MULTIVARIATE FILTER WITH UNOBSERVED COMPONENTS

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Introduction

In its Monetary Programme to 2008 the National Bank of Slovakia defined its monetary policy as inflation targeting under the conditions of the ERM II. At the same time it placed increased emphasis on communication activities together with explaining the causal relations and impacts of individual steps on the implementation of monetary policy. The publication of analytical outputs and their presentation is a component of the effective and transparent functioning of NBS monetary policy. Following the publication of the simple model of the transmission mechanism QPM[1] the National Bank of Slovakia presents another model – **the multivariate filter with unobserved components.** The role of this model, which is a component of the projection and analytical system, is to separate the cyclical and trend component of economic variables from historical data.

The aim of this article is to describe the estimating of the cyclical and trend components of economic variables, such as potential output, equilibrium interest rates and equilibrium exchange rate. The first part focuses on defining the basic indicators of the economic cycle and approaches to estimating potential output. The second part provides a more in-depth description of the model for estimating cyclical and trend components based on a structural system of time series with the **unobserved components** using the Kalman filtration algorithm. The third part presents the results of trend and cyclical component estimates and compares the algorithm with standard approaches.

Economic cycle theory

The basis for correct decision-making in effecting economic policy is knowledge of the current state of the country's economy. This is the starting point for forming forecasts of future development, with potential implications for subsequent economic policy decisions, should the current state appear undesirable. In the most general sense, a state is considered desirable when economic activity is sustainable in the long term and is at an appropriately high and balanced level. Exceeding this level represents a great burden for the economy and induces disequilibria in it. For a monetary policy that is to ensure price stability such disruption of the economy is indicated by the emergence of demand-side pressures and the deviation of inflation from its desired, targeted trajectory. Conversely, when economic activity does not achieve its potential, the economy becomes less effective, inflation falls and also the economy's equilibrium is disturbed.

The output gap, defined as the percentage deviation of an economy's total product from its potential level, is considered a key indicator of the state of an economy. While it is in principle possible to measure an economy's total product (an appropriate and most frequently used indicator is the gross domestic product), potential output is by its very essence a hypothetical, immeasurable variable and must be estimated.

The methods of estimating potential output are closely connected with what is understood by the term "potential". In literature the **equilibrium product** is usually defined as the hypothetical product that is achieved at the optimal allocation in the market of production factors and of goods and services on the market. The economy converges to it under a suitable economic policy. Sometimes it is also termed the potential output, though due to the precise definition of the terms here we shall use only the term equilibrium product. **In the following text the potential output is understood to mean such an equilibrium product that does not accelerate inflation.** If a production gap is positive, this creates pressure for a growth in the rate of inflation. When it is negative, inflation has a tendency to fall. Therefore when the output gap is closed and the economy is in equilibrium, inflation stays at a constant level.

The preceding definition is justified especially for the needs of monetary policy, because it correlates an economy's development in the framework of the economic cycle with changes in inflation. In this way a part of the so-called indirect transmission mechanism channel via which the monetary authority influences the future development of inflation in a desirable direction is identified.

The multivariate filter with unobserved components is one of the instruments for estimating long-term equilibrium trends (and the respective deviations from these trends – gaps) of relevant economic indicators. Inclu-

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ding assumed economic links and using historical data the most probable developments of unobserved states, such as the output gap, potential output, equilibrium interest rates and the equilibrium exchange rate are reconstructed. The greatest emphasis is placed upon the quality of estimates from the most recent period, therefore the methodology used in obtaining these estimates and comparing outputs with estimates using other approaches is important.

In professional literature various methods are used for estimating unobserved states. These include, for example, the Hodrick-Prescott filter, the multivariate generalisation Hodrick-Prescott filter, a method based on structural VAR models, or an estimate from the production function. In recent years the Kalman filter has received much attention, where this uses a state space representation of the model for estimating unobserved components.

