NÁRODNÁ BANKA SLOVENSKA
EUROSYSTÉM

Testing for linear and Markov switching DSGE models

Marián Vávra

Working Paper

3/2013

www.nbs.sk
Imricha Karvaša 1
813 25 Bratislva

reserarch@nbs.sk

December 2013

ISSN 1337-5830

The views and results presented in this paper are those of the authors and do not necessarily represent the official opinions of the National Bank of Slovakia.

NÁRODNÁ BANKA SLOVENSKA
EUROSYSTÉM

Testing for linear and Markov switching DSGE models[1]

Working paper NBS

Marián Vávra[2]

Abstract

This paper addresses the issue related to testing for non-linearity in economic models using new principal component based multivariate non-linearity tests. Monte Carlo results suggest that the new multivariate tests have good size and power properties even in small samples usually available in practice. The empirical results indicate that the use of linear economic models is unsuitable for policy recommendations.

JEL classification: C12, C15, C32
Key words: DSGE model, Markov-switching, Monte Carlo method, principal components, non-linearity testing

[1] I would like to thank Professor Ron Smith from the University of London and Professor Adrian Pagan from the University of Sydney for very useful comments and many suggestions. All remaining errors are only mine.
[2] Marián Vávra, Advisor to the Governor and Research Department of the NBS, marian.vavra@nbs.sk.

1. Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a standard tool in macroeconomics both for forecasting and policy analysis simulations in the last decade. Two versions of DSGE models are predominantly used in the literature: (a) linear DSGE (L-DSGE) models, see Erceg et al. (2000), Christiano et al. (2001), Amato and Laubach (2003), Dib (2003), Smets and Wouters (2003) or Adolfson et al. (2007), Altig et al. (2011), among others; and (b) Markov-switching DSGE (MS-DSGE) models[3], see Davig and Leeper (2005), Sims and Zha (2006), Davig (2007), Liu et al. (2009), Chen and MacDonald (2012), Bianchi (2013), or Foerster (2013), among others. The main advantage of MS-DSGE models is that they can capture empirically observed phenomena without breaking theoretical concepts or imposing unrealistic assumptions. For example, Davig (2007) examines the implications of changing the slope of the Phillips curve for optimal discretionary monetary policy. He shows that significant instability in the inflation rate if the slope parameter of the Phillips curve is subject to Markov switching. Optimal monetary policy is computed subject to the Markov-switching Phillips curve under both ad-hoc and utility-based welfare criteria. The utility-based criterion instructs monetary policy to disregard the switching effect of the Phillips curve and keeps the interest rate constant across regimes. This stands in contrast to the standard rules, which advises monetary policy to change the rates according, regardless of the slope parameter. Note that this type of issues cannot be addressed in linear DSGE models. The main disadvantage of MS-DSGE models is that individual modelling steps (e.g. estimation) are computationally intensive as compared to linear counterparts.[4]

Therefore, the ultimate question for applied researchers is which DSGE model should be used in practice: a linear or a Markov-switching DSGE model? This question is of much practical importance since any model misspecification may lead to misleading inference (e.g. hypothesis testing, impulse-response functions, forecast error variance decompositions, point and interval forecasts, etc.) and serious mistakes in economic policy (e.g. setting the interest rate, etc.). Until now, the choice about the functional form of DSGE models has been made based on an ad-hoc decision. The main task of this article is to fill the gap in the literature and assess formal test statistics, which can be used to test for linear and Markov-switching DSGE models.

[3]Note that there exists other forms of non-linear DSGE models based on a higher-order Taylor approximation of equlibrium conditions, see Schmitt-Grohe and Uribe (2004) or Rudebusch and Swanson (2008), among others. However, MS-DSGE models have become very popular and widely used in empirical macroeconomics. Therefore, we restrict our attention to this particular class of non-linear DSGE models. However, it is worth noting that the test stattistics discussed later on in the paper might have power against other non-linear DSGE models as well.

[4]For instance, Psaradakis and Sola (1998) show that the MLE method of Markov-switching autoregressive models performs poorly in finite sample. Therefore, some jackknife- or bootstrap-based bias correction method is necessary to implement. This method is, however, computationally intensive in multivariate models. Another problem is to find non-restrictive and easy-to-check determinacy conditions for multivariate Markov-switching type models, compare the results in Cho (2011), based on a concept of mean-square stability, and Barthélemy and Marx (2013), based on a concept of bounded stability.

There are two contributions of this paper to applied economic modelling. First, it is shown, using Monte Carlo experiments based on simplified, yet realistic, DSGE models, that the (principal component based) multivariate TSAY and ARCH tests do exhibit good size properties against linear VAR/DSGE models and power properties against Markov-switching VAR/DSGE models even in small samples usually observed in empirical macroeconomics. Second, strong empirical evidence against linear VAR/DSGE models is found for US economic variables.

The paper is organized as follows. A brief description of principal component based multivariate non-linearity TSAY and ARCH tests is given in Section 2. Monte Carlo setup and results are presented in Sections 3 and 4. An empirical application is provided in Section 5.

2. MULTIVARIATE NON-LINEARITY TESTS

2.1 THE NULL AND ALTERNATIVE HYPOTHESIS

Before we proceed to a testing procedure, we state assumptions about stochastic processes under consideration.

Uhlig (1995) shows that a standard linear DSGE model can be written into the following state-space representation

$$\mathbf{y}_t = \mathbf{A}(\boldsymbol{\omega})\mathbb{E}_t(\mathbf{y}_{t+1}) + \mathbf{B}(\boldsymbol{\omega})\mathbf{y}_{t-1} + \mathbf{C}(\boldsymbol{\omega})\mathbf{x}_t, \tag{1a}$$

$$\mathbf{x}_t = \mathbf{R}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t, \tag{1b}$$

where $\mathbf{y}_t$ is a vector of dependent variables, $\mathbf{x}_t$ a vector of exogenous variables (shocks), and $\mathbf{A}(\boldsymbol{\omega})$, $\mathbf{B}(\boldsymbol{\omega})$, $\mathbf{C}(\boldsymbol{\omega})$, $\mathbf{R}$ are matrices of (structural) parameters and $\boldsymbol{\omega}$ is a vector of structural (deep) parameters.[5] It can be shown that under mild conditions, (1) can be further simplified into the form of a VAR(2) model given by

$$\mathbf{y}_t = \boldsymbol{\Phi}_1(\boldsymbol{\omega})\mathbf{y}_{t-1} + \boldsymbol{\Phi}_2(\boldsymbol{\omega})\mathbf{y}_{t-2} + \boldsymbol{\Theta}(\boldsymbol{\omega})\boldsymbol{\epsilon}_t, \tag{2}$$

where $\boldsymbol{\Phi}_1(\boldsymbol{\omega})$, $\boldsymbol{\Phi}_2(\boldsymbol{\omega})$ and $\boldsymbol{\Theta}(\boldsymbol{\omega})$ denote reduced-form parameter matrices. Therefore, without loss of generality, a VAR(P) model can be consider under the null hypothesis. The null hypothesis is stated in the following assumption.[6]

[5] For instance, $\boldsymbol{\omega}$ contains parameters such as habit formation, risk aversion, price indexation, etc.

