The equilibrium interest rate – theoretical concepts and applications

Part 2

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In this part of the text, we apply some of the theoretical concepts, presented in the previous part, to the economy of Slovakia. Not all the theoretical concepts from the previous parts can be applied, because they are vague(Wicksell's definition) or trivial (opportunity cost concept), depend on assumptions and quantities, which are hard to determine, or for whose long-term level it is hard to provide arguments (technological progress in the case of a golden-rule-based definition), or their complexity goes beyond the scope of this paper (interest rate determination by means of the DSGE model).

3 Estimates of the equilibrium interest rate

3.1 Suitability of individual theoretical concepts and application in Slovakia

We have decided to direct our attention at the application of the neutral interest rate concept using two different calculation methods (this concept is probably the most popular one in the literature), we also deal with the issue of the relations of the equilibrium interest rate in the euro area and in Slovakia. In doing so, we modify the deliberations presented in the first part and take into account the catching-up process of the real economy. We also present an experimental model, which applies in practice aspects of the long-tem interest rate as much as possible. For comparison's sake, we also provide the results of a one-dimensional trend extraction.

3.2 Data used

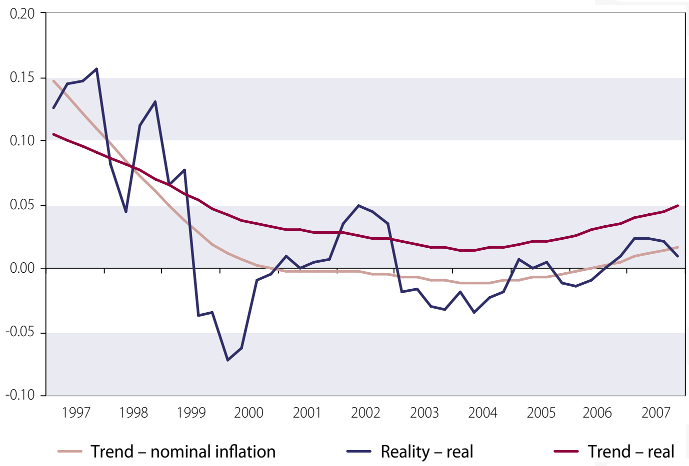
All the time series used are quarterly time series, which include the period from 1997 to 2007. Monthly rates of inflation were first converted to a basic index, which was then recoded to quarterly periodicity using the last period; subsequently we calculated the growth rates. The data stems from publicly available sources, primarily from the Statistical Office of the Slovak Republic, the ECB and the Eurostat, from which we have taken over the aggregate time series of interbank interest rates. The fixed capital stock is an exception – in this case we have used preliminary (unofficial) time series provided by the Statistical Office of the Slovak Republic.

3.3 One-dimensional trend extraction

The actual value of an interest rate can be viewed as the sum of two components – the equilibrium interest rate and a disequilibrium component. Let us assume that the actual interest rate oscillates around the equilibrium interest rate and that the equilibrium interest rate has greater inertia in time. Then we can obtain an estimate of the equilibrium interest rate by smoothing it in an appropriate way. We can use moving averages or, like in our case, a Hodrick-Prescott filter for that purpose.

The trend obtained in this way is shown in Chart 2. It has to be noted that if we filter directly the real interest rate time series (the inflation rate used is calculated from the total consumer price index, ex post), we assume the same cycle both in the nominal interest rate and in the rate of inflation. Alternatively, we can filter the nominal rate and the rate of inflation separately and then subtract the trend values. According to Chart 2, the second approach yields values that more closely follow the actual value at the same values of the smoothing parameter of the HP filter compared

Chart 2 Trends of the interbank interest rate (HP filter, λ=400)



Source: Own calculations.

to a situation where only the real interest rate series is subject to filtering. Additional assumptions and facts are necessary for a definitive evaluation of this phenomenon, but it can be associated either with the time lag (from the time inflation becomes visible to the time of its inclusion in inflationary expectations and in nominal interest rates) or with the generally differing character of inflation and interest rates fluctuations (inflation has outlying observations due to deregulations of some prices).

3.4 Neutral interest rate determined by means of the Kalman filter

A multidimensional filter with unobserved variables (the Kalman filter) is probably the most common method for the calculation of the neutral interest rate corresponding to a state where inflation equals the target and the actual product equals the potential product. Most models are based on the Philips curve (the relationship between the rate of inflation and the production gap, an observation equation) and the IS curve (the relationship between the production gap and the past values of the production gap, the disequilibrium component of the interest rate and possibly also other variables, a state equation). Similarly to the equilibrium interest rate, we consider the production gap an unobserved variable, so that the algorithm of the Kalman filter constructs simultaneous time series of the production gap and decomposes the interest rate. Benati and Vitale also consider inflationary expectations and the NAIRU to be unobserved variables in their version. Antoničová and Huček have constructed such a model also for Slovakia; in addition to the equilibrium interest rate the model also determines the equilibrium exchange rate. In the IS curve, the interest rate and the exchange rate appear jointly in the form of a currency conditions index.

A linear multidimensional filter with unobserved variables is constructed as a system of two groups of stochastic equations: the observation equations describe observable quantities – signals y as a function of the unobservable quantities – and the state equations describe unobservable quantities – states – as a function of their previous values. Both groups of equations can contain exogenous variables x – a fact usually used in practice, but not changing the mathematical essence of the problem. The equations contain matrices of the parameters F, G, M and N and variable components (the signal noise u in the observation equations and the process noise v in the state equations). In matrix terms, we can write this system as:

$$y_t = Fs_t + Mx_t + u_t$$

$$s_{t+1} = Gs_t + Nx_t + v_t$$

If the equations contain unknown parameters, they are estimated using the maximum likelihood method. The actual calculation of unobserved variables consists in a linear combination of the values implied, on the one hand, by the current values of the signals, and, on the other hand, by the past values of the states, so that the result has the minimum squared prediction error. The obtained state values are then adjusted so that all values contain information from the whole period under review (the formulas are specified e.g. by Pollock 2002).

