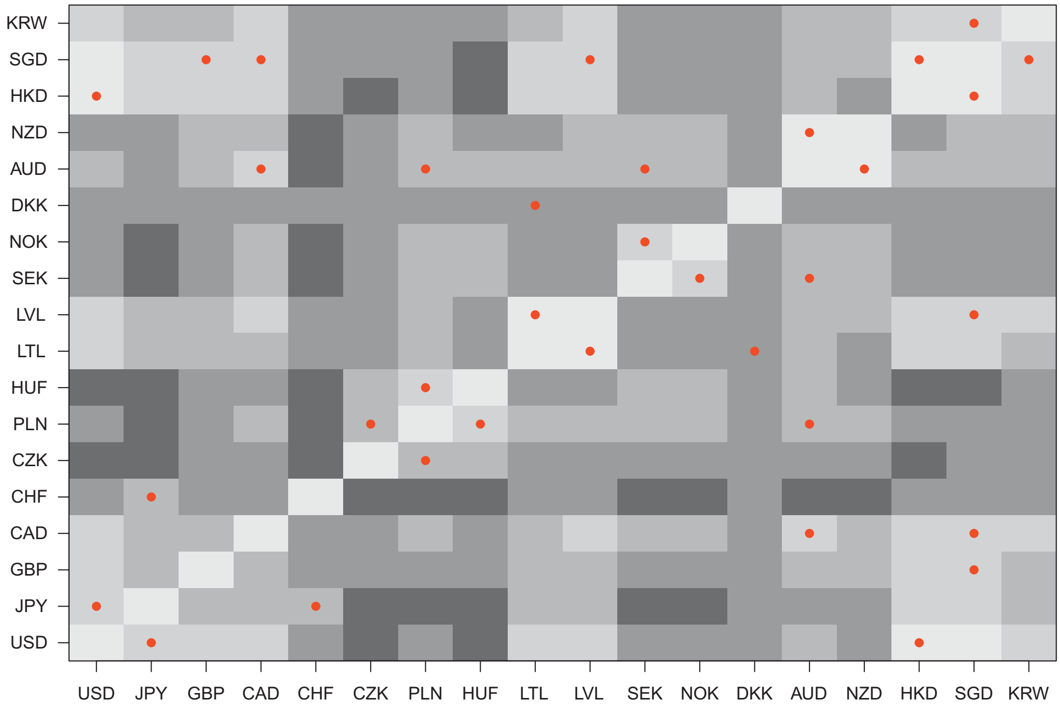
The MST diagram is an appropriate tool to identify the currencies playing a central role in the world currency market as well as the peripheral currencies having limited power to influence the market. In our case, the grouping reflecting the regional variations is rather obvious (see groupings CZK, PLN, HUF; NOK, SEK; AUD, NZD; HKD,

1 *Data were obtained from the Eurostat databases.*

Table 1 List of the investigated world and national currencies with their symbols

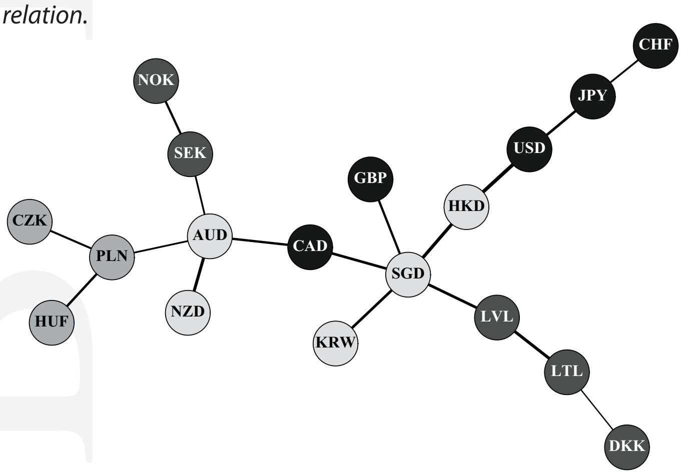
AUD	Australian Dollar
CAD	Canadian Dollar
CHF	Swiss Franc
CZK	Czech Koruna
DKK	Danish Krone
GBP	UK Pound Sterling
HKD	Hong Kong Dollar
HUF	Hungarian Forint
JPY	Japanese Yen
KRW	Korean Rep. Won
LTL	Lithuanian Litas
LVL	Latvian Lat
NOK	Norwegian Krone
NZD	New Zealand Dollar
PLN	Polish New Zloty
SEK	Swedish Krona
SGD	Singaporean Dollar
USD	U.S. Dollar

Figure 1 Correlation matrix of logarithmic returns of currencies calculated using Eq. (1). Higher correlations are depicted by lighter color, smaller correlations are depicted by the darker one. Adjacent vertices in the MST are depicted by red points.



