Dynamic Portfolio Optimization During Financial Crisis Using Daily Data and High-frequency Data

PhDr. František Čech[1]

Institute of Economic Studies, Faculty of Social Sciences, Charles University in Prague

Our work focuses on variance-covariance matrix modeling and forecasting. Majority of existing research evaluates covariance forecasts by statistical criteria. Our main contribution is economic comparison of parametric and non-parametric approaches of covariance matrix modeling. Parametric approach relies on RiskMetrics and Dynamic Conditional Correlation GARCH models that are applied on daily data. In the second approach, estimates of variance-covariance matrix are directly obtained from the high-frequency data by non-parametric techniques Realized Covariation and Multivariate Realized Kernels. These estimates are further modeled by Heterogeneous and Wishart Autoregression. Moreover, our contribution arises from the use of dataset that covers period of financial crisis. Portfolio of assets that is dynamically optimized consists of two highly liquid assets – Light Crude NYMEX and Gold COMEX, and of European asset represented by DAX index. Forecast evaluation results indicate better economic performance of models estimated on daily data. However, we found out that data synchronization procedure is the main driver of the results.

Motivation

Volatility modeling is one of the key issues in the area of financial econometrics. The risk of individual financial instruments is crucial for asset pricing, portfolio selection and risk management. Besides volatility of individual assets knowledge of covariance and correlation structure is of great importance. Accurate forecasts of variance-covariance matrices are particularly important in asset allocation and portfolio management.

Nature of the financial data with dependencies in higher moments of the daily return series motivated the work of Nobel laureate Robert Engle and later Tim Bollerslev. They have developed a new family of parametric univariate conditionally heteroscedastic models represented by widely used Generalized Autoregressive Conditional Heteroscedasticity (GARCH). In the late eighties and nineties numerous multivariate extensions of the GARCH were created. Among all of them let us mention Constant Conditional Correlation GARCH of Bollerslev (1990) further generalized by Engle (2002) into Dynamic Conditional Correlation GARCH. Multivariate GARCH (MGARCH) models are popular in the literature although they suffer from curse of dimensionality problem.

Increased availability of high-frequency data in the last decade resulted in development of the new non-parametric approach of treating volatility, which is an interesting alternative to traditional MGARCH models. Model-free estimator called Realized Volatility that makes volatility observable is proposed in Andersen et al. (2001). Most influential works providing rigorous theoretical background of the concept of realized volatility is Andersen et al. (2003) and Barndorff-Nielsen & Shephard (2004). In Barndorff-Nielsen & Shephard (2004) theory of realized volatility is completed with Realized Covariation. Estimates of variance-covariance matrix that are obtained by realized covariation method do not have to be necessarily positive semi-definite due to market microstructure noise. Therefore Barndorff-Nielsen et al. (2011) introduced Multivariate Realized Kernels estimator guaranteeing the positive semi-definiteness of the variance-covariance matrix.

Once the covariance matrix is estimated from the high-frequency data it needs to be further modeled. There is still ongoing research dedicated to the entire covariance matrices modeling. From the already established methods let us mention Wishart Autoregression of Gourieroux et al. (2009) with numerous extensions presented in Bonato et al. (2009) and Bonato et al. (2012). The use of Cholesky factors further estimated by Vector Autoregressive Fractionally Integrated Moving Average, Heterogeneous Autoregresion or Wishart Autoregression combined with Heterogeneous Autoregression can be found in Chiriac & Voev (2011).

Selection of the assets included in the portfolio that is dynamically optimized is crucial for empirical work. Majority of researchers (Andersen et al. (2003), Bonato et al. (2009), Chiriac & Voev (2011) among others) concentrate on instruments traded mostly on the United States market (S&P 500 index or U.S. treasury bills) and evaluate forecasting performance generally by statistical criteria. However, our main contribution is that we include the European asset in portfolio in order to meet the perspective of European investor whose portfolio includes not only world's most traded assets

1 *Článok je zhrnutím diplomovej práce s rovnakým názvom, za ktorú získal autor druhú cenu v súťaži o cenu guvernéra NBS pre študentov vysokých škôl za najlepšiu prácu v oblasti ekonómie.*

but also the local one. Furthermore, we evaluate covariance forecasts mostly by economic criteria. Economic performance of volatility forecasts is of great importance especially for financial practitioners because it provides direct financial evaluation of their decisions.