In our model, there are four signals: the first two are identities, balancing the decomposition of the GDP logarithm and interest rate into the potential product LYPOT and the production gap LYGAP and into the equilibrium interest rate R and the disequilibrium component RGAP, respectively:

$$LOG(NAGDPSA) = LYPOT + LYGAP$$

$$REALMMIR = R + RGAP$$

The third signal is the core inflation rate *Δ LOG(CPIC)*, defined by the Philips curve as a function of the production gap, the nominal Slovak koruna to euro exchange rate *DLOG(EURSKK(-1))*, its past values and autonomous inflation (approximated by the difference between total and core inflation INFLEX)

$$\Delta LOG(CPIC) = \alpha_0 + \alpha_1 * LYGAP + (1 - \alpha_2 - \alpha_3) * \Delta LOG(EURSKK(-1)) + \alpha_2 * LOG(CPIC(-1)) + \alpha_3 * INFLEX + u_{CPIC},\ u_{CPIC} \sim IN(0, \sigma_1^2)$$

The last signal is loosely derived from the uncovered interest rate parity and contains information on the fact that exchange rate movements are associated with changes in the disequilibrium component of the interest rate:

$$\Delta LOG(1/EURSKK) = RGAP1\text{-}RGAP2 + \beta_1 * T1 - \Delta LOG(CPI\_SK/CPI\_EU) + u_{EURSKK},\ u_{EURSKK} \sim IN(0, \sigma_2^2)$$

Our model contains several states, but most of them are defined only as stochastic processes without a particular economic structure. The only relation based on economic theory is the IS curve describing the production gap (LYGAP) as a function of its past values and of the disequilibrium component of the interest rate. Auxiliary identities, defining the values of the production gap from past periods, also appertain to this equation:

$$LYGAP = \gamma_1 * LYGAP(-1) + \gamma_2 * LYGAP3(-1) + \gamma_3 * RGAP(-1) + v_{LYGAP},\ v_{LYGAP} \sim IN(0, \sigma_3^2)$$

$$LYGAP1 = LYGAP(-1)$$

$$LYGAP2 = LYGAP1(-1)$$

$$LYGAP3 = LYGAP2(-1)$$

The potential product is determined as a sum process with a deterministic drift and autocorrelated increments

$$LYPOT = \delta_0 + LYPOT(-1) + \Delta LYPOT(-1)$$

$$\Delta LYPOT = \eta_1 * \Delta LYPOT(-1) + v_{DPOT}, v_{DPOT} \sim IN(0, \sigma_4^2)$$

The equilibrium interest rate is determined as a cumulative process:

$$R = R(-1) + RSHOCK(-1) + v_R, v_R \sim IN(0, \sigma_5^2)$$

$$RSHOCK = v_{RSHOCK}, v_{RSHOCK} \sim IN(0, \sigma_6^2)$$

The disequilibrium component of the interest rate is determined as a first order AR process. The auxiliary identities define the values from the previous periods, similarly to the case of the production gap:

$$RGAP = \varphi_1 * RGAP(-1) + v_{RGAP}, v_{RGAP} \sim IN(0, \sigma_7^2)$$

$$RGAP1 = RGAP(-1)$$

$$RGAP2 = RGAP1(-1)$$

Because we have had relatively short time series, problems with numerical instability of the estimates have occurred when estimating the model – small changes in the starting values for the individual parameters have caused the estimation not to converge or to converge towards economically inacceptable values (a production gap with values of hundreds of percents, considerable negative interest rates). Laubach and Williams (2003) and other studies propose procedures to get round some of the problems, for example the introduction of a-priori values for the variance ratios of random components. In our case, we have obtained the starting values by estimating the individual relations using the least square method, with putting in values for state variables obtained by a one-dimensional band-pass filter. We have obtained the starting values of the variances either as descriptive statistics or as three times the standard regression error (when we used the standard regression error directly, the estimates often yielded a singular Hessian matrix). When the parameters had strong multicolinearity, we calibrated them so that the results can be interpreted economically as satisfactorily as possible. To calculate the real interest rate, we have used the rate of inflation from the total consumer price index ex post.

Charts 3 and 4 depict the development of the potential product and of the equilibrium interest rate. The values at the beginning of the period under review are not reliable, because they are influenced by converging interim results in the filtration algorithm. A comparison of the developments of the equilibrium values of the product and the interest rates shows that equilibrium values are more stable in time and that the actual values are oscillating around them, which corresponds to our assumptions. The interest rate chart shows great fluctuations, probably caused by slow absorption of inflation expectations in nominal interest rates. In addition, it is obvious that real interest rates are very low in the second half; as a result, the equilibrium interest rates are close to zero in that period.

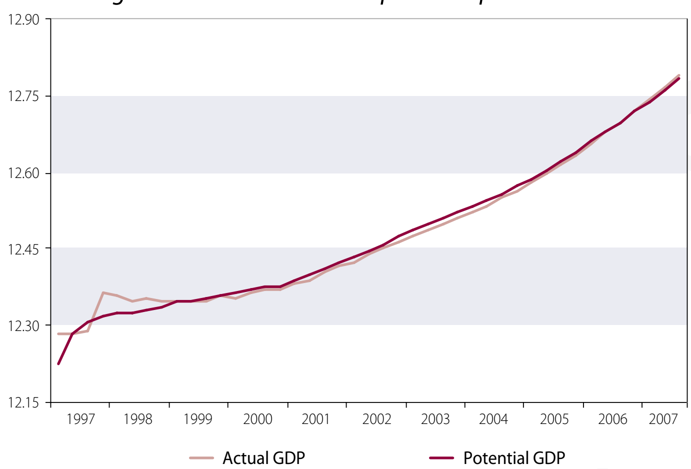
This interest rate corresponds to relations relevant for anticyclical monetary policy. A comparison of the actual and neutral rate shows that the fluctuations of the real interest rate in 1998 to 2000 were caused by a disequilibrium component, which is in line with the situation of the Slovak economy at that time. On the other hand, the growth of the actual values was accompanied by a growth of the equilibrium interest rate, which meant that while inflation decreased in

Table 1 Parameters of the model for the neutral interest rate in Slovakia

Parameter	Value	z-statistic
α_0	0.006	Calibrated
α_1	0.200	7.393
α_2	0.600	4.412
α_3	0.250	1.453
σ_1	0.010	7.773
β_1	0.050	4.982
σ_2	0.015	14.113
γ_1	0.900	2.848
γ_2	-0.070	-2.631
γ_3	-0.057	-3.335
σ_3	0.005	6.433
δ_0	0.012	3.072
η_1	0.200	4.390
σ_4	0.008	5.471
σ_5	0.002	Calibrated
σ_6	0.010	6.309
φ_1	0.400	4.714
σ_1	0.040	6.210
Log likelihood.	279.9503	Observations

