Asymmetric fan chart – a graphical representation of the inflation prediction risk

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A prediction of macroeconomic indicators is the first step in providing information on where the economy is heading. However, any such prediction involves a certain degree of uncertainty or risk.

Asymmetric distribution of the prediction risk

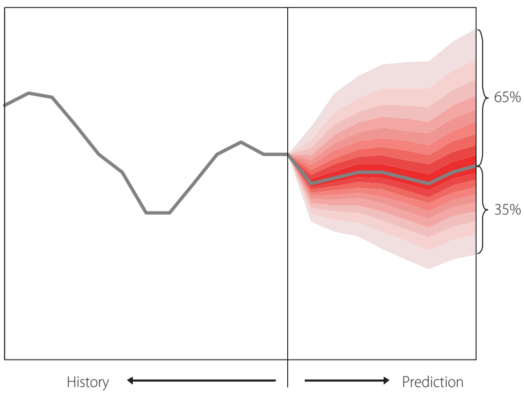
The uncertainty of a prediction is related to the risk in the input assumptions for exogenous variables, to the use of simplified econometric models, but it can be also related, for example, to data revision. All such risks cause a deviation of the actual values for individual indicators from their point predictions. It is therefore obvious that a point prediction does not have to be sufficient to get a complex picture of the expected development of the individual indicators. Instead of point predictions, risks are allowed for in the form of publishing interval predictions in practice. In recent years, a modern information tool for the communication of uncertainty resulting from the prediction of macroeconomic indicators has been a graphical output showing an estimate of the probability distribution of the prediction of the indicator under consideration – the fan chart. Fan charts were used for the first time by the Bank of England in 1996 and several central banks started to use them subsequently thanks to their obviousness and easy to comprehend form.

A fan chart is currently used in the forecasting process of the NBS when presenting an estimate of the development of inflation within a medium-term prediction.[1] It is based on a simple empirical approach, in which the uncertainty of inflation prediction (the width of the interval around the point prediction) is estimated based on historical prediction errors. In this case it is assumed that the probability of reaching higher inflation compared to the point (baseline) prediction is the same as the probability of reaching a lower inflation. In the graphical form, this is represented by the same width of the interval below and above the baseline prediction. This is called symmetric distribution of risk or a symmetric fan chart.

Although a symmetric fan chart resolves the issue of uncertainty, it does not enable to take into account current (real) risks of shocks in the economy (for example a higher then expected foreign demand, changes in oil prices or a higher then expected growth of nominal wages). These are risks that can bring about an asymmetric upward or downward bias of the uncertainty prediction, for there are many real situations, where it is necessary (indispensable) to allow for such risks. A risk factor, for example in the form of a possible faster growth of oil prices as compared to the assumptions of the baseline prediction, would represent a unilateral pro-growth risk in reality. Materialization of such a risk would mean that inflation should be higher than forecasted with a higher (more than 50%) probability. At the same time, this reduces the probability (less than 50%) that inflation will be below its forecast level. There would be also a shift, but not a widening, of the intervals towards higher levels. However, it has to be emphasized that the baseline prediction does not change this way and it will continue to represent the most probable scenario of the development of inflation.[2] It will not be, however, in the middle of the expected interval anymore. A graphical representation of the prediction allowing for such a type of risk represents a shift from a symmetric to an asymmetric fan chart.

Consequently, the main benefit of using asymmetric fan charts should be an extension of the view on existing risks accompanying the predic-

Chart 1 An illustrative example of an asymmetric fan chart with prevailing pro-growth risks

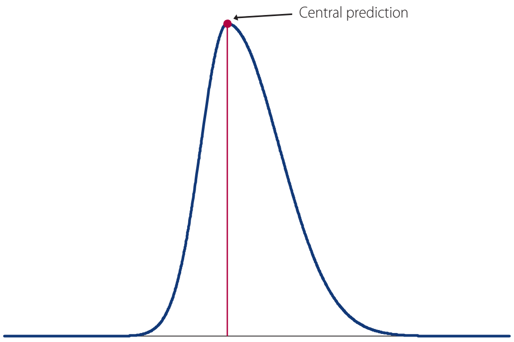


Source: NBS.