Data and Methodology

Portfolio of assets that is optimized consists of highly liquid commodity (Light Crude NYMEX), "safe haven" investment (Gold COMEX) and German stock index DAX. For the analysis we use 5-minutes closing prices from period July 8, 2003 to November 29, 2011. All data were obtained from the Tick Data.

In our work covariance matrix forecasts used for dynamic portfolio optimization are obtained from two "return as input" models represented by Exponentially Weighted Moving Average (RiskMetrics standards) and Dynamic Conditional Correlation (DCC) GARCH and four "covariance as input" models that include Heterogeneous Autoregression (HAR), Cholesky-Heterogeneous Autoregression (Cholesky-HAR), Wishart Autoregression (WAR) and diagonal Wishart Autoregression. Covariances used in second models group are calculated by Realized Volatility and Multivariate Realized Kernels approach. Moreover, we evaluate accuracy of covariance forecasts by one statistical, Root Mean Square Forecasting Error (RMSFE) and three economic criteria – Global Minimum Variance Portfolio (GMVP), Mean-Variance optimization of Markowitz and Value-at-Risk (VaR).

To study effects of financial crisis, the models are estimated on two sub-samples, representing period before crisis and during crisis, and full sample, covering period July 8, 2003 to November 29, 2011. The sub-samples are obtained by dividing whole dataset into two equal parts. Each period is further divided into in-sample and out-of-sample part. In-sample period lasts 713 days for all subsamples. On the other hand out-of-sample period lasts 252 days which represents one year for before crisis and during crisis sub-sample. In case of full dataset, duration of out-of-sample period is 1 217 days. For estimation purposes the rolling window estimator of length 713 days is used.

Discussion of results

In our work we try to answer the following questions: Which model provide us with appropriate forecasts? Do we gain some advantages using more sophisticated models compared to simple ones? What kind of data are to be used in order to minimize the risk of the portfolio?

Overall performance

Forecasting performance of the RiskMetrics is the most stable one. From the Value-at-Risk perspective it is the only model with correctly specified risk level within all examined periods. Results of remaining evaluation methods show similar patterns for all periods, although they are not the best ones. The division into sub samples does not affect performance of RiskMetrics much, making it applicable not only during the stable but also the volatile times.

Second representative of the return based models, DCC-GARCH, shows similar patterns for all evaluating methods except Value-at-Risk. From the RMSFE, GMVP and Mean-variance optimization point of view, DCC-GARCH substantially outperforms RiskMetrics and both WAR models during all the periods. Value-at-Risk performance of DCC-GARCH can be characterized as time dependant. In the short sample, financial crisis does not affect results much, while in the long one, crisis might be the reason of worse performance.

Description of results of covariance based models starts with Heterogeneous Autoregression. Performance of HAR is very similar for during crisis and full sample period. According to Value-at-Risk, model shows the best performance in during crisis period. The risk is specified correctly for both 95% and 99% VaRs. In case of before crisis and full sample period risk is underestimated. According to remaining forecasts evaluation methods, HAR is the model that outperformed almost all the other models.

Cholesky-HAR is the absolute winner if we take into account RMSFE, GMVP and Mean-variance optimization criteria. It also shows the best performance within all time periods. From the Value-at-Risk point of view, similar to HAR, the risk is correctly specified for during crisis period and underestimated in case of before crisis and full sample period.

Wishart Autoregressive model and diagonal Wishart Autoregressive model are the models with the worst forecasting performance. Although for diagonal WAR the lowest variance among all the models is achieved, the results are not conclusive – estimated degrees of freedom fall below minimum level where no density function is specified for the covariance distribution. Generally, WAR models show a bad performance independent on the time-period.

The last part of the section is dedicated to Realized Covariation and Multivariate Realized Kernels comparison. Differences in the performance of both methods are minor. According to results of the RMSFE, GMVP and Value-at-Risk comparisons both methods show similar performance. From the Mean-variance optimization point of view Multivariate Realized Kernels slightly outperform Realized Covariation. If both methods are compared across different time periods, results indicate that the performance of covariance estimates is not affected by financial crisis.

Results of our analysis partially correspond to results of Voev (2009) and Chiriac & Voev (2011) where the Cholesky-HAR shows good forecasting performance. On the other hand, DCC-GARCH was outperformed by diagonal and full WAR which is not in line with our results. In the work of Bonato et al. (2009) where a set of different WAR specifications and the DCC-GARCH are estimated, diagonal WAR outperforms the DCC-GARCH while

Bibliography

Andersen, T.G., Bollerslev, T. & Diebold, F.X. (2005): "Roughing it up: Including Jump Components in the Measurement, Modeling and Forecasting of Return Volatility." *NBER Working Paper Series*, (11775).