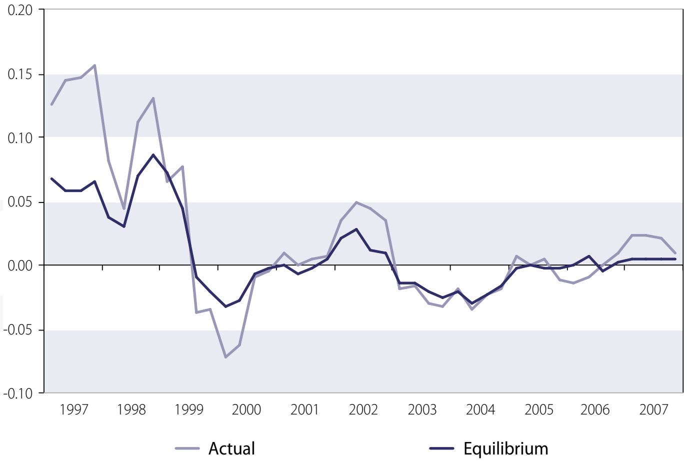
Source: Own calculations.

Chart 3 Logarithm of the actual and potential product



Source: Own calculations.

Chart 4 The actual and neutral real interest rate (Kalman filter)



Source: Own calculations.

1 *We think that the shift in income distribution was rather related to globalization than to technological innovation within the individual countries. However, this issue, as well as the setting of the equilibrium ratio of employee compensation to value added, goes beyond the scope of this study – important for us is the influenze of this phenomenon to interest rates.*

2002 (which was reflected in an increase in the real rates), there were no considerable changes in the GDP growth. The development of the neutral interest rate shows that the rate is positively influenced by positive supply shocks. The rate probably decreases with a growing public and private saving rate as a result of consolidation of public finance and as a result of the introduction of the second pillar (saving) of the pension system. These properties are in line with its assumed tendency to mutually balance investment and saving.

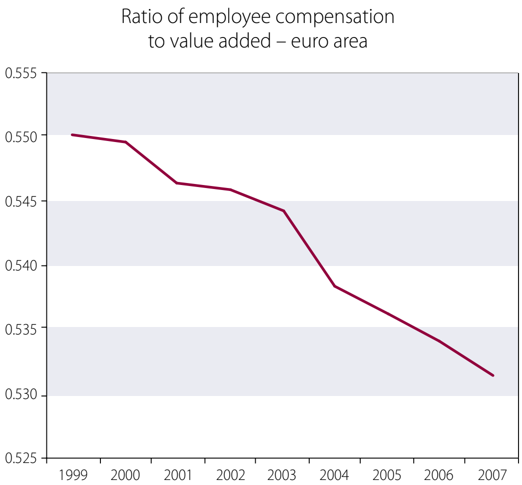
From the theoretical point of view, this method proceeds from the identification of disequilibria (of the production gap and the appertaining disequilibrium component of the interest rate), for which the equilibrium values have been calculated as the difference between the actual values and the disequilibrium components of the interest rate. This is associated with the definition of the neutral interest rate, which is supposed to correspond to stable inflation and a GDP at its potential level, so that economic theory (the IS curve and the Philips curve) actually speaks of disequilibrium components and the equilibrium values are obtained as a supplement to the actual values of the product and of the interest rate. Low values of the equilibrium interest rate are also associated with this procedure from disequilibrium components to equilibrium components, because just like the actual product oscillates around the potential product, the actual interest rates oscillate around the equilibrium interest rates.

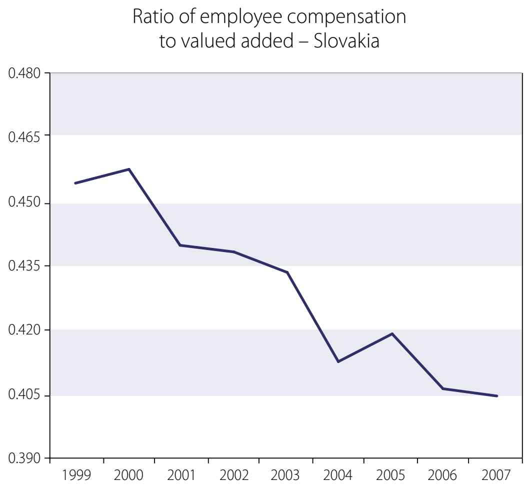
It is, however, obvious for the period under review that the actual values of real interest rates were influenced by a broader range of factors, which brought about their low level and are not included in the model used. The most important factors were probably insufficient inflation elasticity of nominal interest rates and the excess of worldwide saving over the volume of required investment.

It can be assumed that, following a long period of regulated prices, some economic agents do not form their inflationary expectations correctly and orient themselves according to nominal, not according to real interest rates. This leads to a decreased short-term and long-term sensitivity of nominal interest rates to inflation, which can entail a decrease in real interest rates. We assume that such an effect is present in transition economies; we do not expect it to be present considerably in the original EU countries.

The hypothesis of an excess of saving over investment is confirmed by the fact that the distribution of gross values added between employee compensation and gross operating surplus shifted in favor of the operating surplus during the period under review. If economic agents receiving employee compensation have a greater propensity to consume than economic agents receiving the operating surplus, this change leads to higher saving. However, equilibrium investment is determined by capital consumption and by the parameters of the production function and they can be considered constant.[1] If financial mar-

Chart 5 Economic indicators of the economic development of Slovakia





Source: Eurostat, own calculations.

kets were in equilibrium under the original value added distribution, there will be an excess of saving after the said change. The development of the ratio of employee compensation to values added is shown in Chart 5, which demonstrates the presence of the phenomenon both in Slovakia and in the euro area.