The **Hodrick-Prescott filter, HP filter**, is defined as a solution to the dynamic optimisation problem:

$$\min_{y_t^p} \sum_{t=1}^{T} (y_t - y_t^p)^2 + \lambda\left[(y_{t+1}^p - y_t^p) - (y_t^p - y_{t-1}^p)\right]^2 ,\ \lambda > 0,$$

where y_t and y_t^p are actual and trend states of economic variables. The smoothing parameter λ must be set exogenously. Small λ values cause potential output to move closely alongside the real product, and conversely, large λ values cause the potential output to be smooth and closer to the linear trend given by the original data.

The HP filter is often used in estimating not only the potential output, but also other economic and unobserved variables. This popularity is related to its simplicity and possibility to use its to any structure of data.

A serious shortcoming of the HP filter is that it significantly biased estimates at the start and end of the observed period. The deviation is directly related to the definition of the filter – estimates over time use both past and future values, where the latter are not available. Since the precise estimate of the trend at the end of the observed period is the most important (from the economic policy aspect the most essential information is that on the present and future), the method of estimation by way of the HP filter comes to have little practical use. A problem is also setting the value of the smoothing parameter λ, since the economic justification for selecting the appropriate level of interconnectedness of the actual and potential output is not clear.

A further limitation of the HP filter is the fact that it does not inherently contain information on the current position of the economy. Similarly as with other purely statistical filters, the method does not allow any economic links to be used in estimating immeasurable variables. Likewise, it is unable to capture structural changes, which are typical for transitional economies. Therefore, neither is it guaranteed that the estimate of the potential output using the HP filter is an estimate in the sense of the definition, i.e. whether it is actually a product that does not accelerate inflation.

The generalisation of multivariate Hodrick-Prescott filter includes in the dynamic optimisation problem also information on the structure gained from economic theory, for example the residuals ε_t estimated from the Phillips curve:

$$\min_{y_t^p} \sum_{t=1}^{T} (y_t - y_t^p)^2 + \lambda_p\left[(y_{t+1}^p - y_t^p) - (y_t^p - y_{t-1}^p)\right]^2 + \lambda_\varepsilon \varepsilon_t^2,\ \lambda_p, \lambda_\varepsilon > 0.$$

The parameters λ_p and λ_ε, which must be input in advance, set the weightings attached to the individual components. Although the filter adjusted in this way represents a substantial improvement of the estimate against the simple HP filter, it still does not directly solve the problem of skewed estimates at the end of the observed period. Moreover, it can be demanding to solve concurrently the HP filter and estimate the structural relationship. Neither is the choice of weightings for λ_p and λ_ε straightforward.

Methods based on **structural vector autoregression models, SVAR**, are appropriate for (mainly short-term) projections beyond the observed period, because they do not skew initial and end estimates. Furthermore, economic information is directly incorporated in them. Estimation using the SVAR model is relatively complicated and requires a long time series of observations. At first a VAR model is estimated without restrictions and then estimates of coefficients for the VAR structural model are additionally calculated. The transition from the VAR model to the SVAR model requires additional restrictions, because the SVAR model is underidentified. A weakness remains, similarly as in the preceding methods, the fact that the definitions of potential output are not explicitly used – i.e. that inflation is at the targeted level when the real and potential output are at the same level.

A different approach, based on microeconomic principles, is to estimate the potential output (or rather the equilibrium product according to the definition) via the production function:

$$y_t^p = f(l_t^p, k_t^p, c_t^p),$$

where l_t^p, k_t^p and c_t^p are trend components of the labour, capital and productivity factors. In practice the Cobb-Douglas production function is mainly used to its simplicity. An approach via the production function, while it does directly use microeconomic links and estimates the dependence of the potential output on inputs, nevertheless does not solve the problem itself of obtaining the trend components of inputs. Again it is necessary to apply some statistical filter. Furthermore, data for technical progress and capital stocks are not currently available for the Slovak economy.