[6] It is worth noting that not all economic model can be written into a VAR representation. We do not consider this type of models in this paper. However, it is possible to modify the proposed testing procedure to capture this type of models as well.

Assumption 1 *Let us assume the following stationary real-valued finite-order linear VAR model under the null hypothesis*

$$\mathbf{y}_t = \boldsymbol{\xi}_0 + \sum_{i=1}^{P} \boldsymbol{\xi}_i \mathbf{y}_{t-i} + \boldsymbol{\Sigma}\mathbf{a}_t, \tag{3}$$

where $\mathbf{y}_t$ *denotes a* $(k \times 1)$ *vector,* $\{\mathbf{a}_t : t \in \mathbb{Z}\}$ *is a sequence of multivariate WN(*$\mathbf{0}$,$\mathbf{I}$*) such that* $\mathbb{E}\|\mathbf{a}_t\|^4 < \infty$*. Let* $\boldsymbol{\beta} = (\boldsymbol{\xi}_0', \mathrm{vec}(\boldsymbol{\xi}_1)', \ldots, \mathrm{vec}(\boldsymbol{\xi}_p)')'$ *be a* $(k^2P + k \times 1)$ *parameter vector, which is assumed to lie in the interior of the parameter space given by*

$$\mathcal{B} = \{\boldsymbol{\beta} \in \mathbb{R}^{k^2P+k} : \det(\mathbf{I} - \sum_{i=1}^{P} \boldsymbol{\xi}_i z^i) \neq 0 \quad \textit{for all} \quad |z| \leq 1\}.$$

□

The assumption ensures that a given linear process is stationary, parameters do not lie on the boundary, and all moment conditions are satisfied. These conditions are sufficient to ensure consistency of the estimated parameters in $\boldsymbol{\beta}$, the estimated residuals, and subsequently, the non-linearity test statistics.

Special attention is paid to a Markov-switching DSGE model under the alternative hypothesis only. Cho (2011) shows that a MS-DSGE model can be formally written as follows

$$\mathbf{y}_t = \mathbf{A}(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbb{E}_t(\mathbf{y}_{t+1}) + \mathbf{B}(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{y}_{t-1} + \mathbf{C}(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{x}_t, \tag{4a}$$

$$\mathbf{x}_t = \mathbf{R}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t, \tag{4b}$$

where $\mathbf{y}_t$ is a vector of dependent variables, $\mathbf{x}_t$ a vector of exogenous variables shocks, and $\mathbf{A}(\boldsymbol{\omega}_{S_t,S_{t-1}})$, $\mathbf{B}(\boldsymbol{\omega}_{S_t,S_{t-1}})$, $\mathbf{C}(\boldsymbol{\omega}_{S_t,S_{t-1}})$ are matrices of structural (deep) parameters subject to Markov switching, whereas $\mathbf{R}$ is a matrix of fixed parameters.[7] S_t denotes the first-order time-homogenous hidden Markov chain defined on a discrete space with constant (irreducible and aperiodic) transition probabilities. Cho (2011) points out that the closed-form solution of the model, if it exists, can be written into the form of a state-space representation, and, subsequently, into the form of a MS-VAR(2) model given by

$$\mathbf{y}_t = \boldsymbol{\Phi}_1(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{y}_{t-1} + \boldsymbol{\Phi}_2(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{y}_{t-2} + \boldsymbol{\Theta}(\boldsymbol{\omega}_{S_t,S_{t-1}})\boldsymbol{\epsilon}_t, \tag{5}$$

where $\boldsymbol{\Phi}_1(\boldsymbol{\omega}_{S_t,S_{t-1}})$, $\boldsymbol{\Phi}_2(\boldsymbol{\omega}_{S_t,S_{t-1}})$ and $\boldsymbol{\Theta}(\boldsymbol{\omega}_{S_t,S_{t-1}})$ denote the regime-dependent reduced-form parameter matrices. Therefore, without loss of generality, a Markov-switching VAR(P) model

[7] Note that the fact that the vector of deep parameters $\boldsymbol{\omega}$ is effected by both S_t and S_{t-1} is caused by intertemporal optimization of agents in the model.

can be consider under the alternative hypothesis without loss of generality. The alternative hypothesis is stated in the following assumption.[8]

Assumption 2 *Let us assume the following stationary real-valued finite-order Markov switching VAR model under the alternative hypothesis*

$$\mathbf{y}_t = \boldsymbol{\xi}_0(s_t) + \sum_{i=1}^{P} \boldsymbol{\xi}_i(s_t)\mathbf{y}_{t-i} + \boldsymbol{\Sigma}(s_t)\mathbf{a}_t, \tag{6}$$

where $\mathbf{y}_t$ denotes a $(k \times 1)$ vector, $\{\mathbf{a}_t : t \in \mathbb{Z}\}$ is a sequence of multivariate WN($\mathbf{0}$,$\mathbf{I}$) innovations such that $\mathbb{E}\|\mathbf{a}_t\|^4 < \infty$ and s_t denotes the first-order time-homogenous hidden Markov chain defined on a discrete space $\{1, \ldots, q\}$ with constant (irreducible and aperiodic) transition probabilities $\mathbf{P} = [p_{ij}]$, for $i, j \in \{1, \ldots, q\}$. Let $\boldsymbol{\beta}(s_t) = (\boldsymbol{\xi}_0(s_t)', \mathrm{vec}(\boldsymbol{\xi}_1(s_t))', \ldots, \mathrm{vec}(\boldsymbol{\xi}_p(s_t))')'$ be a $(k^2P + k \times 1)$ parameter vector for $s_t \in \{1, \ldots, q\}$, and $\boldsymbol{\beta} = (\boldsymbol{\beta}(1)', \ldots, \boldsymbol{\beta}(q)')'$, which is assumed to lie in the interior of the parameter space given by

$$\mathcal{B} = \{\boldsymbol{\beta} \in \mathbb{R}^{q(k^2P+k)} : \rho(\Psi) < 1\},$$

where $\rho(\Psi)$ denotes a spectral radius of the matrix $\Psi = [p_{ji}\mathbf{D}(s_t = j) \otimes \mathbf{D}(s_t = j)]$, for $i, j \in \{1, \ldots, q\}$ and

$$\mathbf{D}(s_t) = \begin{bmatrix} \boldsymbol{\xi}_1(s_t) & \boldsymbol{\xi}_2(s_t) & \cdots & \boldsymbol{\xi}_{P-1}(s_t) & \boldsymbol{\xi}_P(s_t) \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix}.$$