The glut of saving caused money to "chase" return and both the price of liquidity and the risk premium were accordingly changed. Although the interest rates had reflected the unsustainable character of fiscal policy until 1998, after consolidation they gradually also reached a very low level. Thus, the interest rates reacted to the discordance between saving and investment, but the conditions on financial markets were such that the interest rates took values out of keeping with other aspects of the equilibrium interest rate. It is obvious that it is lower than the marginal product of capital and probably also lower than the subjective discount rate of households[2], so that they should shift consumption from the future to the present. This is not sufficiently included in the model used, so that the explanatory power of the results might be decreased. In addition, the period under review is characterized by a relatively low sensitivity of the output to the interest rates, which can lead to unreliable results. The level of the neutral interest rate obtained in this way has thus to be considered a special artifact of the economic development in the period under review and not a value corresponding to the broader equilibrium. Therefore we have searched for an alternative calculation method, which would not be so dependent on relations within the economic cycle in Slovakia and would directly issue from the optimality conditions, in which the (equilibrium) interest rate occurs.

3.5 Neutral interest rate determined by means of a structural VAR model

In the previous part, we determined the equilibrium interest rate by means of two macroeconomic relations – the IS curve, in simplified terms the production gap as a function of the disequilibrium component of the interest rate, ygap = f(rgap), with the production gap itself not being a function of the neutral interest rate.

Alternatively, this chain of ideas can be approximated by a structural VAR model. Similarly to Brzoza-Brzezina, we construct a two-component SVAR model for the GDP increment (as an approximation of the production gap) and the real interest rate. The aim of the model is to calculate unobserved shocks, of which one corresponds to the equilibrium interest rate and the other one to the disequilibrium component of the interest rate. The first one is supposed to influence only the interest rate, the second one influences both quantities. The model takes the form:

$$\begin{bmatrix} \Delta y_t \\ r_t \end{bmatrix} = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} \Delta y_t \\ r_t \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

Inversion of this model to a so-called MA representation describes the dependent variables as weighted sums of shocks. The sums of the respective weights are long-term multipliers, determining the long-term effect of each shock on each dependent variable. They can be arranged in the form of a matrix, in which the first row corresponds to the GDP increment and the second row to the interest rate. The columns correspond to shocks. If we put the long-term multiplier of the second shock to a GDP increment equal to zero, the matrix of long-term multipliers takes the form:

$$\begin{bmatrix} C_{11} & 0 \\ C_{12} & C_{22} \end{bmatrix}$$

The GDP increment will then be only a function of the first shock in the long run. The interest rate, on the other hand, will be a function of both shocks. Under such restrictions, the first shock will roughly correspond to the neutral interest rate and the second shock will correspond to the disequilibrium component of the interest rate. Out of these structural shocks, time series of the equilibrium interest rate and of the disequilibrium component of the interest rate are subsequently cumulated as weighted averages.

We have calculated the structural shocks according to the Astley-Garratt (1998) study and have cumulated them as the production gap and potential product growth rate in the paper Menashe-Yakhin (2004). In practice, we have carried out the calculation as follows:

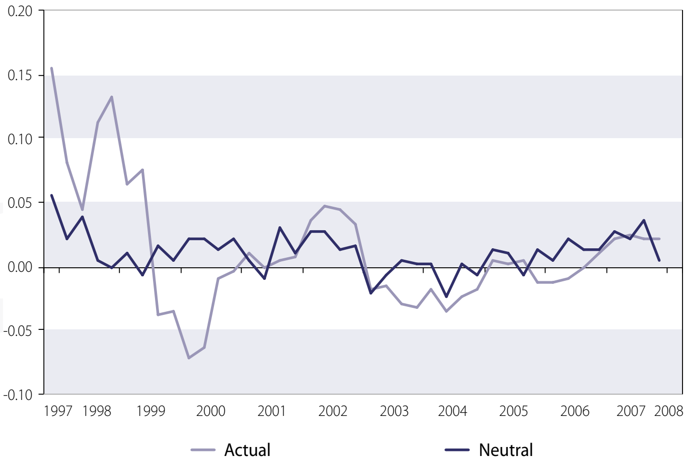
1. We have estimated the (reduced) AR form Ax= ε with the covariance matrix Var(εε')=Ω.
2. We have determined B(1) = Σ Bi from the MA form x = Bε (by summing up the weights).
3. We have calculated auxiliary matrices C(0) = $B(1)^{-1}$ Ω.
4. We have calculated long-term multipliers C(1)= Σ Ci by Cholesky decomposition of the matrix B(1) Ω B(1)'.
5. We have calculated the weights of the structural MA form Ci = Bi C(0), Bi is from the MA form for unit shocks.
6. We have determined the structural shocks e = $C(0)^{-1}$ ε, with the covariance matrix being Var(ee') = 1.
7. We have obtained the increments $\Delta r_t = \Sigma \{C_{12}\}$ i $\{e_2\}^{t-i}$ and $\Delta rgap_t = \Sigma \{C_{11}\}$ i $\{e_1\}^{t-i}$ (the curly brackets mean elements of the matrices Ci and of the matrix of structural shocks e).

8. By cumulating, we have calculated the equilibrium rate r and the disequilibrium component rgap and have added the mean value of actual values of the real interest rate to the time series of the equilibrium rate.

We have chosen a VAR process specification that ensures that the estimated model is stable and a LM autocorrelation test does not refute the null hypothesis for 12 periods. We have used the rate of inflation from the total consumer price in-

2 In this context, relations can be complicated by the changing age structure of the population. Households use to have low saving and higher consumption demand when they are young, later on saving increases even beyond the level of consumption and after reaching the post-productive age the households typically consume their saving. Crespo Cuaresma and Gnan (2007) suggest a possible connection of the decrease in the equilibrium interest rates in the euro area.

Chart 6 The actual and neutral real interest rate (structural VAR)



Source: Own calculations.

dex in the estimation. Due to weak convergence of the weights, the sum of the equilibrium interest rate and of the disequilibrium component did not equal exactly the actual values, so that the results can be only evaluated along general lines. Chart 5 shows the actual and equilibrium values of the real interbank interest rate.

Chart 6 shows that, along general lines, the result is similar to the result from the Kalman filter. This indicates a relatively strong connection between the equilibrium interest rate and the IS curve, which appears in both models, and a relatively weak connection between the equilibrium interest rate and inflation, because the Philips curve is missing in the structural VAR model. The critical remarks at the equilibrium interest rate level mentioned in the previous part also apply to this calculation.

