NÁRODNÁ BANKA SLOVENSKA

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Collective investment via fund management companies, investment companies, and pension funds plays a permanently important role in capital markets. The professional approach of these companies' managers does not occur without the use of portfolio creation methods. With the help of these it is possible to determine in which securities from a selected range it is advantageous to deposit investors funds and concurrently to also set the optimal ratios in individual assets. The work of H. M. Markowitz, published in 1952 in the article "Portfolio selection" is considered the keystone of modern portfolio theory.

Harry Max Markowitz was born on 24 August 1927 in Chicago. His path to economics was a meandering yet absorbing one. During college studies he expressed an interest mainly in physics and astronomy. He also displayed a keen interest in reading the philosophical works of David Hume, in particular parts concerning the theory of probability, and with equal interest also read Darwin's work "The Origin of Species" , where he was mainly interested in the issue of data. His interests led him to enrol for a two-year bachelor studies course at Chicago University, focusing on philosophy.

After completing his bachelor studies in 1947 he decided to continue in a graduate school majoring in economics. His thinking and ideas were influenced by such famous professors of Chicago University as Friedman, Marschak, Savage and Koopmans (also Nobel Prize winners). Via micro and macroeconomics he gradually came to the topic of economic uncertainty, which became his lifelong work. When in 1950 he finished his master's studies, he decided to continue in doctorate studies at Chicago University, which he completed through a defence of his dissertation work on portfolio theory in 1954.

As H. M. Markowitz himself stated on the occasion of receiving the Nobel Prize, at the time when he was defending his dissertation work, the academic soil of Chicago University was none too fertile nor enthusiastic to be inclined to his scientific research. Even professor Milton Friedman at that time said that portfolio theory is not economics and so a scientific degree cannot be granted to the dissertation student for his work. In the end, however, they still awarded him the scientific degree PhD.

The contribution of H. M. Markowitz to economic theory

Prior to Markowitz's portfolio theory several economists had pointed to the need for diversification in investment ("don't put all your eggs in one basket"). This approach, however, was founded on lay observation without due analysis and quantification. As is clear this idea without an appropriate scientific grounding greatly irritated Markowitz and led him to his famous model on the creation of efficient allocation and portfolio creation.His further enthusiasm in research was to a significant extent influenced also by practice. Since 1952 he has worked in many well-known companies, which also created for him the conditions for his further work. In the Fifties and Sixties he worked in Rand Corporation, General Electric Corporation, Cairmand, Consolidated Analysis Centres Inc. etc. In his work he learnt in particular optimisation techniques and methods of linear programming. He enriched logistic simulation models and programming languages through his theoretical knowledge in the field of portfolio theory (at Rand he was a co-creator of the programming language SIMSCRIPT, which in an updated version is still used today).

The level of computer equipment at that time did however to a significant extent limit the creation of various subbranches and links between branches in this scientific field. The current level of computer technology has also removed this barrier.

In the Seventies and Eighties he again returned to academia, working as professor of economics and finance at universities such as the Wharton School, University of Pennsylvania, Rutgers University New Brunswick, but at the same time also in IBM (1974 – 1983).

Of the various awards for his scientific research special attention must be given to the prize from the Operational Research Society of America and the Von Neumann prize from the Institute of Management Sciences for the theory of operational research, which he was awarded in 1989. In 1990 the Swedish Royal Academy of Science awarded him the Nobel Prize for economics.

Markowitz's selection model

As Markowitz states, in the creation of portfolio theory he was inspired by the article "The Theory of Investment Value" by John Burr Williams. J B Williams is known in financial theory for his share price model. Williams in his work states that the price of a share represents the cash flow of future dividends discounted to the net present value. According to this model an investor is interested in future expected dividend flows and thus also the expected share price. Markowitz applied this conclusion also to portfolio theory when he stated that investors are interested in the expected value of a portfolio, where this value may be quantified. Among those values which investors are interested in are, according to him, risk and return.

In creating his selection model Markowitz works from certain abstractions and presumptions:

- investors have an aversion to risk,
- all investors invest at the same time,
- the basis of investment decisions is expected utility,
- investors make their investment decisions on the basis of expected risk and return
- perfect capital markets exist.

What does a portfolio in the financial market represent in the meaning of this theory? A portfolio is a set of various investments, which an investor creates in order to minimise risks connected with investing and also to find the best possible proportion between risks and returns. Since according to Markowitz's theory the investor is risk averse, the investor will create a portfolio with the aim of achieving the largest return for the minimum risk.