□

Although model builders are particularly interested in non-linearity in the structural (i.e. first-order) equations (e.g. see matrices $\mathbf{A}(\boldsymbol{\omega}_{S_t,S_{t-1}})$, $\mathbf{B}(\boldsymbol{\omega}_{S_t,S_{t-1}})$, $\mathbf{C}(\boldsymbol{\omega}_{S_t,S_{t-1}})$ in (4a)), we focus on testing for non-linearity in the reduced-form models. There is a good reason for doing so. It is not possible to consider the functional form of the model in (1) under the null hypothesis when testing for non-linearity since the $\mathbb{E}_t(\mathbf{y}_{t+1})$ term is not observed and the neglected non-linearity tests (i.e. TSAY and ARCH) are considered in this paper.[9]

[8] For the notational simplicity, Assumption 2 is based on the switching variable $s_t = (S_t, S_{t-1})$, where S_t denotes the first-order time-homogenous hidden Markov chain defined on a discrete space with constant (irreducible and aperiodic) transition probabilities. For example, a two state (regime) Markov chain for S_t implies a four state (regime) Markov chain for $s_t = (S_t, S_{t-1})$.

[9] The author thanks Professor Adrian Pagan from the University of Sydney for bringing my attention to this point.

2.2 Why new multivariate tests?

Two problems immediately arise when testing for non-linearity in multivariate dynamic stochastic processes in practice. First, although there exists many different univariate non-linearity test statistics, there are only a few multivariate tests available in the literature. Main attention is paid to statistical properties of the multivariate TSAY and ARCH tests. Our choice of the statistics is based on the fact that the functional form of the reduced-form model under the alternative hypothesis, see equation (4a)), contains non-linearity in the conditional mean (i.e. $\mathbf{\Phi}_1(\boldsymbol{\omega}_{S_t,S_{t-1}})$ and $\mathbf{\Phi}_2(\boldsymbol{\omega}_{S_t,S_{t-1}})$), which motivates the use of the TSAY test, and conditional volatility (i.e. $\mathbf{\Theta}(\boldsymbol{\omega}_{S_t,S_{t-1}})$), which motivates the use of the ARCH test.

Second, those existing multivariate tests often suffer from the dimensionality and multicollinearity problems. In order to make this point clear, let us consider a multivariate version of the TSAY (MTSAY) test proposed by Harvill and Ray (1999). The test is based on running the following auxiliary equation

$$\hat{\mathbf{a}}_t = \mathbf{b}_0 + \mathbf{B}_1\mathbf{z}_t + \mathbf{B}_2\mathbf{v}_t + \mathbf{u}_t, \tag{7}$$

where $\hat{\mathbf{a}}_t$ is a $(k \times 1)$ vector of residuals from a particular VAR(P) model in (3), $\mathbf{z}_t = (\mathbf{y}'_{t-1}, \ldots, \mathbf{y}'_{t-P})'$ denotes a $(kP \times 1)$ vector of aggregated predetermined variables, $\mathbf{v}_t = \mathrm{vech}(\mathbf{z}_t \otimes \mathbf{z}'_t)$ represents an $(s \times 1)$ vector of predetermined variables consisting of all square- and cross-product elements ($s = kP(kP+1)/2$). $\mathbf{b}_0$ denotes a $(k \times 1)$ vector of constants, $\mathbf{B}_1$ represents a $(k \times kP)$ matrix of parameters, and finally, $\mathbf{B}_2$ is a $(k \times s)$ matrix of parameters. The null hypothesis of linearity of the vector $\mathbf{y}_t$ is given by: $H_0 : \mathbf{B}_2 = \mathbf{0}$ versus $H_1 : \mathbf{B}_2 \neq \mathbf{0}$.

The dimensionality problem: due to the construction of the test, the vector $\mathbf{v}_t = \mathrm{vech}(\mathbf{z}_t \otimes \mathbf{z}'_t)$ contains a large number of cross- and square-terms. As a result, the original MTSAY test requires a large number of observations. It is clear from the dimension of the $\mathbf{v}_t$ vector that, for a given set of k variables and the lag order P of a VAR filter under the null hypothesis, the test requires $T > kP(kP+1)/2$ observations. For example, consider a small model consisting of $k = 10$ economic variables and the moderate lag order $P = 4$ of a VAR model, the original MTSAY test requires $T > 820$ observations, which is infeasible to get in applied macroeconomics.

The multicollinearity problem: terms in the vector $\mathbf{v}_t$ are highly collinear, which increases the degrees of freedom of the limiting distribution of a given test statistic, and thus, the critical values for rejecting the null hypothesis of linearity, but actually does not improve the fit in (7). As a result, multicollinearity in the vector $\mathbf{v}_t$ can reduce the power of the multivariate tests even further.