3.6 Long-term equilibrium interest rate determined by the Kalman filter

We have seen in the previous parts that the IS curve and the Philips curve are not sufficient to determine a reliable value of the equilibrium interest rate, if there is excess of liquidity (saving) over demand for liquidity (investment) on the broader financial markets. In this connection, the Ramsey model provides a theoretically exact derivation of and reasons for the links between growth, saving and the interest rate as a function of the parameters of the production function and household preferences, including the population growth rate, but the strict requirements of the model are not fulfilled in reality. The production function, utility function, as well as other relations are mathematical abstractions of actual processes chosen based on their properties and implications. Even if we assume that in each period the production process can be described by a production function and the decision-making process by utility function maximization under a budget restriction, the economy is permanently exposed to a variety of shocks and changes, which cause the individual quantities to differ from their equilibrium values.

In constructing a multidimensional filter with unobserved variables, we had to take into account the following deviations:

- The net domestic product is not equal to is equilibrium value.
- The marginal product of capital is not equal to the equilibrium interest rate – as a result of permanent adaptation processes and as a result of optimization inaccuracy. Typical of transition economies is a capital endowment deficit and the associated high marginal product.
- Deviations of the rate of inflation from its average are reflected in the disequilibrium component of the interest rate.
- The economy does not exhibit a harmonic growth, the ratio of consumption to net production changes and influences the equilibrium interest rate.

The model for the calculation of the long-term equilibrium interest rate takes the form of a Kalman filter with five signals and eight states. Because the demand structure of the net product was changing in the period under review, we used the consumption growth rate and not the total net product to determine the equilibrium interest rate. We used the real interest rate with the ex post GDP deflator growth rate instead of expected inflation. The first signal is the real interest rate REALMMIR decomposed into the equilibrium component R and the disequilibrium component RGAP:

$$REALMMIR = R + RGAP$$

The second signal is the labor productivity growth rate – calculated from the net domestic product (GDP – depreciations) – decomposed into the equilibrium component DLOGY and the disequilibrium component YGAP. The net domestic product and employment were adjusted for short-term fluctuations by means of a HP filter.

$$DLOG((NDP/L),0,4) = DLOGY + YGAP$$

The third signal is the marginal product of capital MPC, calculated from the differences of adjusted net domestic product. In a period, in which thedefinition yielded nonsensical results due to fluctuations of quantities used, we have imputed the value of 0.06 based on the closest economically admissible values. This signal exceeds the equilibrium interest rate R by the liquidity premium PREM – approximated by the difference of the average interest rate on loans and the interbank interest rate-, a part of the disequilibrium component of the interest rate RGAP and the random component u_{MPC}.

$$MPC = \alpha_1 * RGAP + R + PREM + u_{MPC},\ u_{MPC} \sim N(0, \sigma_1^2)$$

The next signal is the deviation of the GDP deflator from its average PYGAP, which is reflected with a negative sign in the disequilibrium component of the interest rate. The equation also contains the signal noise u_{PYGAP}.

$$PYGAP = \Phi_1 * RGAP + u_{PYGAP},\ u_{PYGAP} \sim N(0, \sigma_2^2)$$

The last signal is the growth rate of the ratio of final consumption to net domestic product QC (both series have been seasonally adjusted and smoothed by a HP filter). We define an analogous state variable QCS.

$$QC = QCS + u_{QC},\ u_{QC} \sim N(0, \sigma_3^2)$$

The growth rate of the potential product DLOGY is defined as a sum process with a drift and with autocorrelated increments SHOCK. We have eliminated the multicolinearity of the variance estimate by introducing the ratio to another variance.

$$DLOGY = \beta_1 + SHOCK(-1)$$

$$SHOCK = \beta_2 * SHOCK(-1) + v_{SHOCK},\ v_{SHOCK} \sim N(0, 0.9 * \sigma_7^2)$$

The disequilibrium component of the growth rate of the net product YGAP is a sum process, in which the increment in DYGAP depends on the disequilibrium component of the interest rate RGAP:

$$DYGAP = \gamma_1 * RGAP1(-1) + v_{DYGAP},\ v_{DYGAP} \sim N(0, \sigma_4^2)$$

$$YGAP = DYGAP(-1) + YGAP(-1)$$

The disequilibrium component of the interest rate RGAP is a first order autoregressive process. The auxiliary identity defines the value from the previous period:

$$RGAP = \delta_1 * RGAP(-1) + v_{RGAP},\ v_{RGAP} \sim N(0, \sigma_5^2)$$

$$RGAP1 = RGAP(-1)$$

The auxiliary state for the growth rate of the ratio of consumption to the product QCS is defined as the sum process:

$$QCS = QCS(-1) + v_{QCS},\ v_{QCS} \sim N(0, \sigma_6^2)$$

The equilibrium interest rate is defined by means of parameters corresponding to the subjective discount rate ρ and substitution elasticity θ from the consumption growth rate. It is defined as the sum of the equilibrium growth rate DLOGY and the growth rate of the ratio of consumption to the product QCS. This enables us to allow for the connection between the interest rate and consumption.

$$R = \rho + \theta * (DLOGY(-1) + QCS(-1)) + v_R,\ v_R \sim N(0, \sigma_7^2)$$

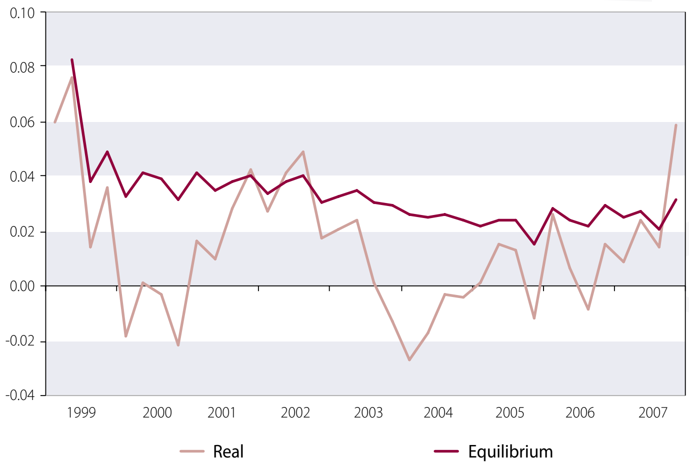
We had to calibrate some parameters in the estimate like we have done it with the neutral interest rate. We have used the paper Mankiw (1985) in the selection of the starting values for the subjective discount rate. The values at the beginning of the period under review are influenced by convergence of some parameters of the applied algorithm and have no sufficient explanatory power. The resulting long-term equilibrium interest rate has a lower variability in time than the actual one. It is determined not based on the disequilibrium component like in the previous case, but based on variables that are tied directly to the equilibrium value (marginal product of capital, consumption growth rate). The development of the long-term equilibrium interest rate is shown in Chart 7. This rate is more probable considering the microeconomic deliberations behind the equilibrium interest rate concept.