In quantifying the yield (return) of a portfolio Markowitz worked at first from determining the expected yield of one instrument and then from the expected return of the whole portfolio.

Expected yield of one instrument (share)

$$E(r_i) = \sum_s p_s r_s$$

where the variable *E(r)* expresses the expected yield of the instrument in question; the variable *p* determines the level of probability with which occurs the yield *r* of the instrument in question

Expected yield of a portfolio

$$E(r_p) = \sum_i w_i \cdot E(r_i)$$

where *w* represents the share of individual instruments in the portfolio. The expected yield of the portfolio is a weighted average of the expected individual fields of individual instruments in the portfolio, where the weightings are the shares of individual investments in the portfolio.

An investor is interested not only in the rate of return but also in the risk. In measuring risk Markowitz at first works from the risk of one asset and then from the risk of the portfolio.

For the individual shares of which the portfolio is comprised he then determined the risk as:

$$\delta = \sqrt{\sum_s [r_s - E(r_i)]^2 \cdot p_s}$$

where p_s is the probability, at which the yield in question is achieved,

δ – standard deviation.

The **risk of a portfolio** however is not simply a weighted average of the risks of individual instruments in the portfolio. The degree of risk of the portfolio is influenced also by other variables, in particular by the mutual relation between the yields of individual instruments, which is expressed by means of a coefficient correlation.

$$\delta^2 = [r_p - E(r_p)]^2 = \overset{N}{\underset{i=1}{E}} \Sigma\, [r_i - E(r_i)]^2 \cdot w_i$$

$$\delta^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \delta_{ij} = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \delta_i \delta_j \rho_{ij}$$

where

(δ_{ij} – is the covariance between the yield of the *i* and *j* security,

(ρ_{ij} – is the correlation between yields of the *i* and *j* securities,

This covariance in relation to the standard deviation of the *i* and standard deviation of the *j* security gives us the **correlation** between the yields of these securities. We can express this as the relation:

$$\rho_{i,j} = \frac{\delta_{i,j}}{\delta_i \delta_j}$$

The coefficients of covariance range from –1 to +1 and express the direction of the correlated movement of investments in the portfolio. If the covariance has a positive value it means that the yields of investments have the same direction of movement. An inverse relationship between the yields of investments exists in the case where the covariance has a negative value. The covariance has a zero value where the yields moving independently.

With help of complex matrices of covariance coefficients Markowitz stated that if an investor invests in a portfolio which perfectly positively correlated yields, then it does not at all lower his risk, because the yields move in only one direction and the investor in such a portfolio can suffer significant losses. The ideally compiled portfolio has negatively correlated yields, i.e. the yields have an inverse movement. To compile such a portfolio however is in practice impossible. Assets with non-correlated yields create a portfolio in which the yields have no relation to one another.

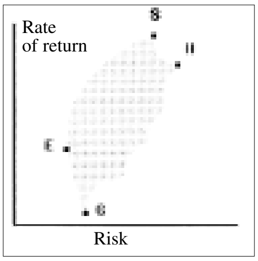
The benefits of **diversification** lie in the fact that a more efficient compensation effect of risk and return will be achieved through an appropriate combination of assets, the correlation of which does not extend to a form of completely positive correlation. In such cases the standard deviation of the yield of a portfolio is less than the weighted average of the standard deviations of the assets in the portfolio. Diversification lowers risk also in the case of a smaller number of securities – first of this risk is lowered quickly, gradually with an increasing amount of securities, the effectiveness declines.

It is thus possible to assess risk in the context of a portfolio. We cannot judge the effective risk of any security in a way that we will examine it in isolation. A part of the uncertainty concerning the yields of a security is "de-diversified" as soon as a security is grouped with others in the portfolio.

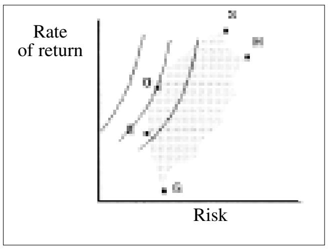
From Markowitz's selection model it thus results that if an investor wants to reduce the overall risk of the portfolio, then he must combine those assets which are not perfectly positively correlated.