As explained in Vávra (2013), both drawbacks of the multivariate non-linearity tests can be bypassed using a principal component analysis. The author shows, using extensive Monte Carlo experiments, that the principal-component based multivariate TSAY and ARCH tests do offer a remarkable dimensionality reduction (in average about 70 $\%$) without any systematic power distortion. For this reason, the principal component based multivariate TSAY and ARCH tests are implemented in this paper.[10]

2.3 Multivariate TSAY test

The modified multivariate TSAY test is based on running the following auxiliary equation

$$\hat{\mathbf{a}}_t = \mathbf{c}_0 + \mathbf{C}_1\mathbf{z}_t + \mathbf{C}_2\mathbf{w}_t + \mathbf{u}_t, \tag{8}$$

where $\hat{\mathbf{a}}_t$ is $(k \times 1)$ a vector of residuals from the filter in (3), $\mathbf{c}_0$ is a $(k \times 1)$ vector of constants, $\mathbf{C}_1$ is an appropriate $(k \times kP)$ matrix of coefficients, and $\mathbf{w}_t$ is a $(n \times 1)$ vector of principal components such that $k \leq n \leq s$.[11] The principal components are calculated from the original vector $\mathbf{v}_t = \mathrm{vech}(\mathbf{z}_t \otimes \mathbf{z}_t')$, where $\mathbf{z}_t = (\mathbf{y}_{t-1}', \ldots, \mathbf{y}_{t-P}')'$ denotes a $(kP \times 1)$ vector of aggregated predetermined variables, using the (Pearson) correlation matrix. The null hypothesis about linearity of the vector $\mathbf{y}_t$ is given by: $H_0 : \mathbf{C}_2 = \mathbf{0}$ versus $H_1 : \mathbf{C}_2 \neq \mathbf{0}$. The appropriate LR based test statistic is given by

$$MTSAY = (T - \tau)(\log(|\mathbf{\Sigma}_r|) - \log(|\mathbf{\Sigma}_u|)) \xrightarrow{d} \chi^2(nk), \tag{9}$$

where $|\cdot|$ denotes the determinant of a square matrix, T stands for the sample size, $\tau = (k + n + 1)/2$ is a small sample correction term recommended by Anderson (2003, p. 321-3), where k is the number of variables, and n represents the number of principal components. The author argues that the small sample correction works well, provided that $k^2 + n^2 < T/3$. Note that the model in (8) can be easily estimated by a multivariate LS method, see Lütkepohl (2005, Ch. 3) for details. The proof of the limiting distribution can be found in Anderson (2003, Ch. 8.5).

2.4 Multivariate ARCH test

It can be shown that the original multivariate ARCH test suffers from the same problems as the TSAY test, see Vávra (2013) for details. Therefore, the modified principal component based multivariate ARCH test is considered here as well. The test is based on running the following

[10] It is worth noting that some other tests can be used as well. For example, Professor Adrian Pagan recommended to apply the multivariate RESET tests. It can be concluded from the Monte Carlo results not reported later on in this paper that both multivariate TSAY and RESET tests produce similar results.

[11] In practice, the maximum number of principal components s might be restricted to run the test. For example, we set $s = \min\{[T/2], kP(kP + 1)/2\}$, where $[\cdot]$ denotes the integer part and $kP(kP + 1)/2$ the number of orginal additional variables in the vector $\mathbf{v}_t$.

auxiliary equation

$$\mathrm{diag}(\hat{\mathbf{a}}_t \otimes \hat{\mathbf{a}}_t') = \mathbf{c}_0 + \mathbf{C}_1\mathbf{w}_t + \mathbf{u}_t, \tag{10}$$

where $\mathrm{diag}(\hat{\mathbf{a}}_t \otimes \hat{\mathbf{a}}_t')$ is a $(k \times 1)$ vector of diagonal elements, $\mathbf{c}_0$ is a $(k \times 1)$ vector of constants, $\mathbf{C}_1$ is an appropriate $(k \times n)$ matrix of coefficients, and $\mathbf{w}_t$ is an $(n \times 1)$ vector of principal components, such that $k \leq n \leq s$.[12] The principal components are calculated using the (Pearson) correlation matrix of the original vector $\mathbf{v}_t = (\mathrm{vech}(\hat{\mathbf{a}}_{t-1} \otimes \hat{\mathbf{a}}_{t-1}')', \ldots, \mathrm{vech}(\hat{\mathbf{a}}_{t-Q} \otimes \hat{\mathbf{a}}_{t-Q}')')'$ denotes an $(s \times 1)$ vector of all the predetermined variables of the test. The null hypothesis of homoscedasticity of the vector $\mathbf{a}_t$, is given by: $H_0 : \mathbf{C}_1 = \mathbf{0}$ versus $H_1 : \mathbf{C}_1 \neq \mathbf{0}$. The appropriate LR-based test statistic is given by

$$MARCH = (T - \tau)(\log(|\hat{\mathbf{\Sigma}}_r|) - \log(|\hat{\mathbf{\Sigma}}_u|)) \xrightarrow{d} \chi^2(nk), \tag{11}$$

where $|\cdot|$ denotes the determinant of a square matrix, $\hat{\mathbf{\Sigma}}_r$ and $\hat{\mathbf{\Sigma}}_u$ represent the estimated variance-covariance matrix of the restricted model, unrestricted model respectively, T is the sample size, $\tau = (k + n + 1)/2$ is a small sample correction term recommended by Anderson (2003, p. 321-3), where k is the number of variables, and n represents the number of principal components. He argues that the small sample correction works well, provided that $k^2 + n^2 < T/3$. Note that the model in (10) can be easily estimated by a multivariate LS method, see Lütkepohl (2005, Ch. 3) for details. The proof of the limiting distribution can be found in Anderson (2003, Ch. 8.5).

It is worth noting that our specification of the multivariate tests, based on the LR principle, is motivated by Harvill and Ray (1999), who consider the F-test based on the Wilks lambda statistic. It can be argued (e.g. Godfrey (1988, Ch. 2)) that for linear regression models such as (8) and (10), the LR-based tests are slightly more powerful as compared to the LM-based tests. In addition, both the LR and LM-based tests are computed in the same way (using the auxiliary equations) in this particular case and have the same limiting distribution, see Davidson and MacKinnon (1999, p. 423–428).[13] Although the LM-based test statistics are slightly more preferred, it is important to point out that there are many other applications of the LR tests in the multivariate time series literature, see Hamilton (1994, p. 296-298, 648-650) for additional examples.

[12] The maximum number of principal components s might be restricted as in the previous case, see Footnote 9

[13] The LM-based test for multivariate systems is discussed in Deschamps (1993). Monte Carlo comparison of both LM and LR-based tests for multivariate systems can be found in Deschamps (1996).