Table 2 Parameters of the model for the long-term interest rate in Slovakia

Parameter	Value	z-statistic
α_1	0.400	11206.150
σ_1	0.130	9319.620
ϕ_1	-1.000	-67005.190
σ_2	0.020	Calibrated
σ_3	0.001	Calibrated
β_1	0.055	Calibrated
β_2	0.300	2.359
σ_4	0.060	1.324
γ_1	-0.010	-1.894
δ_1	0.600	4.452
σ_5	0.022	3.026
σ_6	0.004	Calibrated
ρ	0.030	2.814
θ	0.200	1.576
σ_7	0.012	1.629
Log likelihood.	316.2335	Observations

Source: Own calculations.

Chart 7 Actual and long-term equilibrium interest rate

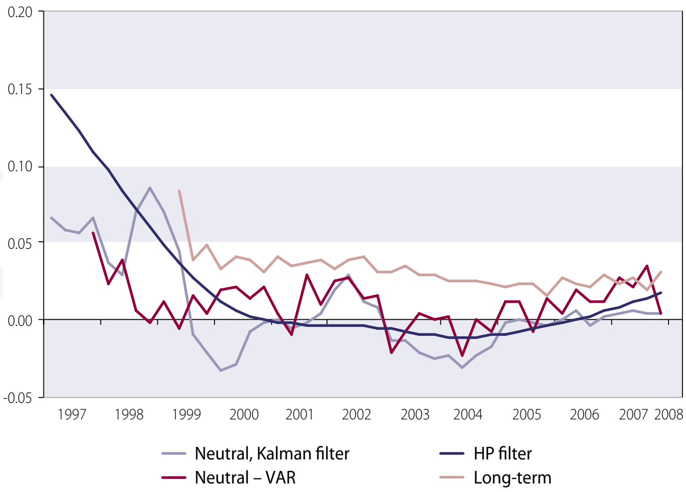


Source: Own calculations.

3.7 A comparison of the estimates of the equilibrium interest rate for Slovakia

Chart 8 compares the various interest rates in Slovakia using various methods – a one-dimensional

Chart 8 Comparison of estimates of the equilibrium interest rate for Slovakia



Source: Own calculations.

Table 3 Parameters of the model for the neutral interest rate in the euro area

Parameter	Value	z-statistic
α_0	-0.0012	Calibrated
α_3	0.0142	4.152
α_1	0.0027	1.556
α_2	0.2509	2.586
α_4	0.3845	3.691
α_5	0.0027	2.621
α_6	0.0015	2.157
σ_1	0.0020	370.797
β_1	0.0390	5.518
σ_2	0.0146	13.160
γ_0	0.0005	Calibrated
γ_1	1.1281	693.154
γ_2	-0.2210	-18.994
γ_3	-0.1061	-156.059
σ_3	0.0023	22.967
δ_0	0.0049	6.759
n_3	0.1290	646.508
σ_4	1.8E-6	Calibrated
σ_5	0.002	Calibrated
σ_6	0.0047	3.175
σ_7	0.0039	20.116
Log likelihood.	502.3798	Observations

Source: Own calculations.

filter, a multidimensional filter with unobserved variables for both the neutral and the long-term rate and the VAR for the neutral interest rate. Although the results cannot be compared without reservation, because we have used another rate of inflation for the calculation of the long-term interest rate, certain conclusions can be drawn.

In the case of the HP filter, a high inertia of the first and second differences of the resulting time series is obvious, so that given the high values at the beginning and low values in the middle of the period under review, the result takes a parabolic course for the real interest rate. This course can be also achieved with the (in our case probably incorrect) assumption that the cyclical components of the nominal interest rates and rates of inflation were in the same phase all the time. Because this result strongly depends on the method used, we consider it is better to define the equilibrium real interest rate as the difference between the filtered values of nominal rates and the rates of inflation. Moreover, this result is similar to the results for the neutral interest rate in most observations.

The chart clearly shows the difference between the assumptions of a neutral and a long-term equilibrium rate. The neutral interest rate is defined, in a fundamental way, by the macroeconomic context and cyclical movements, which imply a disequilibrium component oscillating around zero. The actual values of the real interest rates are relatively low – in addition to the above mentioned glut of saving on financial markets probably also due to the fact that inflation did not pass through to nominal interest rates as expected by theory. These relatively low actual values, along with the disequilibrium components oscillating around zero, imply relatively low values of the equilibrium rate. The long-term interest rate is closer to normative microfoundations, which determine it directly (not by means of the disequilibrium component), so that the rate is higher, which is also associated with the fact that the equilibrium values are higher than the actual values in most observations. This means that although the disequilibrium component has a non-zero mean value, this calculation provides results that, out of the presented values, can be interpreted best.

3.8 Neutral interest rate in the euro area

To compare the equilibrium interest rates in Slovakia and the euro area, we attempted to estimate a similar model with a Kalman filter for the euro area like for the determination of the neutral interest rate in Slovakia. In our estimate, we have used data only from 1996 onwards. Other studies use also older data, but in our opinion the individual euro area countries functioned as independent countries in earlier periods, so that including those observations in the estimate would hardly correspond to the model assumption that there is a uniform IS curve and Philips curve for the whole area. At the end of the 1990s, the euro area countries went through a process, in which the interest rates in the individual countries converged due to expected introduction of a common currency. Therefore we think that we can use those observations already, because here the economic reality was influenced by the introduction of the euro already.