1 For more information on the issue of symmetric fan charts and their application at the NBS see the article: T. Oparty, M. Gavura: Estimating the probability distribution of an inflation forecast. The fan chart – a graphical presentation, BIATEC, volume 13, 5/2005, Národná banka Slovenska.

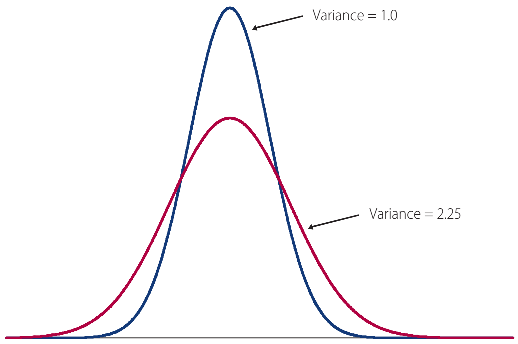
2 The baseline prediction represents the mode of the assumed probability distribution.

Chart 2 The density of a skewed normal distribution



Source: NBS.

Chart 3 Density of a normal distribution with various variances



Source: NBS.

tion. These are primarily risks that increase or decrease the probability of reaching higher or lower inflation as compared to the baseline prediction. This approach should also contribute to a comparison of the risks between two predictions. The extension of the fan chart used so far by adding risk analysis should contribute to increased transparency in the area of formation of expectations. It enables to provide information on expected asymmetric risks in a more comprehensible form. In addition to a graphical form, it is, of course, possible to present numeric values of the probability intervals or the probability of meeting or failing to meet the inflation target.

A fan chart, as a way of graphical representation of the development of confidence intervals for the forecast of inflation by quarters in time, is actually a kind of simplification of the actual probability distribution. When preparing an asymmetric fan chart, the primary issue to be dealt with is the construction of an asymmetric probability distribution of the inflation forecast in one point of time. For simplicity and a wide use, an asymmetric normal distribution is used, the construction of which is based on a normal symmetric distribution. To put it simply, it is necessary to estimate how the uncertainty (risk) of the prediction is distributed below and above the baseline prediction. Three main parameters have to be known for the construction of the asymmetric fan chart: the central prediction, the prediction uncertainty and asymmetry, which are actually economic interpretations of a non-symmetric normal distribution with the parameters μ-mode, σ-volatility estimate – the so-called standard deviation, and γ-skewness of the distribution.[3]

The **central prediction** is a forecast based on the baseline scenario (of the central prediction, Chart 2). In our case, the central prediction is a forecast of year-on-year inflation from the medium-term forecast. In terms of probability, it is a prediction, which is considered the most probable, i.e. from a mathematical point of view it is the mode of the distribution (μ). However, this does not rule out the alternative that the actual value of future inflation will not be higher or lower. It is always assumed, however, that the central prediction represents an inflation development consistent with the most probable paths of economic factors. The idea that the central prediction is the mode is based above all on the process of preparation of the medium-term forecast, where, after several meetings, the final prediction is set as the alternative assumed to be the most probable one. If the prediction process had a different direction, for example if several possibilities were considered – because there is the conviction that each of those alternatives will have its weight in future inflation, it would be more suitable to take the prediction as a weighted average of alternatives. The resulting prediction could be considered an approximation of the mean value of an asymmetric normal distribution.

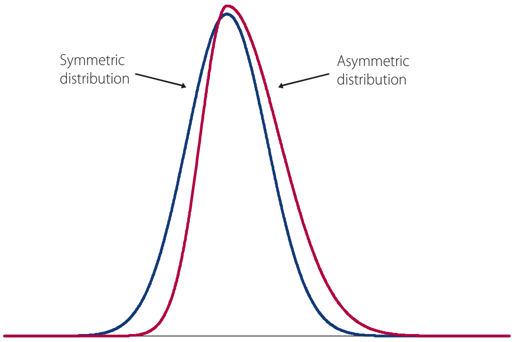
The **prediction uncertainty** gives the probability that actual inflation will differ from the forecast. The higher the uncertainty, the higher the growth of probability that the difference between the actual value of inflation and central prediction will be large (Chart 3). The forecast uncertainty is approximated by the variance (σ) of the probability distribution and is estimated by means of the standard deviation of the variance of past prediction errors (RMSE – root mean square error)[4]. There are also other possibilities of how to determine the risk of prediction, for example using an econometric model for the simulation of the inflation development and the subsequent estimation of the risk of inflation. The disadvantage of such a procedure is the risk of error of the model itself, in which the simulations are performed. Based on its simplicity, as well as the fact that the variance of inflation prediction errors does not depend on the model applied, it has been decided to use it as the method to measure prediction uncertainty.

