Modeling the development of the Slovak economy using a basic DSGE model[1]

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In recent years, DSGE modeling has been a mainstream in the economic modeling literature. One of the main advantages of these models is the fact that they are based on microeconomic foundations and thus are resistant to the Lucas critique. Equations of the model have been obtained by optimizing the behaviour of the participating agents – households, firms, the central bank and the foreign sector. This article presents a basic New Keynesian DSGE model for a small open economy, which is adapted to the Slovak data using the calibration and estimation method. Finally, the article presents the reactions of selected variables to exogenous shocks.

Introduction

The aim of this article is to present a basic dynamic stochastic general equilibrium model (DSGE) for a small open economy and to calibrate and estimate its parameters for Slovakia.

The gist of DSGE modeling is the optimization of the behavior of an economy's individual agents; this is the basis for the derivation of the individual equations of the model. The first article that started the era of DSGE models was published by Kydland and Prescott in 1982. It was an RBC (***real business cycle***) model. The model assumes that fluctuations of real quantities are caused by real shocks only – by technological progress or government spending shocks. Nominal quantities have no impact on real economy. New Keynesian DSGE models which unlike the said RBC models include nominal rigidities (sticky prices, sticky wages) are used more frequently today. The result of using such rigidities is that nominal shocks also cause fluctuations of real quantities. These models are widely used in central banks in practice, whether in simulations of the development of economic quantities or in their forecasts (e.g. Great Britain, Finland, and the Czech Republic).[2]

In the following part, we are going to describe the basic features of the presented model; subsequently we will calibrate it and will estimate the selected parameters. In the final part, we will use the calibrated model and will present the impacts of a nominal interest rate shock and a real exchange rate shock on selected quantities.

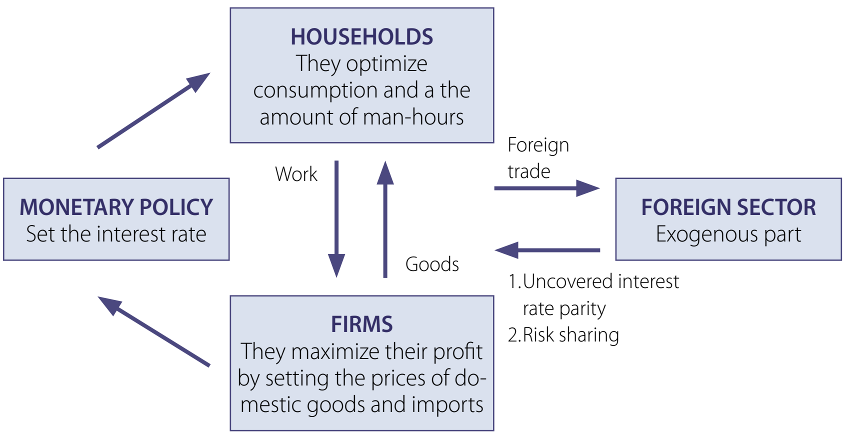
The Model

We start out from a standard New Keynesian model for a small open economy, which has been used in several earlier papers (see Monacelli, 2005; Liu, 2006; Vašíček and Musil, 2006). The last one of these papers provides a very detailed derivation of the individual equations of the model.

The model assumes the existence of two economies. On the one hand, there is the domestic economy, designated as a small and open economy, and on the other hand, there is a foreign economy, considered considerably bigger compared to the domestic one and relatively closed. These features influence their mutual relations and it holds that the domestic economy does not influence the development in the foreign economy. However, the said simplification is justified, because it enables to monitor the relationships between the Slovak economy and for example the euro area. In our case, we are modeling relations for the development in the domestic economy. The rest of the world, i.e. the foreign economy, enters the model exogenously.

As mentioned earlier, unlike with RBC models, nominal rigidities also enter New Keynesian models. Our model includes one type of real rigidity, specifically the external habit formation in consumption and two types of nominal rigidities, inelasticity of goods produced in the domestic economy and stickiness of the prices of imported goods.

Fig. 1 Diagram of the DSGE model used



[1] *Dynamic stochastic general equilibrium models are designated by the abbreviation DSGE.* *We thank František Brázdik from the Czech National Bank, as well as Ľudovít Ódor and Martin Šuster from Národná banka Slovenska for comments.*

[2] *A more detailed overview of using DSGE models in central banks can be found in the article Senaj, 2007 [10].*

The simplified diagram (Figure 1) makes orientation between the individual agents and their mutual relationships easier.