SGD, KRW; LVL, LTL, DKK). On the other hand, the currencies with the node degree higher than two are considered to be central. The examples are PLN, AUD, SGD. As we know these were least affected by the 2007-2008 financial crisis that has harmed economic growth during the total period of interest (all other investigated countries besides Hong-Kong felt in recession). The central position of PLN could be caused by lower openess of Polish economy in comparison with other countries in the region of Central Europe, so the PLN is less affected by the economic crisis. From

Figure 2 MST representation obtained using the correlations of the logarithmic returns of currencies. Geographically related currencies and world important currencies are displayed by the same color. The thickness of lines is proportional to the currency pair correlation.



the MST construction is also clearly visible the central position of the China-related economies such as Hong-Kong and Singapore. There is now sufficient evidence to assume that growing divergences within Europe are intensified by the specific evolution of CHF, DKK, NOK, CZK, HUF and GBP graph's leaf nodes (peripheral positions whose degree is one), which are located in the opposite branches of the MST graph.

The above given interpretation of the MST outputs is quite obvious, however there are systematically leaved unmentioned correlations (edges), which are likely to be essential for the economic changes. This may be seen, for instance, by exploiting relatively old but not obsolete technique of Sammon mapping. Suppose the distance matrix between objects – currencies is known. Due to natural reasons the visualisation requirement leads us to introduce two extra dimensions with two real-valued coordinates $x_i, y_i; i = 1, 2, \dots, n$ in order to characterize the positions of nodes. The key point in such projection is introducing the distance $d_{ij}^* \equiv d_{ij}^*(x_i, y_i, x_j, y_j)$ between the nodes i and j. For this task it is natural to use the standard Euclidean metric

$$d_{ij}^* = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \qquad (3)$$

In order to judge the quality of the approximation $d_{ij} : d_{ij}^*$ the error formula is introduced

$$E\left(\{x_i, y_i\}_{i=1}^n\right) = \frac{1}{\sum_{i<j,i=1}^{n} d_{ij}^*} \sum_{i<j;i=1}^{n} \frac{\left(d_{ij} - d_{ij}^*\right)^2}{d_{ij}^*}. \qquad (4)$$

Figure 3 The comparison of two geometrically motivated methodologies applied to daily time-series data corresponding to the currencies depicted as graph nodes. Sammon mapping provides a two--dimensional view of the situation.[2]

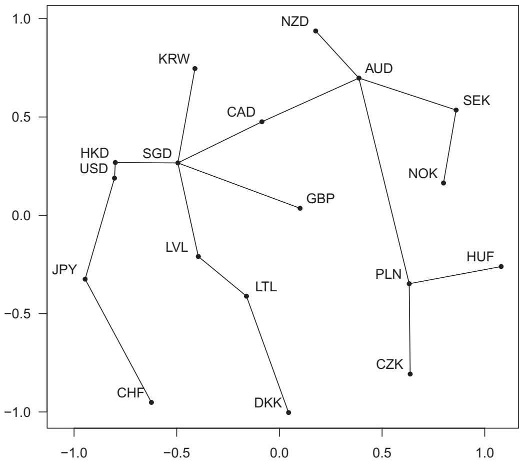
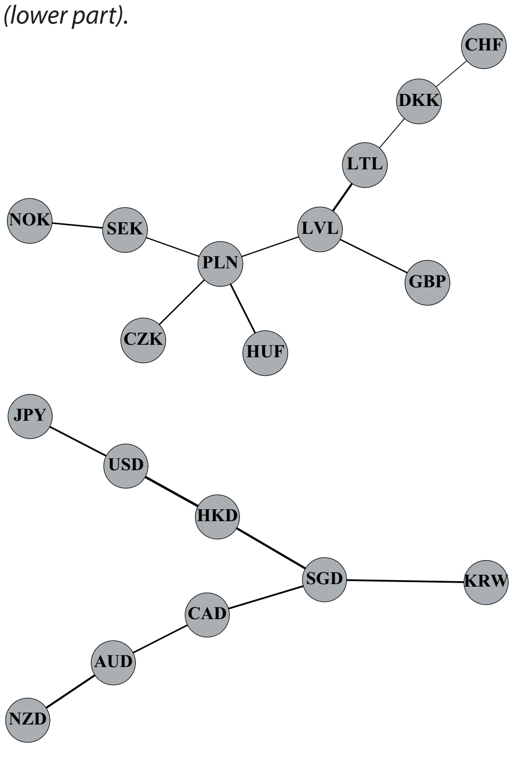