Prediction asymmetry defines how uncertainty is distributed below and above the central prediction, i.e. whether there is higher probability that inflation will be higher or lower than determined by the medium-term forecast (Chart 4). Asymmetry can be set in two equivalent ways – using lower probability or using an alternative prediction. The following properties, which the

3 A simple and elegant way to introduce skewness is to define a two-piece normal distribution (TPN), which is composed of two normal distributions with the same mean value and different variances. More on this can be found in a separate technical note at the end of this article.

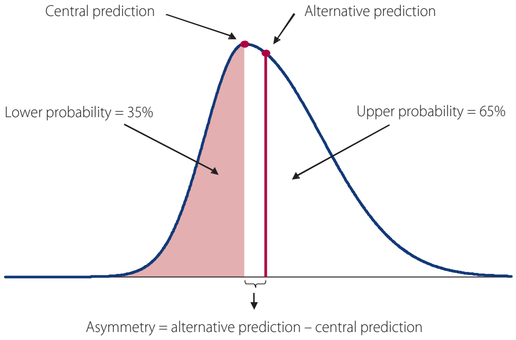
4 Because the fan chart is set up 8 quarters ahead of the period of creation of the current medium-term forecast, the standard deviation has to be estimated for each probability distribution in the given quarter. For the first 4 future quarters, the standard deviation is always estimated with the corresponding time lag, i.e. the estimation of the deviation for the 1st future quarter is computed from the standard deviations of errors of past predictions, which were performed for a period of 3 months; the estimate for the 2nd quarter is computed from the deviations of the errors of past predictions for a 6 month period etc. The remaining estimates for the 5th to 8th quarter are approximated by a logarithmic trend. This procedure has been selected, because the highest degree of uncertainty occurs in the first four quarters. For the remaining periods, it is assumed that behavioral bonds considerably decrease the growth of uncertainty.

Chart 4 Density of a normal distribution



Source: NBS.

Chart 5 Density of a skewed normal distribution – alternative prediction and lower probability



Source: NBS.

desired estimate should have, have been decisive for the concrete selection of one of the ways. First of all, the parameter should be comprehensible, because the setting of asymmetry requires primarily an expert estimate on the distribution of risk in the future. The second property needed for the estimate is the relative simplicity of the transformation of information from the parameter to the asymmetry of the probability distribution. In both cases, the data has to be gained from several respondents by means of a questionnaire[5].

For the lower probability, it is necessary to know how to determine the probability, with which inflation will be below or above the central prediction. In the case of symmetry, the risk is distributed equally below and above the central prediction (meaning that in 50% of the cases, inflation will be below the central prediction and in 50% of the cases above the central prediction). In the case of asymmetry, uncertainty is not distributed in an equal way, e.g. it is possible that 35% of the inflation realizations are below the central prediction and 65% are above it. Hence the lower probability is 35%. This way of determining asymmetry is, at first sight, less clear, because does not provide a particular respondent a direct idea of the impact or level of inflation in the event of materialization of the risk, i.e. the respondent has to know the answer to the question: "What is the probability of inflation being above or below the central prediction of inflation provided that there is a shock not included in the central prediction?"

A more appropriate way of measuring asymmetry is to use an alternative prediction, which is actually an inflation forecast based on an asymmetric shock[6]. For the respondent, the alternative prediction thus represents the answer to the question: "How the inflation forecast will change provided that there is a shock not included in the central prediction?". This method is more transparent and easier to identify for the respondent, because the alternative prediction can be determined by means of an expert estimate or by simulation from a model. The technical simplicity of this method for the subsequent derivation of the particular shape of the asymmetric normal distribution has to be also pointed out.

Asymmetry expressed by means of an alternative prediction can be translated to a lower probability and vice versa, asymmetry expressed by means of lower probability can be translated to an asymmetry characterized by the alternative prediction (Chart 5). It is possible to create a conversion table, which will translate the individual percentages of the lower probability at a fixed uncertainty to a better readable estimate – namely to an alternative prediction.