For monetary policy decisionsthe interconnection of consumer prices with the real economy is essential, as is the defining of the non-inflationary product, i.e. the product in which the development of prices is also balanced, which in the case of inflation targeting, means that inflation is found at its target value. The most appropriate approach therefore seems to be where the definition of the potential output is incorporated directly. This is made possible by the structural model system with unobserved states. Similarly as with the SVAR model, it is founded on relations from economic theory, but can also contain unobserved variables that can be estimated by means of the recursive Kalman filtration algorithm.

The Kalman filter

The benefit of the Kalman filter is the recasting of the estimated dynamic system to a special form, the so-called State-Space Representation. The Kalman filter progressively updates the linear projection system.

The **State-Space Representation** of a dynamic system is composed of two groups of equations. In the first group the measurable components y_t observed at the time t are described with the state-space variables ξ_t, exogenous (known in advance) variables x_t and random deviations w_t. The state-space variables don't need be measurable. The matrix notation of the first block of equations is termed the **matrix of observations** and is written:

$$y_t = A_t x_t + H_t \xi_t + w_t \,.$$

The second block, the **state-space equation**, describes state space variables as a vector autoregressive process:

$$\xi_{t+1} = F_t \xi_t + B_t x_t + v_t \,.$$

The vectors w_t and v_t are white noises and are not correlated:

$E(w_t, w_\tau^t) = R_t$ for $t = \tau$, 0 otherwise,
$E(v_t, v_\tau^t) = Q_t$ for $t = \tau$, 0 otherwise,
$E(v_t, w_\tau^t) = 0$ for any t a τ,

where R_t and Q_t are variation-covariation matrices of the disturbances w_t and v_t.

The matrices A_t, B_t, H_t, F_t, R_t and Q_t are matrices of the model's parameters that need not be constant over time. Their values can be estimated with the help of the Kalman filter using a Maximum Likelihood method.

A detailed description of the Kalman filter with other requirements is given, for example, by Hamilton (1994) or Harvey (1989).

Model estimate of the output gap

In the following part the **Multivariate Filter with Unobserved Components (MVF-UC)** is described in more detail, where this tool is a supporting instrument of the prediction and analytical system of monetary policy. Its basic philosophy and structure is consistent with another model, the quarterly prediction model (QPM). The MVF-UC is designed so that it contains the same specification of relevant equations as the QPM. Furthermore, it enables the statistical properties of the behaviour of individual variables to be identified in more detail, with the possibility of estimating some parameters. This approach uses economic links, whose validity is assumed by theory, the expert judgment used in calibration of the core model and at the same time a statistical estimate of the remaining parameters.

Basic macroeconomic relations form the starting point for formulating the model. It is clear that the potential output is, via economic links, tied to the real product, i.e. that which is actually produced in the whole economy. Gross domestic product at fixed prices is the commonly used indicator of real product. A further economic variable with which potential output is linked is (by definition) inflation. Expectations also have an important role here. Inflation expectations have a significant influence on the real economy via wage bargaining; they can help or hinder the attainment of a declared inflation target. Expectations are measured as a weighted average of adaptive and rational expectations.

From the perspective of monetary policy, the greatest importance is given to the link between monetary-policy variables, such as interest rates and the exchange rate, and the real economy. Based on UIP conditions there does exist a certain equilibrium trajectory for real interest rates and the real exchange rate. Deviations of real rates and the exchange rate from the equilibrium development create pressure for a deviation of the real product from the potential and can lead to a change in the size of the output gap. For simplification the interest rate and the exchange rate gap are aggregated into one indicator, the real monetary conditions index, which is a weighted average of them.