We have based our construction of the model on the model for Slovakia described above, but we had to make several modifications to it. The first signal is the logarithm of the real GDP for the 15 member euro area, divided into the potential product LYPOTEU and the production gap LYGAPEU.

$$LOG(GDP\_EU) = LYPOTEU + LYGAPEU$$

The second signal is the real rate RELMMIREU, calculated from the money market rate and the consumer price index. These quantities apply to the whole euro area, taking into account its enlargement[3].

$$REALMMIREU = REU + RGAPEU$$

The third signal is total inflation from the consumer price index CPI_EU, determined by the Philips curve. On the right-hand side, it contains the values from previous periods, the production gap LYGAPEU and Brent oil price P_BRENTF (1 month forward).

$$\Delta LOG(CPI\_EU) = \alpha_0 + \alpha_1 * \Delta LYGAPEU2 + \alpha_2 * \Delta LOG(CPI\_EU(\text{-}2)) + (1 - \alpha_2 - \alpha_3 - \alpha_4) * LOG(CPI\_EU(\text{-}4)) + \alpha_3 * \Delta LOG(P\_BRENTF) + \alpha_4 * \Delta LOG(CPI\_EU(\text{-}3)) + \alpha_5 * T2 + \alpha_6 * T4 + u_{CPI},\ u_{CPI} \sim N(0, \sigma_1^2)$$

The last signal is the relative change in the average real euro to US dollar exchange rate as a function of the disequilibrium component of the interest rate RGAPEU2 and a binary dummy variable.

$$(\Delta LOG(EURUSD,0,4) + \Delta LOG((CPI\_EU/CPI\_US),0,4))/4 = RGAPEU2 + \beta_1 * (TIME>37) * (TIME<50) + u_{RER},\ u_{RER} \sim N(0, \sigma_2^2)$$

The production variable LYGAPEU is a state variable that depends on its previous values and the disequilibrium component of the interest rate RGAPEU. The other identities define auxiliary states:

$$LYGAPEU = \gamma_0 + \gamma_1 * LYGAPEU(\text{-}1) + \gamma_2 * LYGAPEU3(\text{-}1) + \gamma_3 * RGAPEU(\text{-}1) + v_{LYGAP},\ v_{LYGAP}$$

$$LYGAPEU1 = LYGAPEU(\text{-}1)$$

$$LYGAPEU2 = LYGAPEU1(\text{-}1)$$

$$LYGAPEU3 = LYGAPEU2(\text{-}1)$$

The potential product LYPOTEU is defined as a cumulative process with a drift and autocorrelated DPOT increments

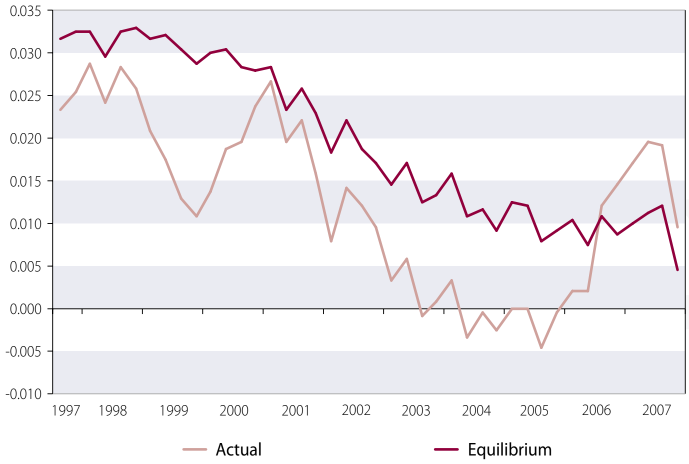
$$LYPOTEU = \delta_0 + LYPOTEU(\text{-}1) + \Delta LYPOT(\text{-}1)$$

$$\Delta LYPOT = \eta_1 * \Delta LYPOT(\text{-}1) + v_{DPOT},\ v_{DPOT} \sim N(0, \sigma_4^2)$$

The equilibrium interest rate is summed up from the increments.

$$REU = R3(\text{-}1) + RSHOCK(\text{-}1) + v_{REU},\ v_{REU} \sim N(0, \sigma_5^2)$$

Chart 9 Actual and neutral real interest rate in the euro area



Source: Own calculations.

$$RSHOCK = v_{RSHOCK},\ v_{RSHOCK} \sim N(0, \sigma_6^2)$$

The disequilibrium component RGAPEU is also a cumulative process:

$$RGAPEU = RGAPEU(\text{-}1) + v_{RGAPEU},\ v_{RGAPEU} \sim N(0, \sigma_7^2)$$

Finally there are auxiliary identities, which complete the model giving it the standard form:

$$RGAPEU1 = RGAPEU(\text{-}1)$$

$$RGAPEU2 = RGAPEU1(\text{-}1)$$

$$R1 = REU(\text{-}1)$$

$$R2 = R1(\text{-}1)$$

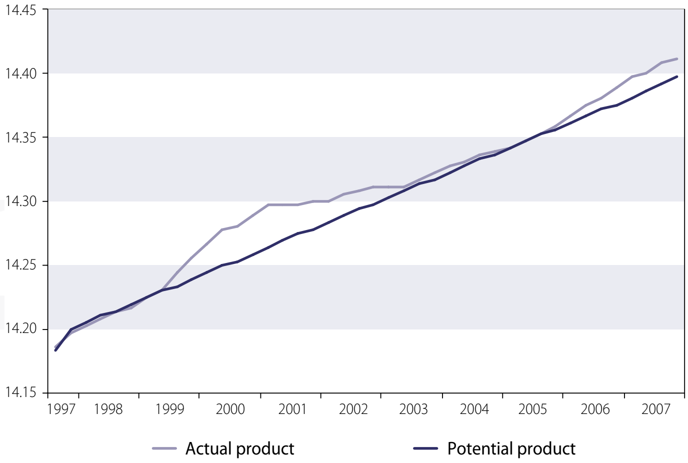
$$R3 = R2(\text{-}1)$$

When estimating the model parameters, we had to calibrate some parameters. This model is characterized by considerable numerical instability. This can be associated with the fact the euro area is not one state, but a grouping of states with different fiscal policies. Calculation problems are also related to the fact that the production gap is not centered around zero, but in most cases take positive values. Estimates of the equilibrium interest rate in the euro area differ. Crespo Cuaresma et al. (2004) suggest values around 1 percent, Giammarioli and Valla (2003) around three percent (but different methods are used). Our results are gradually decreasing from above three percent down to below one percent. The estimated production gap in the euro area is shown in Chart 9.