Evaluation of the prediction risks MTF-2009Q2

To determine possible risks of inflation prediction, a questionnaire has been set up. It serves for getting an expert estimate of the distribution of risk (asymmetries) of prediction of future inflation in the medium run. It contains selected indicators (from domestic and foreign environment), which directly or indirectly influence inflation. Average impacts of inflation in individual indicators are calculated from the filled-in questionnaires. The impacts for the individual years are subsequently

5 In the case of the NBS, these are primarily the employees of the monetary policy department (involved directly or indirectly in the preparation of the forecast).

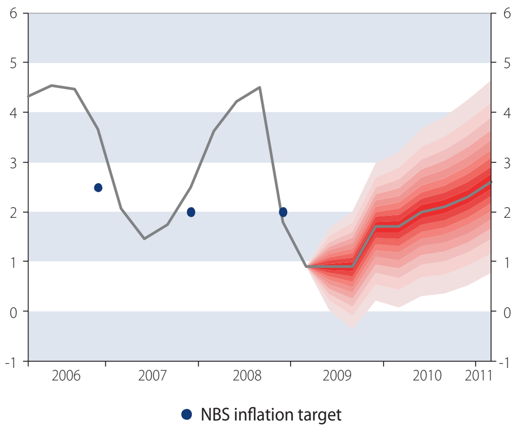
6 In terms of mathematics, the alternative prediction is the mean value in this case.

Table 1 Central prediction and alternative prediction

	III. 2009	VI. 2009	IX. 2009	XII. 2009	III. 2010	VI. 2010	IX. 2010	XII. 2010	III. 2011
Central prediction	0.90	0.90	0.90	1.70	1.70	2.00	2.10	2.30	2.60
Alternative prediction	0.90	0.85	0.82	1.60	1.64	1.99	2.14	2.38	2.71

Source: NBS.

Chart 6 Asymmetric risk distribution, fan chart – Medium-term HICP inflation forecast MTF-2009Q2 (%)

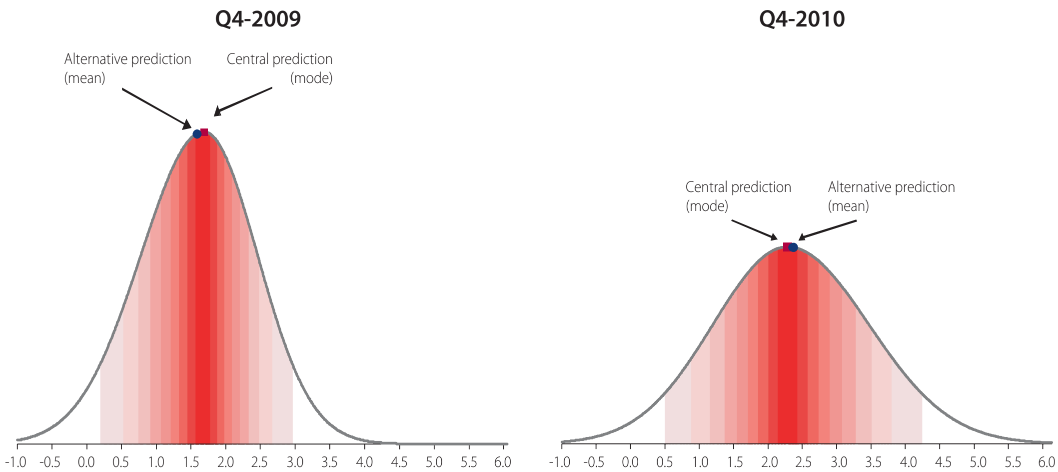


Source: NBS.

added up and they thereby make up the resulting impact (i.e. how much the predicted baseline inflation would change, if all these risks would materialize). Information obtained this way is used directly to construct an asymmetric fan chart using an alternative prediction. In addition, the questionnaire evaluation itself will provide much information for a deeper analysis of the risks of individual factors influencing inflation.