The first step in estimating the overall state of the economy is the estimate of the setting of real monetary conditions, which influence the behaviour of the whole economy. In other words it is necessary to ascertain whether monetary policy is having a tightening or stimulating effect on the economy. An essential step is to estimate the equilibrium trends of real interest rates, the real exchange rate and the risk premium. Firstly it is necessary to determine, or estimate, the equilibrium level of the foreign real interest rate which features in the model for the Slovak economy as an explanatory (exogenous) variable. Here it is possible to choose several approaches, for example to consider the equilibrium foreign rate as constant and to estimate it as an average of the real rates for the period observed, or to estimate it, for example, via the HP filter. The model uses the estimation approach via the univariate Kalman filter:

(1) $r_t^f = \bar{r}_t^f + \hat{r}_t^f$
(2) $\Delta\bar{r}_t^f = \varepsilon_t^{\bar{r}^f}$
(3) $\hat{r}_t^f = \varepsilon_t^{\hat{r}^f}$

The difference between the real foreign interest rate r_t^f and the equilibrium rate $\bar{r}_t^f$ by definition equals the gap in the foreign interest rate $\hat{r}_t^f$. Equations (2) and (3) define the statistical properties of both estimated variables.

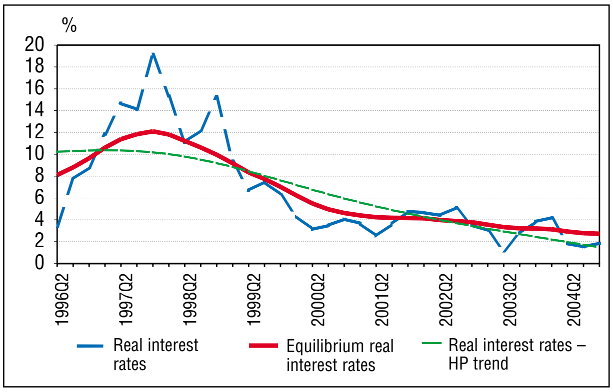
In the next step the equilibrium foreign interest rate now features as an exogenous variable:

(4) $r_t = \bar{r}_t + \hat{r}_t$
(5) $z_t = \bar{z}_t + \hat{z}_t$
(6) $q_t = \bar{q}_t + \hat{q}_t$
(7) $-4\Delta\bar{z}_t = \bar{r}_t - \bar{r}_t^f - \bar{q}_t$
(8) $\Delta\bar{r}_t = \varepsilon_t^{\bar{r}}$
(9) $\Delta\bar{z}_t = \bar{\rho} + \varepsilon_t^{\bar{z}}$
(10) $\hat{r}_t = \varepsilon_t^{\hat{r}}$
(11) $\hat{z}_t = \varepsilon_t^{\hat{z}}$
(12) $\hat{q}_t = \varepsilon_t^{\hat{q}}$

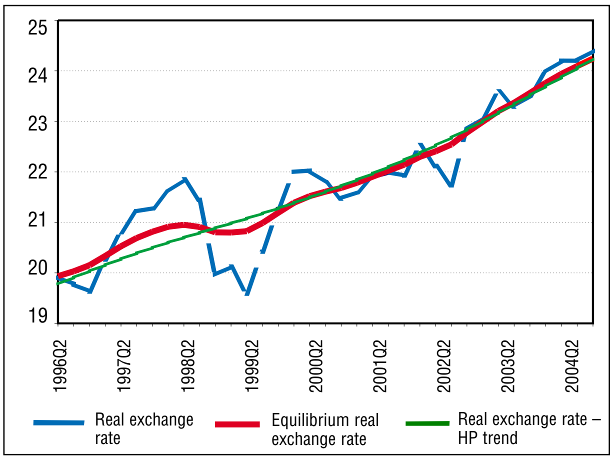
The first three equations (4) – (6) are identities for the real domestic interest rate r_t, the real exchange rate z_t and the real risk premium q_t. The equilibrium values of the variable x are marked $\bar{x}$, and the their gaps have the indicator $\hat{x}$.