2.5 PRINCIPAL COMPONENT ANALYSIS

Although a principal component analysis can reduce, or even completely eliminate, the dimensionality problem, there is still an ultimate question of how many components to retain. Therefore, some comments on the basics about principal component analysis are in order. A principal component analysis (PCA) is concerned with explaining the variance-covariance/correlation structure of the original variables by a few components, which are linear combinations of the original variables. It is important to point out that there is no one-to-one mapping between the roots calculated from the variance-covariance matrix and the correlation matrix. A problem is that, unlike the correlation matrix, the variance-covariance matrix is not scale invariant and hence, neither the calculated roots. Therefore, comfortable or not, the use of the correlation matrix is often recommended, especially for heterogenous data sets and/or indicators originally measured in different units, see Jackson (1991, p. 64–65) for details. For this reason, the correlation matrix is used in this paper unless otherwise stated. One of the main advantages of using PCA is that the calculated principal components are uncorrelated linear combinations of the original variables due to orthogonality of the estimated eigenvectors. The advantage of this feature is that principal components eliminate multicollinearity from a testing procedure. Formally, the principal components are defined as follows

$$w_{jt} = \mathbf{e}_j' \mathbf{v}_t, \quad \text{for} \quad j = 1, \ldots, s, \quad t = 1, \ldots, T, \tag{12}$$

where w_{jt} is the jth-principal component at time t, $\mathbf{e}_j$ is a particular eigenvector associated with the eigenvalue λ_j estimated from the variance-covariance or correlation matrix. For instance, $\mathbf{v}_t$ takes the following form for the MTSAY test based on a VAR(P) filter

$$\mathbf{v}_t = \text{vech}(\mathbf{z}_t \otimes \mathbf{z}_t'),$$

where $\mathbf{z}_t = (\mathbf{y}_{t-1}', \ldots, \mathbf{y}_{t-P}')'$ is a vector of predetermined variables. Obviously, the vector $\mathbf{v}_t$ for the MARCH test is defined in a similar way, see Section 2.4.

The number of principal components to retain is determined by the so called stopping rules. These can split into four basic categories: (i) purely statistical rules (e.g. a Bartlett test); (ii) graphical rules (e.g. a scree plot); (iii) rule-of-thumb stopping rules; and finally (iv) bootstrap-based rules. The interested reader is referred to Peres-Neto et al. (2005) for a comprehensive survey. Among those rules successfully applied in the literature, the following three are implemented in this paper:[14]

1. The information criterion rule: The number of principal components can be determined using an automatic selection procedure based on minimizing an appropriate information

[14] See Vávra (2013) for arguments why not to implement more sophisticated stopping rules in the context of non-linearity testing.

criterion. Blake and Kapetanios (2003) show, using Monte Carlo experiments, that the BIC approach produces superior results as compared to other methods considered in their paper.

2. The variance rule: Another popular way of selecting the number of principal components is to use the first n components attributing $100\eta\%$ of total variance of the original set of variables. The usually recommended proportion of total variance in multivariate analysis is $\eta = 0.9$.

3. The Kaiser rule: This rule is based on the fact that the average root calculated from the correlation matrix is equal to 1. For this reason, the rule suggests to retain all the first eigenvalues larger than 1.

Note that the testing procedure cannot be carried out if no principal component is chosen by the stopping rule. We, therefore, do not consider this case and start with a minimum of k principal components for the multivariate tests.

3. MONTE CARLO SETUP

The statistical properties of the principal component based multivariate non-linearity TSAY and ARCH tests are assessed via Monte Carlo experiments using simple, yet realistic, DSGE models. Each model describes the behaviour of three agents in the economy (i.e. households, firms, and government). The closed-form of the model consists of three equations (variables): the aggregate demand (the Euler equation) equation (output, denoted as y), the aggregate supply (the New Keynesian Phillips curve) equation (the inflation rate, denoted as π), and the monetary policy rule (the interest rate, denoted as r). The size properties of the tests (i.e. the null hypothesis of linearity is true) are assessed using a simple linear DSGE model, denoted as "L", whereas the power properties (i.e. the alternative hypothesis is true) are examined using two Markov-switching DSGE models, denoted as "MS1" and "MS2". In particular, MS1 is a Markov-switching DSGE model where only parameters of the monetary policy rule are subject to a change, whereas MS2 is a model where both the monetary policy parameters and the Phillips curve parameters are subject to a change. Put differently, the MS1 model is a model with dominance of linear structural equations (2 equations are linear 1 equation is non-linear), whereas the MS2 model is a model with dominance of non-linear structural equations (2 equations are non-linear 1 equation is linear). Using this model configuration, one can easily check the robustness of the above mentioned multivariate tests against different degree of non-linearity in the structural equations. A complete description of linear and Markov-switching DSGE models can be found in Appendix A.

Note that variables in DSGE models are assumed to be deviations from steady-states, whereas the growth rates (first differences) are usually used when testing for non-linearity in (nonstationary) economic indicators.[15] In order to check the robustness of the multivariate tests against the data transformation, the following set of data transformations is considered for a given vector of variables (the inflation rate π, output y, the interest rate r) in this paper: (a) "transformation 1" denotes variables expressed as deviations from steady-states – (the inflation gap $\hat{\pi}$, the output gap $\hat{y}$, the interest rate gap $\hat{r}$); (b) "transformation 2" denotes variables expressed as a mix of deviations from steady-states and the growth rates (first differences) – (the inflation gap $\hat{\pi}$, the growth rates of output $\Delta\hat{y}$, the interest rate gap $\hat{r}$); (c) "transformation 3" denotes variables expressed as the growth rates and/or first differences – (the first difference of the inflation rate $\Delta\hat{\pi}$, the growth rate of output $\Delta\hat{y}$, the first difference of the interest rate $\Delta\hat{r}$).[16]

All model innovations are drawn from a multivariate Gaussian distribution with zero means and unit variances.[17] The sample size is set to $T \in \{150, 300\}$ and the number of repetitions is set to $R = 1,000$.[18] The number of principal components is determined using the above defined stopping rules (i.e. BIC, variance, Kaiser). Principal components are calculated from the Pearson correlation matrix. The following notation is used for the principal component based multivariate tests: (i) "M(BIC)" stands for the multivariate test (i.e. ARCH or TSAY) with the automatically selected number of principal components n using the BIC approach; (ii) "M(0.9)" denotes the multivariate test with the number of components determined by the variance rule with the cutoff 0.9; (iii) "M(K)" is the multivariate test with the number of components determined by the Kaiser (root) rule with the cutoff 1.0.

In order to compare the performance of the principal component multivariate tests, the results from the standard univariate TSAY and ARCH test statistics are reported as well. A description of these univariate tests can be found in Tsay (1986) and Engle (1982). Automatic lag order selection procedure proposed in Ng and Perron (2005) is implemented to determine the lags of both AR and VAR models to filter out the conditional mean.