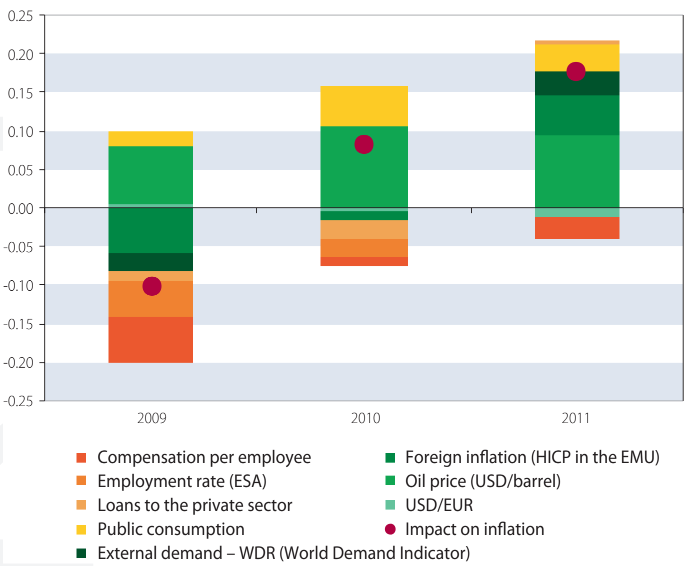
The outputs of the symmetric and asymmetric risk distribution can take the form of a (numerical) table and a graphical form. While the numerical form primarily represents values obtained by the questionnaire survey, the graphical form provides a vivid idea of the distribution of risks around the central prediction, emphasis being put on the fact that the baseline prediction is the most probable level of inflation.

Chart 7 A cross-section of the fan chart for the Q4-2009 and Q4-2010



Source: NBS.

Chart 8 Impact on inflation by risk factors (%)



Source: NBS.

It can be assumed based on the evaluation of the inflation prediction questionnaire from MTF-2009Q that the inflation forecast is realistic, when no higher risks have been identified in any of the selected factors. All risks can be considered low or even negligible with a minimum impact on inflation. Overall, at the end of 2009, the risk factors could increase inflation slightly (by 0.10 p.p.), and in 2010 and 2011 by contrast there are moderate pro-growth risks (0.08 p.p. at the end of 2010 and 0.18 p.p. at the end of 2011).

Table 2 describes the differences in the probabilities of reaching inflation values in the case of a baseline prediction and in the case that prediction risks, i.e. the risk scenario for defined cases, would materialize. The values, as well as the charts, confirm a higher probability of a decrease in inflation as compared to the central prediction at the end of 2009 and of a growth at the end of 2010.

In 2009, the most significant, albeit low, risks are considered to be the employment, employee

Table 2 Probabilities of reaching selected inflation values

MTF-2009Q2	Q409		Q410	
Distribution of risks	Asymmetric risk	Symmetric risk	Asymmetric risk	Symmetric risk
Probability of exceeding 2%[7]	32%	36%	63%	60%
90% confidence interval	(0.22; 2.97)	(0.32; 3.08)	(0.52; 4.25)	(0.44; 4.16)
Risk of inflation decreasing below the central prediction	54%	50%	48%	50%
Deflation risk	3.2%	2.1%	1.6%	2.1%

Source: NBS.

7 Reference value ECB.

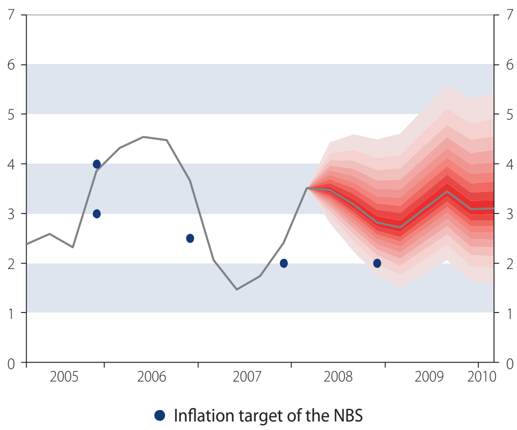
compensations and foreign inflation with a damping impact on inflation. The oil price represents the highest pro-growth risk among all selected factors. In addition to the oil prices, public consumption is also a pro-inflation factor. The two factors together influence the disinflation influence of the other factors only to a low extent.

In 2010, the oil price and public consumption should be the greatest, albeit still low, pro-inflation risks. Their risk level will outweigh all other disinflation factors in the given year. The result is an overall pro-growth risk.

In 2011, a considerable majority of factors will be already pro-growth factors with respect to inflation. The highest risk for inflation growth might consist in the oil price and in foreign inflation.