Equation (7) is the **equilibrium UIP condition** (*Uncovered Interest Rate Parity*). It says that in equilibrium the exchange rate depreciates (appreciates) proportionately to the positive (negative) interest-rate differential, adjusted by the risk premium. The last five equations (8) – (12) describe the statistical properties of individual variables, where it is assumed that their residuals are white noises.

Graph 1 Estimate of equilibrium interest rates



Graph 2 Estimate of the equilibrium exchange rate



The results of the estimate of the real equilibrium interest rate and real equilibrium exchange rate are shown in graphs 1 and 2.

An interesting result of this part of the model is the parameter $\bar{\rho}$, which may be understood as the trend appreciation of the equilibrium exchange rate. The model assumes that from the long-term aspect the appreciation of the equilibrium interest rate is constant, where its level is estimated. Among other things, it is possible to estimate also the respective long-term equilibrium interest rate.

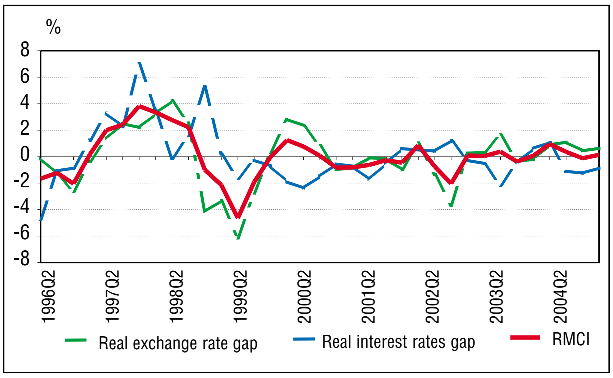
From the estimate of gaps for the domestic interest rate and the exchange rate there is calculated the **real monetary conditions index** $rmci_t$ as a weighted average (Graph 3):

(13) $rmci_t = \beta\hat{r}_t + (1 - \beta)\hat{z}_t$

The parameter β is set on an expert basis in the core model and takes account of the weights by which monetary policy operates via individual monetary-policy variables on the real economy.

The core of the output gap estimate are the equations describing the links between inflation, real product, potential output and the output gap and the reaction of these variables to set monetary conditions, where $rmci$

Graph 3 Estimate of the real monetary conditions index (RMCI) and its components



features in this part as an exogenous variable. They are equations for aggregate demand (IS curves) and aggregate supply (modified Phillips curve).

In view of the regressive development of the gross domestic product it is clear that fiscal policy had a significant role in its formation, particularly in the years 1996 – 1998. The model does not explicitly take account of fiscal development, because monetary policy does not have a direct influence on it. Therefore it is necessary to separate GDP into two components. The first, the private sector y_t^{prv}, will according to assumptions react to interventions in monetary policy. The second component, the public sector y_t^{gov}, does not react to changes in monetary policy. Government investment and end consumption by the general government sector are classified in the public sector. The private sector is calculated as the difference between GDP and the public sector.

Each sector has its potential level (marked by a bar) and a gap (marked by a cap). Equations (14) and (15) express these identities:

$$(14)\ y_t^{prv} = \bar{y}_t^{prv} + \hat{y}_t^{prv}$$
$$(15)\ y_t^{gov} = \bar{y}_t^{gov} + \hat{y}_t^{gov}$$

Equation (16) cleans net inflation, excluding fuels π_t^{netexe}, of the influence of random shocks. Net inflation is marked by a wavy line:

$$(16)\ \pi_t^{netexe} = \tilde{\pi}_t^{netexe} + \varepsilon_t^{\pi^{netexe}}$$

The block of the following five stochastic equations and two identities describes the behaviour of all unobserved variables. It is the core of the whole model.