4. MONTE CARLO RESULTS

The Monte Carlo results are presented in Tables 1–2 in Appendix D. The tables present the average rejection frequency of both the univariate and multivariate non-linearity tests. The results

[15]For instance, the output gap is used in DSGE models, whereas the real GDP growth rates are used when testing for non-linearity.

[16]Note that the first difference of a deviation of real output from the steady state (i.e. $\Delta\hat{y}_t$) is equivalent to the growth rate of real output (i.e. $\Delta \log y_t$), provided that the constant growth rate of the steady-state of real output is considered. The same analogy holds for the interest rate and the inflation rate as well.

[17]Our choice is based on arguments in Vávra (2013) who shows that the size and power properties of the principal component based multivariate TSAY and ARCH tests are insensitive to the configuration of the variance-covariance matrix of model innovations.

[18]For example, Smets and Wouters (2007) use the data set spanning the period 1966Q1 – 200Q4 (i.e. 156 observations), Liu and Mumtaz (2011) use the data set spanning the period 1970Q1 – 2009Q1 (i.e. 157 observations), among others.

reveal the following: (i) Both univariate and multivariate ARCH tests have a good size even in small samples (i.e. $T = 150$). Nevertheless, the multivariate TSAY tests are systematically slightly oversized, whereas the univariate TSAY tests are slightly downsized. However, a size distortion is not of the magnitude to disqualify either of the tests from using them. Put differently, both univariate and multivariate non-linearity tests do produce good results, provided that a model is linear; (ii) The power results of both multivariate TSAY and ARCH tests are very good even in small sample (i.e. $T = 150$) and are insensitive to the data transformation. The power of the principal component multivariate tests improves significantly with the number of non-linear structural equations in the model and as the sample size T increases. For example, the average rejection frequency of the principal component based MTSAY (MARCH) test for the MS2 model ranges from 0.58 to 0.87 (from 0.74 to 0.87) in the sample $T = 150$, depending on the stopping rule and the data transformation; (iii) The average number of principal components determined by individual rules ranges from 3 to 9. From this point of view, the Kaiser rule might be preferred because of a very intuitive setup and its simplicity; (iv) The univariate tests do suffer from a high power variation and a systematic power loss as compared to their multivariate counterparts. What is more, the multivariate tests do produce superior results as compared to the univariate tests in almost all cases.

5. Sensitivity analysis

In order to avoid criticism that the Monte Carlo results are based on a particular model parameter configuration, a sensitivity analysis is conducted. A standard sensitivity analysis approach is implemented, which means that one parameter is subject to a change, whereas the other model parameters are kept constant and set to their benchmark values. Special attention is paid to the MS1 model configuration (i.e. only the policy rule parameters are subject to Markov-switching).[19] We consider the following two parameter configurations:

(i) The parameter $\phi_y(S_t) \in \{0.5, 0.1\}$ is kept constant, whereas

$$\phi_p(S_t) \in \{(1.50, 1.0), (1.75, 1.0), (2.00, 1.0), (2.25, 1.0), (2.50, 1.0)\};$$

(ii) The parameter $\phi_p(S_t) \in \{2.0, 1.0\}$ is kept constant, whereas

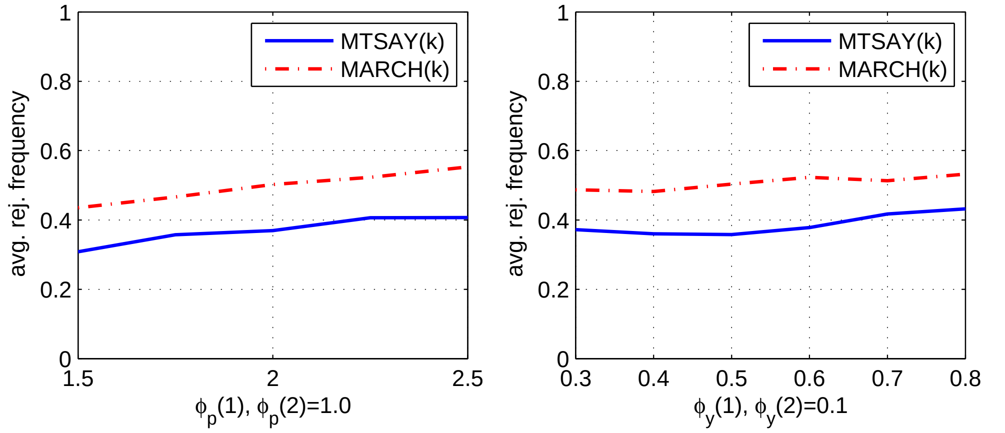
$$\phi_y(S_t) \in \{(0.3, 0.1), (0.4, 0.1), (0.5, 0.1), (0.6, 0.1), (0.7, 0.1), (0.8, 0.1)\}.$$

All other model parameters take their benchmark values as described in Appendix C. We focus on the behaviour of the multivariate tests with the number of principal components determined by the Kaiser stopping rule (i.e. MTSAY(K) and MARCH(K)). The sensitivity results, based on data transformation 2 (see Section 3), are presented in a graphical form in Figure 1. Al-

[19] Similar results are obtain for the MS2 model as well. The results are available from the author upon request.

though the MARCH test is slightly more powerful as compared to the MTSAY test, the power of both multivariate tests is highly robust against variation in the monetary policy rule parameters $\phi_p(S_t)$ and $\phi_y(S_t)$. Interestingly, the rejection frequencies of the multivariate tests are similar for both parameter configurations. For example, the rejection frequency of the MARCH(K) test is around 0.5 for both parameter configurations.

Figure 1: Sensitivity analysis of the multivariate tests: $T = 150$



(a) (b)

Note: "MTSAY(K)" and "MARCH(K)" denote the principal component based multivariate test with the number of components selected automatically by the Kaiser rule.