Andersen, T.G., Bollerslev, T., Diebold, F.X. & Labys, P. (2001): "The Distribution of Realized Exchange Rate Volatility." *Journal of the American Statistical Association*.

Andersen, T.G., Bollerslev, T., Diebold, F.X. & Labys, P. (2003) "Modeling and Forecasting Realized Volatility." *Econometrica*, Vol. 71(2), pp. 579-625.

Andersen, T.G., Bollerslev, T., Christoffersen, P. & Diebold, F.X. (2007): "Practical Volatility and Correlation Modeling for Financial Market Risk Management." In Carey, M. & Stulz, R.M. *The Risks of Financial Institutions*. University of Chicago Press. pp. 513-548.

Back, K.(1991): "Asset pricing for general processes." *Journal of Mathematical Economics*, pp. 371-395.

Barndorff-Nielsen, O.E., Hansen, P.R., Lunde, A. & Shephard, N.(2011): "Multivariate realised kernels: Consistent positive semi-definite estimators of the covariation of equity prices with noise and non--synchronous trading." *Journal of Econometrics*, 162(2), pp. 149-169.

Barndorff-Nielsen, O.E. & Shephard, N. (2004): "Econometric Analysis of Relaized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics." *Econometrica*, 72(3), pp. 885-925.

Bauwens, L., Laurent, S. & Romboust, F.(2006): "Multivariate GARCH Models: A Survey." *Journal of Applied Econometrics*, 21, pp. 79-109.

Bollerslev, T. (1986): "Generalized Autoregressive Conditional Heteroscedasticity." *Journal of Econometrics*, 31, pp. 307-327.

Bollerslev, T. (1990): "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized Arch Model." *The Review of Economics and Statistics*, 72(3), pp. 498-505.

Bonato, M. (2009): "Estimating the degrees of freedom of the Realized Volatility Wishart Autoregressive model." *Working paper*.

Bonato, M., Caporin, M. & Ranaldo, A. (2009): "Forecasting realized (co)-variances with a block structure Wishart autoregressive model." *Working paper*.

Bonato, M., Caporin, M. & Ranaldo, A. (2012): "Risk spillovers in international equity portfolios." *Swiss National Bank Working Papers, 2012*.

Brown, A. (2008): "Private Profits and Socialized Risk – Countepoint: Capital Inadequacy." *Global Association of Risk Professional*, (42), pp. 19-26.

Corsi, F. (2009): "A Simple Approximate Long-Memory Model of Realized Volatility." *Journal of Financial Econometrics*, 7(2), p. 174–196.

score of full WAR is the worse. Possible sources of differences in the results are the estimation time periods and the assets chosen for the purpose of analysis. In the above mentioned works, the period up to 2008 is considered for analysis while in our work financial crisis 2008/2009 is analyzed. Assets used in Voev (2009) and Chiriac & Voev (2011) include six S&P 500 constituents. Two currencies and two bonds are used in Bonato et al. (2009). Within both asset groups similar characteristics (mean, standard deviation ...) are observed for all assets while in our work data are much more volatile.

Simple or sophisticated model?

In an ideal world, the more sophisticated model we use, the better performance of the forecasts we get. However, situation in real world is more complicated and the previous statement might not be necessarily true. Easy interpretation and implementation with low time and computing demands speak in favour of simple models. On the other hand, more sophisticated models based on advanced economic and mathematical theory perform well during simulation studies. However, software implementation, difficult economic interpretation of the estimated parameters, high time and technology requirements are their major disadvantages.

Simple models presented in our work are RiskMetrics, HAR and Cholesky-HAR. Except Cholesky-HAR, where the economic interpretation of the coefficient is ruled out by Cholesky decomposition, all above mentioned advantages can be found in the group. The major advantage is duration of the estimation and forecasting procedure. All results are obtained within a minute.

DCC-GARCH and both WAR specifications belong to sophisticated models group. The main disadvantage in case of DCC-GARCH and full WAR is their time-consumption. The rolling window estimation for period of 713 days estimated for 252 consecutive days can take more than half an hour. Diagonal WAR, restricted and simplified version of full WAR, reduce time necessary for estimation to the level of simple models. Another disadvantage of these models is their software implementation. To our best knowledge there is no software with directly implemented WAR models.