Finally, we will present a log-linearised version of the model, in which no concrete quantities in the levels appear, but their deviations from steady state values, which we will designate by lowercase letters. That means that in the final version of the model we will work with variables in the following form:

$$c_t = \ln(C_t) - \ln(C_{ss}),$$

Where c_t stands for deviation of consumption C_t from its steady state value C_{SS}. Quantities transformed like that are used in most of the models.

Households

The economy is formed by representative households, which maximize their discounted expected utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \{ \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \},$$

where C_t stands for household consumption, N_t are man-hours. The element H_t describes the formation of consumption habits, and the identity $H_t = h * C_{t-1}$ holds, h being the habit formation parameter.

The meaning of the other parameters is as follows: β is the discount factor, σ is the inverse elasticity of intertemporal substitution and φ the inverse elasticity of labor supply.

When maximizing their utility, the households take into account their own budget constraint, which says that consumption spending (P_tC_t) and bond purchases (D_{t+1}/R_t) should not exceed the sum a household obtains for its work (W_tN_t) plus the payment of the bonds held (D_t). R_t designates the gross nominal interest rate[3]. The budget restriction of a representative household can be written mathematically as follows:

$$P_tC_t + \left(\frac{D_{t+1}}{R_t}\right) \leq D_t + W_tN_t.$$

The results of optimization of the utility function are the following two first order conditions, which are called the Euler equations. The second one of them is known as the IS curve:

$$(C_t - hC_{t-1})^{-\sigma} \frac{W_t}{P_t} = N_t^{\varphi}$$

$$1 = \beta R_t E_t + \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+1}}.$$

Firms

We assume that there are two types of firms in the modeled economy – domestic producers and importers.

The domestic producers use the work they obtain from the households to produce goods. Hence their production function takes the following form:

$$Y_t = A_t N_t,$$

where Y_t is the output of domestic economy and A_t stands for technological progress. It is an unobservable quantity and, in the model, it appears as an exogenous variable, whose development is described by a stochastic random walk equation with a shift:

$$\ln(A_t) = \ln(A_{ss}) + \ln(A_{t-1}) + \varepsilon_t^A,$$

where ε_t^A is white noise.

The producers try to minimize the production costs. The result of their optimization is the real marginal cost relation:

$$MC_t = \frac{W_t}{P_{h,t} A_t},$$

where $P_{h,t}$ is the price index of domestic goods. After further adjustment, this relation becomes part of the resulting model.

The prices of goods are set by the producers, who maximize their profit this way. Price rigidity is introduced to the model by a mechanism described by Calvo (1983). The mechanism consists in regular price changes by the firms on the basis of a Poisson process. The probability that a firm changes its prices is constant in each period and equals $(1 - \theta_h)$. Since these price adjustments are random, the interval between the changes is a random quantity for each firm. If the firm does not change the prices, it keeps the price from the previous period. By using the Calvo mechanism, we get the following New Keynesian Phillips curve for the development of the domestic inflation ($\pi_{h,t}$):

$$\pi_{h,t} = \beta(1-\theta_h)E_t\pi_{h,t+1} + \theta_h\pi_{h,t-1} + \frac{(1-\theta_h)(1-\beta\theta_h)}{\theta_h} mc_t.$$

The second type of firms are the importers of foreign goods, which are subsequently sold by them in the domestic economy. We assume that each firm specializes in the import of a certain good and thus has the possibility to set an optimum sales price, which maximizes the profit of the firm and which is, in general, different from the import price. This assumption enables to describe the fact observed in the data that, in the short-run, changes in prices, for which firms buy the imported goods, do not have to pass through in full to the prices, at which they are sold by those firms (incomplete pass through). Hence, the Law of One Price (LOOP) does not have to hold. Let us define the LOOP gap Ψ_t as follows:

$$\Psi_t = \frac{P_t^*}{\varepsilon_t P_{f,t}},$$

[3] The gross interest rate equals the nominal interest rate plus 1.

where ε_t is the exchange rate of the Slovak koruna and the foreign currency, defined in the form: how many units of the foreign currency we need to buy one Slovak koruna (so that koruna appreciation means an increase in ε_t). $P_{f,t}$ is the price index of imported goods and P_t^* is the price index of goods in the foreign economy. If the rule of one price holds, then $\Psi_t = 1$.