6. EMPIRICAL ANALYSIS

In this section, the univariate and principal component based multivariate TSAY and ARCH tests are applied to a set of US quarterly economic variables:[20] the growth rate of real GDP (Y), the growth rate of real consumption (C) and the growth rate of real investment (I), the CPI inflation rate (P), the 3M treasure bill rate (R), the growth rate of nominal hourly wage (W) and growth rate of hours worked (L).[21] In order to check the robustness of the results against the size of the model (i.e. the number of economic variables in the model), three different combinations of variables are considered: (1) $\mathbf{y} = (Y, P, R)'$; (2) $\mathbf{y} = (Y, P, R, W, L)'$; (3) $\mathbf{y} = (P, R, W, L, C, I)'$.[22] All variables span the period 1961Q1 to 2010Q4 (i.e. $T = 200$ observations). Following Assumption 1, a linear VAR/DSGE model[23] is considered as an adequate model under the null hypothesis.[24] As explained in Section 2, both TSAY and ARCH test require the specification of lag orders P and Q. The automatic lag order selection procedure recom-

[20] All relevant variables are seasonally adjusted.
[21] Note that the growth rate is defined as the log-difference.
[22] A similar data set is considered in Smets and Wouters (2007).
[23] Only for simplicity, a DSGE model is assumed to be exactly identified, which means that the same number of shocks as the number of dependent variables is considered in this exercise.
[24] A simple AR model is considered for univariate tests.

mended by Ng and Perron (2005) is implemented here. The procedure indicates $P \in \{3, 4\}$ for the multivariate TSAY tests and $Q = 2$ for multivariate ARCH tests, depending on the sub-set of variables. The maximum number of principal components is restricted to $s = T/2 = 100$.

We are interested in answering the following two questions: (i) ”Is a linear VAR/DSGE model adequate for the selected groups of the US economic variables?”; (ii) ”How similar are the results obtained from the univariate and multivariate non-linearity tests when using the selected groups of the US economic variables?”

The results are presented in Table 3. The table presents the estimated p-values of both univariate and multivariate tests for various sub-sets of US variables. The results suggest the following: (i) The null hypothesis of linearity is clearly rejected by almost all multivariate tests at the significance level 0.05, regardless of the stopping rules used to determine the number of components and the group of economic variables. Put differently, a linear VAR/DSGE model is clearly not an adequate representation for any sub-set of US variables. It is also worth pointing out that the degree of rejecting the null hypothesis increases with the number of variables included in the model (compare the results for the first and the last sub-set of US variables); (ii) As in the case of Monte Carlo experiments, the univariate non-linearity tests do produce misleading results, which can be easily illustrated using the last group of variables $\mathbf{y} = (P, R, W, L, C, I)'$. In this case, the null hypothesis of linearity is clearly rejected by all multivariate tests at the significance level 0.05, regardless of the test configuration, whereas only in 3 out of 6 variables when using univariate tests. This is a nice example illustrating how difficult it might be to interpret the results from univariate non-linearity tests in the multivariate context.

7. Conclusion

This paper addresses the issue of testing for linear and Markov-switching (non-linear) DSGE models. Our Monte Carlo results show that univariate tests might not be adequate for testing for non-linearity in economic variables, which are dependent (correlated/co-integrated) in nature. The univariate tests suffer from a serious power distortion, and thus, may easily lead to misleading inference. On the contrary, principal component based multivariate non-linearity TSAY and ARCH tests exhibit good size and power properties for a given set of DSGE models. Empirical results, based on three different sub-sets of US economic variables, indicate that the null hypothesis of linearity is clearly rejected by all principal component based multivariate non-linearity tests. Therefore, it can be concluded that the use of linear DSGE models such as in Smets and Wouters (2007) is in sharp contrast with our findings, and, thus, unsuitable for policy recommendations.

REFERENCES

Adolfson, M., Laséen, S., Lindé, J., Villani, M., 2007. Bayesian estimation of an open economy DSGE model with incomplete pass-through. Journal of International Economics 72 (2), 481–511.

Altig, D., Christiano, L. J., Eichenbaum, M., Linde, J., 2011. Firm-specific capital, nominal rigidities and the business cycle. Review of Economic Dynamics 14 (2), 225–247.

Amato, J. D., Laubach, T., 2003. Estimation and control of an optimization-based model with sticky prices and wages. Journal of Economic Dynamics and Control 27 (7), 1181–1215.

Anderson, T., 2003. An Introduction to Multivariate Statistical Analysis. Wiley.

Barthélemy, J., Marx, M., 2013. Determinacy conditions for markov switching rational expectations models. Unpublished manuscript.

Bianchi, F., 2013. Regime switches, agents' beliefs, and post-world war ii us macroeconomic dynamics. The Review of Economic Studies 80 (2), 463–490.

Blake, A., Kapetanios, G., 2003. A radial basis function artificial neural network test for neglected nonlinearity. Econometrics Journal 6 (2), 357–373.

Chen, X., MacDonald, R., 2012. Realized and optimal monetary policy rules in an estimated markov-switching dsge model of the united kingdom. Journal of Money, Credit and Banking 44 (6), 1091–1116.

Cho, S., 2011. Characterizing Markov-switching rational expectations models. Working Paper.

Christiano, L., Eichenbaum, M., Evans, C., 2001. Nominal rigidities and the dynamic effects of a shock to monetary policy. NBER working paper 8403.

Davidson, R., MacKinnon, J. G., 1999. Econometric Theory and Methods. Oxford University Press.

Davig, T., 2007. Phillips curve instability and optimal monetary policy. Federal Reserve Bank of Kansas City Working Paper Series 4/2007.

Davig, T., Leeper, E., 2005. Generalizing the Taylor principle. National Bureau of Economic Research 11874.

Deschamps, P., 1993. Joint tests for regularity and autocorrelation in allocation systems. Journal of Applied Econometrics 8 (2), 195–211.

Deschamps, P., 1996. Monte carlo methodology for lm and lr autocorrelation tests in multivariate regression. Annales d'Economie et de Statistique, 149–169.

Dib, A., 2003. An estimated canadian dsge model with nominal and real rigidities. Canadian Journal of Economics 36 (4), 949–972.

Engle, R., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50 (4), 987–1007.

Erceg, C., Henderson, D., Levin, A., 2000. Optimal monetary policy with staggered wage and price contracts. Journal of monetary Economics 46 (2), 281–313.

Foerster, A., 2013. Monetary Policy Regime Switches and Macroeconomic Dynamics. Federal Reserve Bank of Kansas City Research Working Papers 4/2013.

Godfrey, L. G., 1988. Misspecification tests in econometrics. Cambridge University Press.

Hamilton, J., 1994. Time Series Analysis. Princeton University Press.

Harvill, J., Ray, B., 1999. Testing for nonlinearity in a vector time series. Biometrika 89 (3), 728–834.

Jackson, J., 1991. A User's Guide to Principal Components. Wiley.

Liu, P., Mumtaz, H., 2011. Evolving macroeconomic dynamics in a small open economy: An estimated markov switching dsge model for the uk. Journal of Money, Credit and Banking 43 (7), 1443–1474.