Figure 4 The different views provided by two independent MSTs: the MST of European currencies (upper part) and the MST of world currencies (lower part).



The non-uniqueness in the choice of the pair x_i, y_i is in part eliminated by the optimization of $E(\{x_i, y_i\}^n_{i=1})$, which yields

$$[x_i, y_i]_{min} = \arg\min_{[x_i, y_i]} E(\{[x_i, y_i]\}). \qquad (5)$$

Technically, the minimum is achieved in an iterative way. It might be also noted that the main advantage of the nonlinear Sammon projection as compared to the known principal component analysis is that it preserves the topological structure.

Our experience shows us that above mentioned approximate distance conservation property is crucial when considering the MST results. An output of the Sammon's method is presented in Figure 3. Here, the South-East part of the resulting map consists of European countries while the North-West part comprises the remaining countries. The location of the European currencies seems to be rather diversified and cover larger area than the currencies of the rest of the world. On the other hand, some of the currencies, which occupy remoted noncentral positions of the MST are located rather close to each other when projected via the Sammon technique (CHF vs. DKK and HUF vs. NOK are just some of examples).

For the present, perhaps the best solution of the puzzle might be a separate construction of two MSTs for European and the remaining world currencies and the identification of their diversification characteristics. The results are given in Figure 4.

As an overall diversification indicator might be chosen the normalized tree length

$$\bar{L}_{MST} = \frac{1}{n-1} \sum_{[ij]\in MST\,graph} d_{ij}. \qquad (6)$$

In such a way, normalized tree length of the European currencies involved in the MST is 2.17, while for the rest of the world we obtained the value 1.37. The finding coincides with an observation of the high diversity of European currencies uncovered by the Samon mapping. In the new diagram, the distances of HUF vs. NOK as well as CHF vs. DKK are substantially shortened as well. The central positions of the PLN, LVL and SGD currencies are persistent.

4. Conclusion

The currency market application which combines the MST method (Mantegna, 1999) and Sammon method is presented in the paper. The interesting graph structures justified by the couplings between geographic locations for the currencies with preferentially central and highly stable economies have been uncovered. On the other hand, some apparent contradictions between the MST and Sammon mapping occurred, which can be mitigated by providing intuitive splitting of the original complete MST into two non-overlapping parts. The first part includes the MST of the European currencies, while the world currencies are comprised in the second part. Further methodological practice will require the innovative formulation and implementation of the rules on how to partition the graph uniquely by using information from the MST and Sammon mapping. It should be also mentioned that the construction of the MST is built on the pair correlations between currencies which can reflect not only economic relations but also possible speculations in the currency market. The paper was elaborated within the project VEGA 1/0973/11.

2 The smaller the distance the larger is the correlation. The links represent correlations between currencies obtained by the MST construction.

References:

1. BONANNO, G., CALDARELLI, G., LILLO, F., MANTEGNA, R. N. Topology of correlation-based minimal spanning trees in real and model markets. In Physical Review. 2003, vol. 68.
2. MANTEGNA, R. N. Hierarchical structure in Financial Markets. In The European Physical Journal B. 1999, vol. 11, pp. 193-197.
3. MICCICHÉ, S., BONANNO, G., LILLO, F., MANTEGNA, R. N. Degree stability of a minimum spanning tree of price return and volatility. In Physica A. 2003, vol. 324, pp. 66–73.
4. ONNELA, J. P., CHAKRABORTI, A., KASKI, K., KERTÉSZ, J., KANTO, A. Asset Trees and Asset Graphs in Financial Markets. In Physica Scripta. 2003, vol. T106, pp. 48-54.
5. SAMMON, J. W. Jr. A Nonlinear Mapping for Data Structure Analysis. In IEEE Transactions on Computers. 1969, vol. C-18, no. 5, pp. 401-409.