$$(17)\ \Delta\bar{y}_t^{prv} = \phi_1 \Delta\bar{y}_{t-1}^{prv} + (1-\phi_1)\bar{\mu}_1 + \varepsilon_t^{\bar{y}^{prv}}$$

$$(18)\ \Delta\bar{y}_t^{gov} = \phi_2 \Delta\bar{y}_{t-1}^{gov} + (1-\phi_2)\bar{\mu}_2 + \varepsilon_t^{\bar{y}^{gov}}$$

$$(19)\ \hat{y}_t^{prv} = \alpha_1 \hat{y}_{t-1}^{prv} + \alpha_2 rmci_{t-1} + \alpha_3 \hat{y}_{t-1}^{f} + \alpha_4 income + \varepsilon_t^{\hat{y}^{prv}}$$

$$(20)\ \hat{y}_t^{gov} = \alpha_1 \hat{y}_{t-1}^{gov} + \varepsilon_t^{\hat{y}^{gov}}$$

$$(21)\ \tilde{\pi}_t^{netexe} = \gamma_1(\pi_{t-1}^{m} + \Delta_4 \bar{z}_t) + (1-\gamma_1)[\tilde{\gamma}_2 \pi_{t-1}^{netexe} + (1-\gamma_2)(E_{t-1}\pi_t)] + \gamma_3 \hat{y}_{t-1} + \varepsilon_t^{\tilde{\pi}^{netexe}}$$

$$(22)\ \hat{y}_t = (1-\theta_t)\hat{y}_t^{prv} + \theta_t \hat{y}_t^{gov}$$

$$(23)\ E_{t-1}\pi_t = \kappa_1[\kappa_2 \pi 4_{t+3} + (1-\kappa_2)\, \pi 4_{t+3}^{netexe}] + (1-\kappa_1)[\kappa_2 \pi_{t-1} + (1-\kappa_2)\pi_{t-1}^{netexe}]$$

Equations (17) and (18) describe the growth in potential for the private sector $\Delta\bar{y}_t^{prv}$ and the analogous growth in potential for the public sector $\Delta\bar{y}_t^{gov}$. The level of long-term equilibrium growths $\bar{\mu}_1$ and $\bar{\mu}_2$ are estimated from the model.

Equations (19) and (20) are the **IS curves** for the private and public components of the product. The difference between the two is that the private sector reacts to changes in real monetary conditions and also a link to foreign demand is assumed here. Furthermore, in the private sector there is seen the *income* effect, marked as income, resulting from price deregulations which influence real disposable income. Together with the real monetary conditions index $rmci_{t-1}$ they feature in the equation exogenously. The parameters α_i are calibrated in the core model.

Equation (21) is a **modified Phillips curve** describing the development of net inflation excluding fuels cleansed of random deviations $\tilde{\pi}_t^{netexe}$ depending on the development of prices abroad, domestic regulated prices, inflation expectations and the state in the economic cycle. The parameters γ_i are calibrated in the core model. The homogeneity of parameters used in the inflationary influences guarantees that any value of inflation is compatible with an economic equilibrium.

The specification of the Phillips curve ensures that in the case of an economy's overheating pressures for a growth in inflation are created, and conversely in the case of a negative output gap inflation will have a tendency to fall. The potential output estimated from the model is then that equilibrium product that does not accelerate inflation.

Equation (22) defines aggregate demand $\hat{y}_t$. It is expressed as the weighted average of the gap of the private and public sectors. The parameter θ_t representing the public sector's weighting depends on the share of the public sector potential to the total GDP potential.