For imported inflation, the New Keynesian Phillips curve applies, where θ_f designates the proportion of importing firms, which keep the prices from the last quarter in any quarter[4]:

$$\pi_{f,t} = \beta(1-\theta_f) E_t \pi_{f,t+1} + \theta_f \pi_{f,t-1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f}\psi_t.$$

Besides the nominal exchange rate, we also use the definition of the real exchange rate:

$$Q_t = \frac{\varepsilon_t P_t}{P_t^*},$$

The relation between imported and domestic inflation – called the terms of trade – expresses the competitiveness of domestic producers.

$$S_t = \frac{P_{f,t}}{P_{h,t}},$$

Because the households consume goods produced both in the home country and abroad, it then holds for the total inflation in the log-linearised form:

$$\pi_t = (1-a)\pi_{h,t} + a\pi_{f,t},$$

where the parameter a is the ratio of imported goods to total consumption and expresses the extent of openness of the country.

Monetary policy

The consequence of the presence of rigidities in price formation is that monetary policy can influence real quantities. Monetary policy is an instrument, by means of which the central bank sets interest rates in the domestic economy. Various modifications of the Taylor rule are used in the literaturÐ2 We have chosen a Taylor rule that reacts to deviations of domestic inflation and production from their steady state values. By setting the weights Φ_π and Φ_y one can set the importance attributed by the central bank to the quantities under review At the same time, the applied Taylor rule contains the interest rate from the previous quarter with the parameter ρ, which ensures smoothing of the domestic interest rate; it holds that the higher the parameter ρ, the stronger the smoothing effect.

$$r_t = \rho r_{t-1} + (1-\rho)(\Phi_\pi \pi_{h,t} + \Phi_y y_t)$$

Foreign sector

As already mentioned, the foreign sector is described by exogenous variables in this model. Deviations of the output (y_t^*) and real interest rates (r_t^*) from their steady state values are described by first order autoregression processes. Relations between the domestic and foreign economy are part of two equations. The first one is the uncovered interest parity equation. It expresses the fact that the return on domestic bonds equals the expected return on foreign bonds:

$$R_t = R_t^* \frac{\varepsilon_t}{E_t(\varepsilon_{t+1})}.$$

Assuming a complete securities market, equality between the Euler equation in the domestic and foreign economy[5] applies. The result is the following risk sharing equation:

$$(C_t - hC_{t-1}) = \zeta (C_t^* - hC_{t-1}^*)\, Q^{-1/\sigma}$$

Closing the model

We will close the model by assuming that all goods produced in the domestic economy will be consumed in the given period, whether in the home country or abroad (market clearing). This condition can be written as follows:

$$Y_t = (1-a)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t + a\left(\frac{\varepsilon_t P_{h,t}}{P_t}\right)^{-\eta} C_t^*.$$

The parameter η expresses the elasticity of substitution between imported goods and goods produced in the home country.

Stationarisation and loglinearisation of the model

The said articles using the same model for a small open economy (Liu, 2006; Vašíček and Musil, 2006) assume that all variables are stationary. This assumption, however, contradicts empirical data. Some variables are nonstationary in our model. This nonstationarity is caused by the variable A_t, which is nonstationary by construction. Therefore we have stationarised the presented model similarly to Del Negro and Schorfheide (2006), Andrle et al. (2007), Vašíček and Musil (2005) by transforming the nonstationary variables as follows:

$$\hat{Y}_t = \frac{Y_t}{A_t}, \quad \hat{C}_t = \frac{C_t}{A_t}, \quad \hat{MC}_t = \frac{MC_t}{A_t}.$$

We have carried out a similar transformation with the price indices, which are also nonstationary. By means of this transformation the model became stationary and consequently a steady state of the model exists.[6] We then log-linearise all equations around this steady state and thereby obtain a system of linear equations, in which percentage deviations from the steady state appear instead of the original quantities.

The final version of the stationarised model in the log-linear form is shown in box 1.

Calibration and Bayes parameter estimates

Two strategies are normally used to determine the parameters of New Keynesian DSGE models.

[4] ψ_t is the deviation of the LOOP gap Ψt from the steady state value.

[5] Another assumption is that the habit formation parameter is the same both in the domestic and in the foreign economy.