Besides covariance forecasting models, Realized Covariation and Multivariate Realized Kernels were used in the thesis. Realized Covariation can be characterized as easy to implement technique with straightforward interpretation of the estimation procedure, although theory behind it requires deep mathematic knowledge. In contrast, implementation, interpretation of estimation procedure as well as theory of Multivariate Realized Kernels is rather complicated.

Final choice of preferred methods for obtaining covariance forecasts is complicated. It always depends on needs, requirements and limitations of individual investors.

Daily or High-frequency data?

The choice between daily and high-frequency data might be extremely difficult. The main advantage of daily data is that they are freely available and the major drawback is that the information about prices is limited and not suitable for intraday trading. On the other hand, high-frequency data provide us with more information and also the intraday trading is not a problem. Using high-frequency data for covariance forecasting is problematic when individual portfolio assets are traded during not fully overlapping hours. By synchronization of the dataset considerable amount of information might be lost resulting in poor performance of forecasts compared to daily data.

Concluding remarks

Our analysis shows that the performance of models highly depends on datasets and also on chosen assets. Here we present comments on the portfolio selection.

Assets included in the portfolio have to be chosen according to certain criteria. If the daily data are used for optimization, the most important thing we have to care for is similarity of the assets. The more similar assets are used, chance to obtain better results increases. By similarity of assets, the statistical properties like mean and standard deviation are meant. On the other hand, if assets from different risk levels are used (variances of the assets are significantly different), asset weights of global minimum variance portfolio are highest (more than 50 %) for the least risky one. It might happen that the entire portfolio consists of only one asset in an extreme case.

By using high-frequency data, besides similarity of assets, we have to add one more constraint. In order not to throw away significant amount of data by synchronization procedure, trading hours of all assets have to be (almost) the same.

Delbaen, F. & Schachermayer, W. (1994): "A general version of the fundamental theorem." In *Mathematische Annalen*. Springer-Verlag. pp. 463-520.

Engle, R.F. (1982): "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica*, 50(4), pp. 987-1007.

Engle, R.F., (2002): "Dynamic Conditional Correlation – A Simple Class of Multivariate GARCH Models." *Forthcoming Journal of Business and Economic Statistics 2002*.

Engle, R.F. & Kroner, K.F. (1995): "Multivariate Simultaneous Generalized Arch." *Econometric Theory*, 11(1), pp. 122-150.

Engle, R.F. & Sheppard, K. (2001): "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *NBER Working Paper Series*, p. Working Paper 8554.

Gourieroux, C., Jasiak, J. & Sufana, R. (2009): "The Wishart Autoregressive process of multivariate stochastic volatility." *Journal of Econometrics*, 150, pp. 167-181.

Hautsch, N. (2011): *"Econometrics of Financial High-Frequency Data."* Springer.

Chiriac, R. & Voev, V. (2011): "Modelling and Forecasting Multivariate Reralized Volatility." *Journal of Applied Econometrics*, pp. 922-947.

J.P.Morgan & Reuters (1996): *"RiskMetricsTM – Technical Document."* 4th ed.

Jorion, P. (2007): *"Value at Risk: The New Benchmark for Managing Financial Risk."* 3rd ed. McGraw-Hill.

Kempf, A. & Memmel, C. (2006): "Estimating the Global Minimum Variance Portfolio." *Schmalenbach Business Review*, 58, pp. 332-348.

Kupiec, P.H. (1995): "Techniques for Verifying the Accuracy of Risk Measurement Models." *The Journal of Derivatives*, pp. 73-84.

Markowitz, H. (1952): "Portfolio Selection." *The Journal of Finance*, 7(1), pp. 77-91.

Merton, R.C. (1972): "An Analytic Derivation of the Efficient Portfolio Frontier." *The Journal of Financial and Quantitative Analysis*, 7(4), pp. 1851-1872.

Müller, U.A. et al. (1993): "Fractals and Intrinsic Time – A Challenge to Econometricians." In *International Conference of the Applied Econometrics Association (AEA), Real Time Econometrics — Submonthly Time Series*. Ascona (Switzerland), 1993.

Protter, P.E. (2004): *"Stochastic Integration and Differential Equations."* Second Edition ed. Springer.

Voev, V. (2007): "Dynamic Modelling of Large Dimensional Covariance Matrices." *CoFE Discussion Paper 07-01*.

Voev, V. (2009): "On the Economic Evaluation of Volatility Forecasts." *CREATES Research Paper 2009-56*.

Watkins, D.S.(2002): *"Fundamentals of Matrix Computations."* 2nd ed. New York: John Wiley & Sons, Inc..