Chart 9 shows that although the equilibrium interest rate at the beginning of the period under review was around three percent – which can approximately correspond to microeconomic optimality conditions – it gradually also took values close to zero. This shows the global nature of the pressure to decrease interest rates, a probable global saving glut, which is also confirmed by

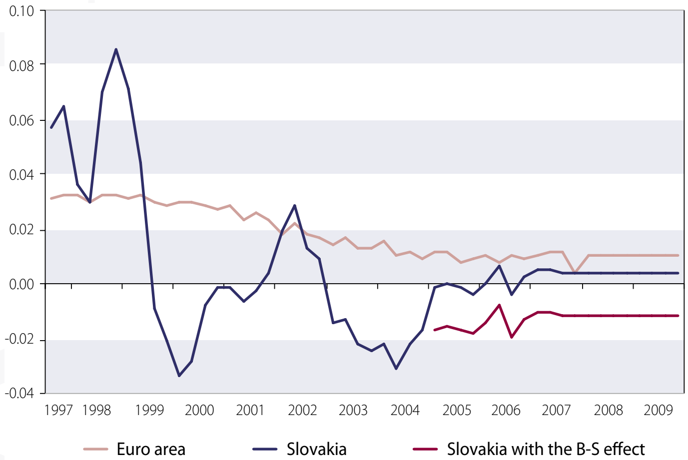
3 For the output, we had to use a fixed number of countries, because changes in the definition would collide with the calculation of the production gap.

Chart 10 Actual and potential product (logarithm) in the euro area



Source: Own calculations.

Chart 11 Neutral real interest rate in the euro area and in Slovakia



Source: Own calculations.

4 The gist of the Balassa-Samuelson effect is that the growth of productivity in the tradable sector under a common labor market within a national economy leads to divergence of unit wage costs in the tradable and non-tradable sector. That, in turn, leads to a systematic appreciation of the real exchange rate. For details on this issue see Šuster et al. (2006).

Chart 5. We therefore think that although the levels of the neutral interest rates obtained by means of the IS curve and Philips curve cannot be evaluated in absolute terms, due to the analogous nature of the distortion they can be compared for various countries.

3.9 Assessment of the influence of the Balassa-Samuelson effect on neutral interest rates after euro area accession

After Slovakia's euro area accession, substantial factors defining the financial market (the exchange rate and its consequences – the risk premium, currency exchange fees) in Slovakia will be eliminated, and the Slovak financial market will be integrated in the common financial market of the euro area. It can be expected that the interest rates in Slovakia will approach the interest rates in the euro area. It has to be remembered, however, that a process of convergence with the original European Union countries is going on in Slovakia and that within the process the so-called Balassa-Samuelson effect[4] arises, causing pressure on the real exchange rate. Due to the impossibility of appreciation of the nominal exchange rate following the introduction of the euro, these pressures will cause inflation in Slovakia to increase. Šuster et al. (2006) mention several estimates of an additional increase in inflation as a result of the Balassa-Samuelson effect. For the purposes of the assessment of the influence of the monetary union accession, let us assume that the economies of the euro area and of Slovakia operate at their potential levels and at stable inflation. Let us further assume that following the monetary union accession, an interest rate equal to the neutral rate in the euro area increased by the (target) inflation in the euro zone will be introduced and that inflation in Slovakia after the introduction of the euro will be 1.5 percent higher under the influence of the Balassa-Samuelson effect, irrespective of the production gap. The differential in equilibrium inflation will then pass through to an interest rate differential. Chart 11 shows the assumed neutral interest rates in the euro area and in Slovakia. From 2008 onwards, we extrapolated them using expert estimates based on the last values counted.

Chart 11 shows that the common neutral interest rate will decrease below zero after allowing for the Balassa-Samuelson effect. That can cause a certain conflict for monetary policy, because the implied interest rate will be too expansive. On the other hand, monetary policy and economic policy in general is a broad set of instruments, so that we expect that it will be possible to compensate the expansive influence of interest rates by other instruments. It also has to be noted that the expected advantages of the introduction of the euro will be higher than the expected cost associated with a loss of independent monetary policy.

4 SUMMARY AND CONCLUSION

This paper treats equilibrium real interest rates. The most important concepts are the neutral real interest rate concept and the long-term real interest rate concept. The neutral interest rate is derived based on macroeconomic deliberations on the economic cycle and inflation. It is the real interest rate that corresponds to the state where the economy operates at its potential level and inflation equals the target value. The determination of the neutral interest rate issues from a particular national economic environment, which can change over the period under review. The long-term interest rate is a theoretical concept corresponding to the ideal general equilibrium state with a stable structure of the economy. On the one hand, it is determined by technology, reflected in the value of the marginal product of capital, and on the other hand by household preferences as set by the Ramsey-Keynes Rule.

We present the calculations of the neutral real interest rate both for Slovakia and the euro area, as well an approximation of the long-term equilibrium interest rate as allowed by changes within the catching-up process. The results of the individual estimates show that the values of equilibrium interest rates strongly depend on the assumptions applied. In the period under review, the neutral real interest rates showed a tendency to fall and in the years 2006 and 2007 they took low values, sometimes even close to zero. These values are, with the utmost probability, connected with particular macroeconomic factors and are not equal to the long-term equilibrium interest rate. Under the existence technological progress, the long-term equilibrium interest rate should be higher than the subjective household discount rate. In this study, we formulate a model, by means which we attempt to derive the long-term interest rate from its microfoundations. The results also show a decreasing trend, but they are higher than the neutral the neutral interest rate. If our models are correct, the differences between the neutral interest rate and the long-term interest rate can correspond to the medium-term influence of national and world saving.

Following Slovakia's accession to the monetary union, the will be a discrepancy between the neutral interest rate in the euro area and in Slovakia, the main reason being the Balassa-Samuelson effect. A comparison of the relevant quantities shows that the expansive influence of interest rates probably will have to be offset by other instruments of economic policy. It is not impossible, however, that the neutral interest rate will be converging slowly to the higher level of the long-term interest rate.

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