[6] For the above-mentioned transformation, a steady state is a state, in which real quantities Y, C and MC are growing at a constant speed ASS, the price indices at the speed PSS and other quantities are constant.

Box 1

Equations Summary

The following relations hold in the steady state:

1. Euler equation II + risk sharing

$$A_{ss}c_t - hc_{t-1} = A_{ss}E_t y^*_{t+1} - hy^*_t - \frac{A_{ss}-h}{\sigma}E_t q_{t+1} + A_{ss}E_t a_{t+1} - ha_t - \frac{A_{ss}-h}{\sigma}(r_t - E_t\pi_{t+1} - E_t p^*_{t+1})$$

2. Marginal cost

$$mc_t = \frac{\sigma}{A_{ss}-h}(A_{ss}c_t - hc_{t-1} + ha_t) + \varphi y_t + \alpha s_t$$

3. Domestic inflation

$$\pi_{h,t} = \beta(1-\theta_h)E_t\pi_{h,t+1} + \theta_h\pi_{h,t-1} + \frac{(1-\theta_h)(1-\beta\theta_h)}{\theta_h}mc_t$$

4. Imported inflation

$$\pi_{f,t} = \beta(1-\theta_f)E_t\pi_{f,t+1} + \theta_f\pi_{f,t-1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f}\psi_t$$

5. Total inflation

$$\pi_t = (1-\alpha)\pi_{h,t} + \alpha\pi_{f,t}$$

6. Definition of the LOOP gap

$$\psi_t = -(q_t + (1-\alpha)s_t)$$

7. Terms of trade

$$\Delta s_t = \pi_f - \pi_h$$

8. Uncovered interest rate parity

$$E_t\Delta q_{t+1} = r^*_t - (r_t - E_t\pi_{t+1}) - e\_q$$

9. Market clearing condition

$$y_t = (1-\alpha)c_t + \alpha y^*_t + \alpha\eta\psi_t + \alpha\eta(2-\alpha)s_t$$

10. Monetary policy

$$r_t = \rho r_{t-1} + (1-\rho)(\Phi_\pi\pi_{h,t} + \Phi_y y_t) + e\_r$$

11. Technological progress factor

$$a_t = e\_a$$

12. Prices in foreign economy

$$p^*_t = e\_p^*$$

13. Foreign sector

$$y^*_t = \rho_{r^*}y^*_{t-1} + e\_y^* \qquad r^*_t = \rho_{y^*}r^*_{t-1} + e\_r^*$$

The first strategy is the calibration of all parameters of the model (Monacelli, 2005). The second alternative is a combination of calibration and estimation of selected parameters, the estimates being mostly based on the Bayes approach (Dib, 2001 and Adolfson et al., 2007). The main advantage of this alternative is a better adaptation of the model to the conditions in the given economy.

We have chosen a second strategy, which means that we have calibrated a part of the parameters and have estimated the other part using empirical data. For Slovakia, however, calibration is difficult due to a lack of microeconomic studies, based on which it would be possible to determine the individual elasticities, therefore we have taken over some of the parameters from studies dealing with comparable models for small open economies.

Based on the goods structure of imports, we have approximately determined the degree of openness of the economy, meaning that the parameter α equals 0.4. The interpretation of the parameter is that 40% of the goods consumed in Slovakia stem from imports. The discount factor β has been calibrated to the value 0.99. Stationarisation of the model has required a fixed setting of the value for the coefficient σ at the level of 1. The other parameters have been taken over from calibrations of similar models (e.g. Vašíček and Musil, 2007; Liu, 2006). We are going to mention some of the most interesting parameters. We have ca-

Box 2

Summary of the Variables and Parameters

Variables		Parameters	
q_t	Real exchange rate	α	Degree of openness of the economy
ψ_t	LOOP gap	β	Discount factor
s_t	Terms of trade	h	Habit formation parameter
π_t	Total inflation	η	Elasticity of substitution between domestic and foreign goods
$\pi_{h,t}$	Domestic inflation	σ	Inverse elasticity of intertemporal substitution
$\pi_{f,t}$	Imported inflation	φ	Inverse elasticity of labor supply
y_t	Domestic economy output	θ_h	Price setting parameter (domestic prices)
c_t	Total consumption	θ_f	Price setting parameter (imported prices)
mc_t	Marginal cost	ρ	Parameter of inertia of domestic interest rates
r_t	Nominal interest rate	ρ_{y^*}	Autoregression coefficient y^*
r^*_t	Real interest rate in the foreign economy	ρ_{r^*}	Autoregression coefficient r^*
y^*_t	Foreign economy output	**Shocks**	
a_t	Technological progress	e_r	Monetary shock
A_{ss}	Steady state of A_t	e_q	Real exchange rate shock
p^*_t	Price growth abroad	e_a	Technology shock
P^*_{ss}	Steady state of P^*_t	e_p^*	Foreign inflation shock
		e_y^*	Foreign output shock
		e_r^*	Foreign real interest rate shock