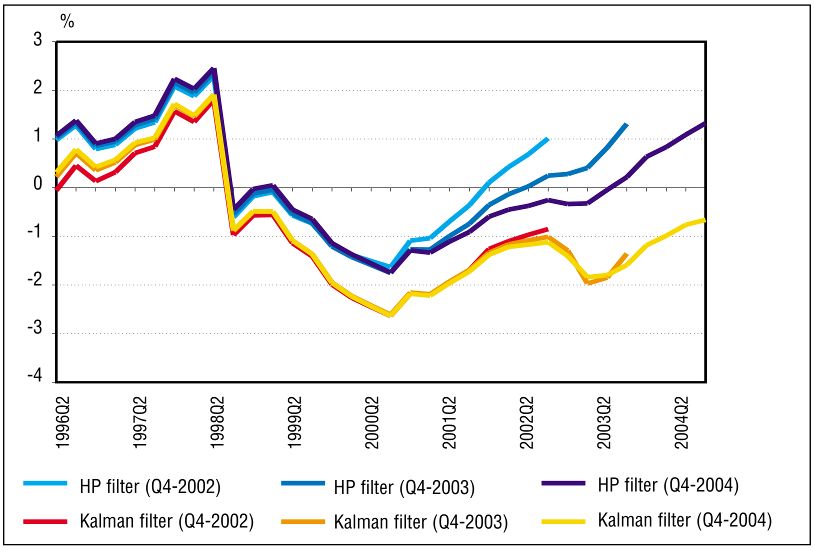
The formation of **inflation expectations** $E_{t-1}\pi_t$ is presented in the equation (23). Expectations are a combination of adaptive and rational expectations. The parameters are calibrated.

Estimate results

The estimates of the model variables, gained with the help of the MVF-UC, were obtained using seasonally-adjusted time series of the respective economic indicators from the first quarter of 1995 through to the end of 2004. For the purpose of comparison there is also given the estimate with the aid of the simple HP filter from seasonally-adjusted GDP for the period from the start of 1993 through to the end of 2004. In order to examine the sensitivity of the end estimates of the output gap, both methods were used also in the shortened database of the time series to the end of 2003 and 2002.

The results indicate that the SR economy is found in the growth phase of the economic cycle, there are

Graph 4 Comparison of production function estimates (HP filter vs. Kalman filter)



however evident differences in identifying its current position. While the HP filter records overheating since 2003, in the case of estimation using the MVF-UC the economy has still not reached its potential. The negative output gap indicates that despite high GDP growth, there are not present demand-side pressures that could lead to a growth in inflation. Conversely, low inflation confirms that the economy is not utilising its capacities optimally and can continue further on a growth trajectory without risk of overheating.

The results in Graph 4 support the presumptions of better stability of the end estimate using the Kalman filter. The HP filter in the case of the estimates in the shortened time series indicated an overheating of the economy from 2002 and in the case of using the whole length of the time series the estimate of the economy's position at the end of the monitored period was significantly deviated in comparison with the estimate using the shortened time series. For the needs of monetary policy, where current values of the output gap are crucial, the Kalman filter provides more stable results. Even in the estimate in the shortened time period the algorithm identified a GDP gap close to the original estimates and thereby confirmed the advantages of the Kalman filter over ordinary statistical approaches.

Conclusion

The estimate of the output gap enables economic policy makers to monitor phases of the economic cycle and thereby diagnose in time undesirable fluctuations in the national economy. This article has described the approach of the estimate using the Kalman filter. Based on assumed economic links and using historical data the Kalman filter reconstructs the development of unobserved variables such as the output gap, potential output, the real monetary conditions index, equilibrium interest rates and the equilibrium exchange rate. In comparison with statistical approaches it takes into account the nature of the economy, its structural changes and with an increase in observations there is no deviation in the estimate at the end of the monitored period, which is crucial for monetary policy.

A not insignificant benefit of the approach used is the better interpretation of the resultant estimates, because thanks to the economic links used in the estimates, it is possible to describe consistently the historic development of basic economic indicators and confront it with the opinions and views of experts as to what has been happening in the country's economy.

A monetary authority can influence the development of the output gap by means of interest rates and indirectly influence consumer price inflation. This effect is called the indirect transmission mechanism channel. For monetary policy the GDP gap is then a useful indicator of future inflationary or deflationary pressures and together with other estimated indicators (for example, the real monetary conditions index) they form a substantial information base in its decision-making process.

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