Because at the first sight, the low risks in MTF-2009Q2 do not provide a vivid idea of asymmetry, we are adding Chart 9 from the Medium-term forecast MTF-2008Q2, where more considerable risks (also in terms of graphical representation) towards higher inflation have been identified, which is shown by a broader zone above the baseline of the prediction. This means that in the prediction horizon, there has been a higher probability that inflation will reach rather high than low values. The expected risks have ultimately materialized in that HICP inflation was estimated to be 2.8% at the end of 2008, but it reached a value of 3.5% in reality.

Chart 9 Asymmetric risk distribution, fan chart – Medium-term HICP inflation forecast MTF-2008Q2



Source: NBS.

Conclusion

In conclusion, it can be said that the extension of the probability distribution by adding asymmetry provides the National Bank of Slovakia with a step forward in terms of transparency and formation of expectations for the professional public. It provides a more vivid and more real picture of the distribution of risks, enables to include ex-post new information, not available at the time of setting up the prediction, in the prediction. In addition to a graphical representation, it also provides the possibility to express the probability of reaching inflation within a certain interval or above or below a certain value, if possible risks materialized. Last but not least, it has to be pointed out that the said procedure with examples can be also applied to other macroeconomic variables, which would also contribute to a more vivid idea of the uncertainty and distribution of prediction risks.

Based on the said advantages, demonstrated using illustrative and concrete examples, the National Bank of Slovakia has decided to publish an asymmetric fan chart as part of its medium-term prediction from the second half of 2009 onwards.

Technical note

Derivation of the TPN distribution

This part describes the method of derivation of a TPN (two-piece normal) distribution from the estimated parameters. In our methodology, the asymmetric distribution has been created by putting together two symmetric normal distributions with the mean value μ and the variances σ_1 or σ_2. To get

$$\int_{-\infty}^{\infty} f(x)dx = 1, \quad (1)$$

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both density functions must be reweighted in an appropriate way. Thus the random variable X has a continuous TPN distribution, if its density function takes the shape (2), where μ does not have to be the mean value now anymore.

$$f(x)=\sqrt{\frac{2}{\pi}}\cdot\frac{1}{\sigma_1+\sigma_2}\cdot\exp\left[-\frac{(x-\mu)^2}{2\sigma_1^2}\right],\quad if\ \ x<\mu$$
$$f(x)=\sqrt{\frac{2}{\pi}}\cdot\frac{1}{\sigma_1+\sigma_2}\cdot\exp\left[-\frac{(x-\mu)^2}{2\sigma_2^2}\right],\quad if\ \ x\geq\mu \qquad (2)$$

In general, it is more correct to consider μ to be the mode, according to the definitional formula for the mode of a continuous distribution (3) σ_1, σ_2 are the remaining parameters of the TPN distribution, which contain information both on the variance and on asymmetry.

$$\left.\frac{df(x)}{dx}\right|_{x=\mu}=0 \qquad (3)$$

A special case is the absence of asymmetry, if $\sigma_1 = \sigma_2$. Then the TPN distribution transforms into a "classical" normal distribution with a variance $\sigma_1 = \sigma_2$ and the mode and mean value of the distribution are identical. In the case of asymmetry, this is not the case and the mean value, mode and median are not equal, as shown by the chart.

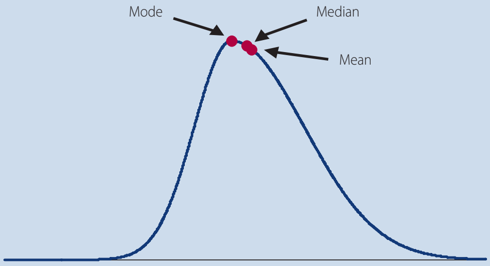
If the upper risk dominates, it will hold mean > median > mode, and in the case of lower risk, by contrast, the sequence will be reversed (mean < median < mode). The mean value and the 2nd and 3rd central moment of the TPN distribution can be written according to (4)[8]:

$$E(x)=\mu+\sqrt{\frac{2}{\pi}}\cdot(\sigma_2-\sigma_1) \qquad (4)$$
$$Var(x)=\left(1-\frac{2}{\pi}\right)\cdot(\sigma_2-\sigma_1)^2+\sigma_1\sigma_2$$
$$T(x)=\sqrt{\frac{2}{\pi}}\cdot(\sigma_2-\sigma_1)\left[\left(\frac{4}{\pi}-1\right)(\sigma_2-\sigma_1)^2+\sigma_1\sigma_2\right]$$