Note: These are deviations from equilibrium values

librated the coefficient of domestic interest rate inertia to the value of 0.6, meaning that 60% of the current deviation of the interest rate is passed through to the deviation in the following time period. We have determined the value 0.8 for the habit formation parameter.

Historical data of variables appearing in the model have been used in estimating the remaining parameters. We have used quarterly data from the 1996 to 2006 period for these variables:

Y – the gross domestic product in Slovakia at constant prices, seasonally adjusted
Y^* – the gross domestic product in the euro area at constant prices, seasonally adjusted
C – household consumption at constant prices, seasonally adjusted
P – consumer price index in Slovakia
q – real EUR/SKK exchange rate, deviation from the steady state
s – terms of trade, deviation from the steady state
r^* – real interest rate in the euro area, deviation from the steady state.

The parameters η, θ_h, θ_f, ρ_{y^*} have been estimated using the Bayesian techniques. The prior parameter distributions and the medians of their posterior distributions are shown in table 1.

The model's reaction to shocks

In the following part, we will present one of the practical applications of DSGE models, which is a simulation of the impact of shocks on the economy. The simulation can help the central bank in its decision making. We will describe the reactions of selected quantities of the presented model on two shocks – on a nominal interest rate shock and a real exchange rate shock.

Nominal interest rate shock

Figure 2 shows the effect of a one-time 1% increase in the nominal interest rate over the following

Table 1 Estimates of selected parameters

Parameter	Prior			Posterior
	Distribution	Mean	Standard error	Median
η	gamma	1.00	0.30	0.43
θ_h	beta	0.50	0.25	0.54
θ_f	beta	0.50	0.25	0.46
ρ_{y^*}	beta	0.70	0.20	0.91