Effective limit and selection of the optimal portfolio

Markowitz worked from the assumption that in the selection of a portfolio the investor can select within the framework of the market various combinations of securities with various yields and risks. In other words he assembled a so-called **feasible set** of all possible combinations of investment, which an investor is faced with in the market. The typical shape of a feasible set of portfolios has entered financial theory under the title of an "umbrella shape", which is depicted in the graph.



From the set of Pareto optimal combinations of expected yields and variances investors will according to Markowitz select portfolios which: give the maximum expected rate of return at various levels of risk or offer minimum risk in the case of various levels of expected rates of return. In the graph it can be seen and that given conditions fulfil the combinations *S*, in the case of which the investor will achieve a maximum yield and *E* with the lowest risk. The set of portfolios fulfilling these two conditions is known as the **efficient set** or **efficient frontier**. This limit depicts the points with the maximum rate of return for a given level of risk, and which are measured by the standards deviations of the portfolio's yields. From the graph it can also be established that the efficient set will be located between points *E* and *S*.



To this efficient frontier he also applied **indifference curves**, which from the aspect of the theory of frontier utility express the various combinations, in the case of which an investor tries to achieve the same utility. As Markowitz states, indifference curves have a different slope in the case of a risk-averse investor and that of a risk seeking investor. The indifference curves of an investor seeking risk have a more moderate slope and will move closer to point S, where they will also touch the efficient frontier. Since in his selection model he gives preference to the risk-averse investor, the contact of his indifference

Most known works of H. M. Markowitz

To be a pioneer in economic theory is today very difficult. As can be seen also from Markowitz's extensive publications he managed to make a connection between economic. thought and the use of mathematical-statistical methods and was not afraid even to delve into programming economic processes and creating programming languages. His scientific research, which he has presented also through extensive publications, has always led to the verification of his ideas in practice. Of his extensive publishing activity are known primarily these publications:

Books:

1. Portfolio Selection: Efficient Diversification of Investments, Wiley, 1959.
2. A Simulation Programming Language (with B. Hausner, H. Kerr), Prentice-Hall. 1963.
3. Studies in Process Analysis: Economy-wide Production Capabilities (with A. S. Manne), Wiley, 1963.
4. The Simscript H. Programming Language (with P. Kiviat, R. Villaneuva), Prentice-Hall, 1969.
5. The EAS Programming Language (with A. H. Malhotra, D. P. Pazel), IBM Thomas j. Watson Res. Center, 1981.

Articles:

1. Portfolio selection, J. Fin, 7 March 1952.
2. The utility of wealth, JPE, 60, April 1952.
3. Industry wide, multi-industry and economy-wide process analysis, The structural independence of the economy, od. T. Barna, Wiley 1954.
4. The optimisation of a quadratic function subject to linear constraints, Naval res. Logistics Q., 3, 1956.
5. The elimination form of the inverse and its application to linear programming, Manag. Sc., 1957.
6. Investment for the long run: new evidence for an old rule, J. Fin., 31, Dec. 1976.
7. Approximating expected utility by a function of mean and variance (with H. levy), AER 69 June 1979.
8. Simscript, Encyclopaedia of Computer Science and Technology, 13, eds, J. Belzer, A.G. Holzman. A. Kent, Marcel Dekker, 1979.
9. Mean variance verses direct utility maximisation (with H. Levy, Y. Kroll), J. Fin, 39, March 1984.
10. The ESA-E application development system: principles and language summary (with A. H. Malhotra, D. P. Patel), Communications of the ACM, 1985.

curve with the efficient frontier of the portfolio creates the **optimal portfolio** for the investor.

Conclusion

Every portfolio manager recognises the value of the innovative approach of H. M. Markowitz in this field. All his theoretical conclusions have become the basis and springboard for the development of other theoretical analyses in the field of portfolio theory. A link to Markowitz's theory of searching for the optimal portfolio was made in 1958 by J. Tobin, who enhanced it also introducing in combining individual assets in the portfolio the so-called risk-free asset, which has a guaranteed yield and at the same time it's standard deviation is zero. Here an investor can invest not only in this security but also for a given interest rate also borrow on the market and make short sales. He thus perfected also Markowitz's theory on the so-called capital market outline and also defined in more detail the search for the optimal portfolio.

In 1964 Markowitz's theory was enhanced also by another famous American economist, W. F. Sharp through his capital asset pricing model, which significantly simplified the process of determining the rate of return and the yield of a portfolio by means of the so-called coefficient b. The key concept here is also the so-called market portfolio.