Because T(x) is proportional to E(x) – μ, – this part also defining its sign (T(x) can be also written as (E(x) – μ).k, k>0), asymmetry can be also determined from the difference E(x) – μ. This way of characterization of asymmetry has been used in the following, because it is easier to work with than with the third central moment. For the construction of the distribution, it is necessary to determine the values of μ, σ_1, σ_2 from the system

$$\begin{array}{l} E(x)-\mu=\sqrt{\frac{2}{\pi}}\cdot(\sigma_2-\sigma_1) \\ Var(x)=\left(1-\frac{2}{\pi}\right)\cdot(\sigma_2-\sigma_1)^2+\sigma_1\sigma_2 \end{array},\quad \sigma_1,\sigma_2>0 \qquad (5)$$

Location of the statistics in case of asymmetric distribution



Source: NBS.

μ is estimated as the central prediction. Provided that the central prediction is the mode, the mean value in a discrete case is the weighted probability average of all possible realizations of inflation for the given alternative shock scenario. The mean value is estimated by means of an alternative prediction. There are two possibilities for setting up the alternative prediction:

1. The alternative prediction is the only estimate of inflation in the case of the shock variant. In such a case, the prediction also represents an estimate of the mean value of the distribution. (An example is a single expert estimate of the alternative prediction or a consensual estimate.)
2. For a given type of shock, there can be several possible estimates of impacts on inflation (several expert estimates from the questionnaire). In such a case, the alternative prediction is determined as the weighted probability average of all predictions. It is to be weighed whether all probability weights will be the same or higher probability is assumed for some inflation realizations. An alternative prediction determined using this procedure also represents an estimate of the mean value of the distribution.

The variance of the distribution is approximated by the variance of past prediction errors, as has been described in the part on asymmetric risk distribution of the prediction. The system's solution is then a quadratic equation (5), from which it is possible to calculate a solution for σ_1. The common condition for the solvability of an equation in real numbers and for strict non-negativity of the solutions yields the formula (6).

$$\sigma_1^2+\sigma_1\cdot\sqrt{\frac{\pi}{2}}(E(x)-\mu)+k\frac{\pi}{2}(E(x)-\mu)^2- -Var(x)=0,\ \text{where}\ k=1-\frac{2}{\pi} \qquad (6)$$

$$|E(x)-\mu|\cdot\sqrt{\frac{\pi}{2}-1}<\sqrt{Var(x)} \qquad (7)$$

8 John, S. (1982): „The three Parameter Two-Piece Normal Family of Distributions and Its Fitting", Communications in Statistics – Theory and Methods, 11(8), 879-885.

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Strict non-negativity is required both for the solutions of the equations and for the solution of the system (5). Condition (7) means that the parameters for the construction of the TPN distribution cannot be chosen arbitrarily, but, on the contrary, it is necessary that this formula comply with in the estimation. Equivalently, the condition can be understood in such a way that under a fixed variance there is an open interval (E_{min}, E_{max}), within which the estimate of the mean value by means of an alternative prediction must fall. In practice, however, an even somewhat narrower interval of alternative predictions has to be chosen. E_{min} or E_{max} are determined so that they to a lower probability of 65% or 35%, respectively. The restriction is done because a too large asymmetry makes the central prediction untrustworthy.

The asymmetry formula can be derived in a similar way by means of lower probability. If we understand $P(x \le X)$ as a cumulative distribution function of a TPN, the lower probability can be understood as $P(x \le \mu)$. If lower probability is denoted Z, it can be expressed:

$$z = \int_{-\infty}^{\mu} f(x)dx = \frac{\sigma_1}{\sigma_1 + \sigma_2} \qquad (8)$$

If Z is obtained according to an expert estimate, both relevant standard deviations for the construction of the TPN distribution can be derived using the variance formula under (5) and the Z formula under (8). After setting σ_1, σ_2 this way, a corresponding alternative prediction from the mean value formula under (5) can be assigned to each lower probability. If, by contrast, the standard deviations are set by means of an alternative prediction, then it is possible to assign to it the corresponding lower probability from equation (8).

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