Source: Calculations of the authors

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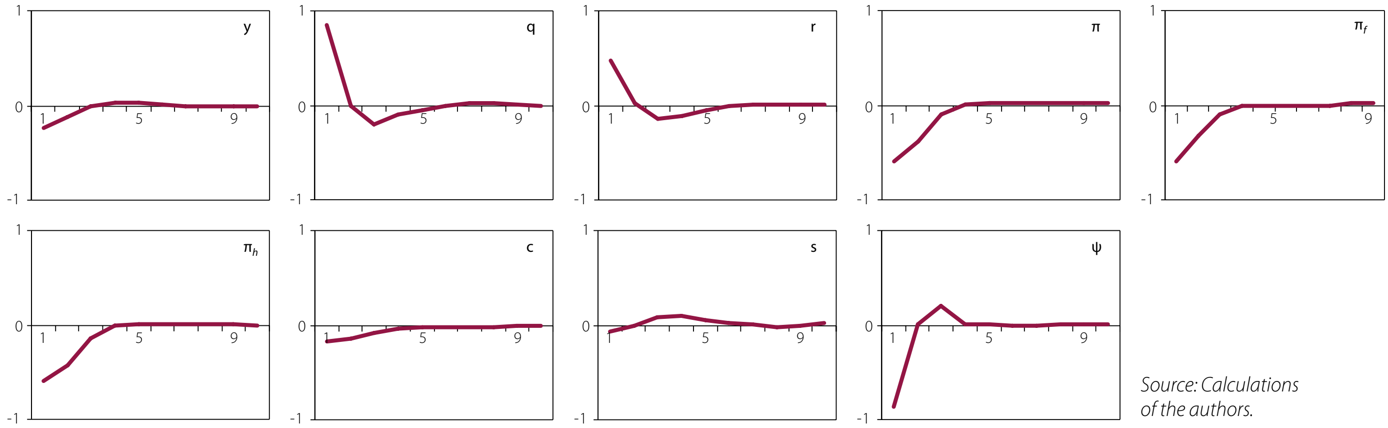
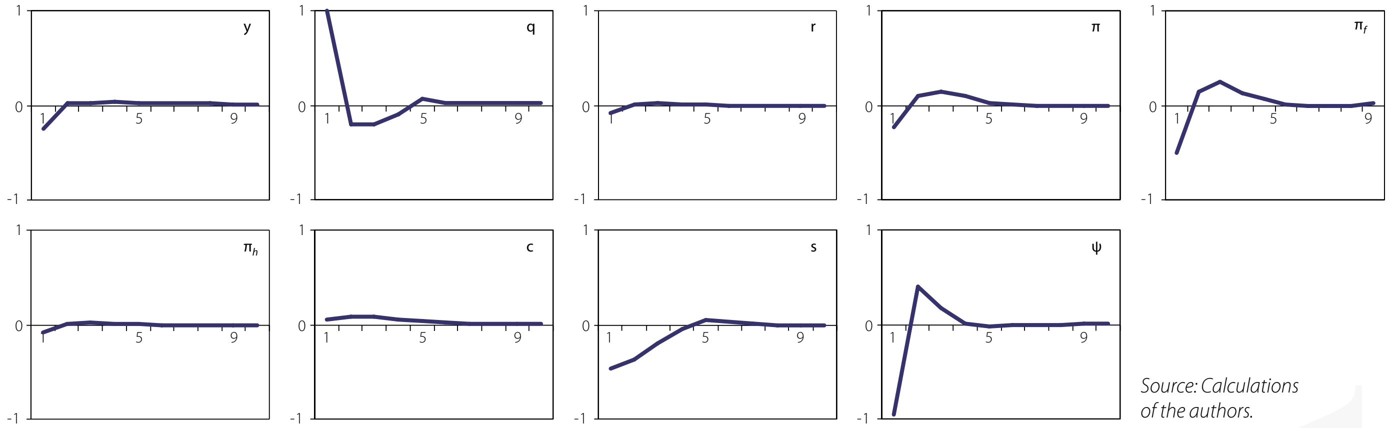
Fig. 2: Effect of a nominal exchange rate increase by 1%



Source: Calculations of the authors.

Fig. 3: Effect of an exchange rate appreciation by 1%



Source: Calculations of the authors.

ten quarters. Since this represents a tightening of monetary policy, both inflation and the output will fall – inflation by 0.7% and the output by 0.2%. Because these changes do not occur immediately, the monetary authority simultaneously reacts (through the Taylor rule) by decreasing the interest rate. As a result, a one-percent initial increase in the interest raáv decreases to 0.4%.

The real exchange rate appreciates by 0.8% (through the uncovered interest parity), which has an impact on a decrease in the prices of imported goods, i.e. imported inflation decreases. The stronger exchange rate, however, entails a fall in domestic production and, through the Phillips curve, also a fall in domestic inflation.

The higher interest rate has a negative impact on consumption, because future consumption becomes more attractive. The decrease in consumption, which is reduced by household habits, is minimal.

Real exchange rate shock

We have also examined the effect of a 1-percent appreciation of the real exchange rate (fig. 3). An exchange rate appreciation is reflected in export prices, which increase, so that the demand for exported goods decreases, which also leads to a decrease in domestic production. The lower production influences lower domestic inflation. At the same time, imported inflation is decreased due to the stronger exchange rate. The decreased production and lower inflation force the monetary authority to cut the interest rate. This effect, however, is very small, because it is compensated by the reaction of the interest rate according to the uncovered interest parity equation.

Conclusion

In this paper, we have derived a dynamic stochastic model of a small open economy for Slovakia. The model is based on microeconomic foundations, i.e. all equations have been obtained on the basis of optimum behavior of the participating agents – households, firms, the central bank and the foreign sector within the given restrictions.

A part of the model parameters has been calibrated; the rest of the parameters has been estimated using the Bayesian techniques, which combines information from preliminary estimates and from historical data.

The ability of the model to describe the dynamic development of the economy has been tested by means of impulse-response functions. According to the prevailing opinion, the qualitative response of the model to a monetary shock and an exchange rate shock is comparable to the response of real economy.

It has to be emphasized that this model is not very suitable for a quantitative analysis. In order to use it for forecast purposes, it has to be extended, which will be subject to further research.

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