Liu, Z., Waggoner, D., Zha, T., 2009. Asymmetric expectation effects of regime shifts in monetary policy. Review of Economic Dynamics 12 (2), 284–303.

Lütkepohl, H., 2005. New Introduction to Multiple Time Series Analysis. Springer.

Ng, S., Perron, P., 2005. A note on the selection of time series models. Oxford Bulletin of Economics and Statistics 67 (1), 115–134.

Peres-Neto, P., Jackson, D., Somers, K., 2005. How many principal components? stopping rules for determining the number of non-trivial axes revisited. Computational Statistics & Data Analysis 49 (4), 974–997.

Psaradakis, Z., Sola, M., 1998. Finite-sample properties of the maximum likelihood estimator in autoregressive models with markov switching. Journal of Econometrics 86 (2), 369–386.

Rudebusch, G., Swanson, E., 2008. Examining the bond premium puzzle with a dsge model. Journal of Monetary Economics 55, 111–126.

Schmitt-Grohe, S., Uribe, M., 2004. Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of Economic Dynamics and Control 28 (4), 755–775.

Sims, C., Zha, T., 2006. Were there regime switches in us monetary policy? The American Economic Review 96 (1), 54–81.

Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the euro area. Journal of the European Economic Association 1 (5), 1123–1175.

Smets, F., Wouters, R., 2007. Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach. American Economic Review 97 (3), 586–606.

Tsay, R., 1986. Nonlinearity tests for time series. Biometrika 73 (2), 461–466.

Uhlig, H., 1995. A toolkit for analyzing nonlinear dynamic stochastic models easily. Federal Reserve Bank of Mineapolis, Discussion Paper (101).

Vávra, M., 2013. Testing non-linearity in multivariate stochastic processes. NBS Working Paper Series 2/2013.

APPENDIX A: WORKHORSE DSGE MODEL

We follow Liu et al. (2009) and consider a simple, yet realistic, DSGE model with nominal and real frictions. The model describes the behaviour of three agents in the economy (i.e. households, firms, and government). It is assumed that the economy is populated by infinitely lived households who consume and supply labour to firms. Households are assumed to maximize an intertemporal CRRA utility function given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t A_t \left[\log(C_t - b\bar{C}_{t-1}) - \frac{\Psi}{1+\xi} L_t^{1+\xi} \right], \tag{13}$$

subject to the following budget constraint

$$\bar{P}_t C_t + B_t = W_t L_t + (1 + r_{t-1}) B_{t-1} + \Pi_t, \tag{14}$$

where C_t denotes real consumption, $\bar{C}_{t-1}$ lagged aggregate real consumption, $0 < b < 1$ represents habit formation, L_t denotes labour, A_t denotes a preference shock, B_t denotes a state-contingent nominal bonds and r_t the nominal interest rate, $\bar{P}_t$ denotes the price level, W_t denotes the nominal wage, and Π_t the profit from firms. The parameter $0 < \beta < 1$ represent the discount rate, $\xi > 0$ the inverse Frisch elasticity of labour supply, and $\Psi > 0$ the relative weight of labour in the utility function. The preference shock $\log(A_t)$ is assumed to follow an AR(1) process for convenience: $\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t^a$ such that $0 < \rho_a < 1$ and $\epsilon_t^a \sim IID(0, \sigma_a^2)$.

The final consumption good is produced in the perfectly competitive sector using differentiated intermediate goods as inputs using the Dixit-Stiglitz aggregation technology

$$C_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}, \tag{15}$$

with constant elasticity of substitution $\theta > 1$. The perfect competition environment implies that the aggregate price index is given

$$\bar{P}_t = \left[\int_0^1 P_t(j)^{1-\theta} \mathrm{d}j \right]^{\frac{1}{1-\theta}}. \tag{16}$$

Firms are assumed to produce intermediate goods using a simple production technology given by

$$Y_t(j) = Z_t L_t(j)^{\alpha}, \tag{17}$$

where $0 < \alpha < 1$ is the production function parameter and Z_t is an aggregate technology progress following a random walk with a drift $\log(Z_t) = \log(\lambda) + \log(Z_{t-1}) + \log(v_t)$, where λ is a measure of deterministic trend of technological progress and $\log(v_t)$ is a technological innovation following a simple AR(1) process $\log(v_t) = \rho_v \log(v_{t-1}) + \epsilon_t^v$ such that $0 < \rho_v < 1$ and $\epsilon_t^v \sim IID(0, \sigma_v^2)$. Firms are assumed to be price-takers in the input market and monopolistic competitors in the product market. Only for simplicity of the model, firms are assumed to follow a Calvo pricing mechanism. In each period, there is a probability ζ that firms cannot adjust their output prices. However, firms that cannot adjust their prices are allowed to re-optimize their prices using a simple indexation rule

$$P_t(j) = \pi_{t-1}^{\gamma} \pi^{ss(1-\gamma)} P_{t-1}(j), \tag{18}$$

where $0 \leq \gamma \leq 1$ denotes the degree of indexation. If firms can reset their prices, the new price $P_t(j)$ follows from optimizing the expected discounted stream of dividends given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \zeta^i D_{t+i} \left[P_t(j) \chi_{t+i} Y_{t+i}(j)^d - W_{t+i} \left(\frac{Y_{t+i}(j)^d}{Z_{t+i}} \right)^{1/\alpha} \right], \tag{19}$$

subject to the demand function

$$Y_t(j)^d = \left(\frac{P_t(j)}{\bar{P}_t} \right)^{-\theta} C_t. \tag{20}$$

The term χ_{t+i} comes from the price updating rule: $\chi_{t+i} = 1$ for $i = 0$ and $\chi_{t+i} = \pi_{t+i+1}^{\gamma} \cdots \pi_t^{\gamma} \pi^{ss(1-\gamma)i}$ for $i > 0$. Note that $D_{t+i} = \Pi_{j=1}^{i} (1 + r_{t+j})^{-1}$ represents the stochastic discount factor.

The monetary authority is assumed to follow a simple policy rule in the model

$$r_t = r + \phi_p \log(\pi_t / \pi^{ss}) + \phi_y \log(Y_t / Y_t^{ss}) + u_t, \tag{21}$$

where π^{ss} and Y_t^{ss} denote steady states, ϕ_p and ϕ_y denote policy parameters, u_t represents a monetary policy shock following an AR(1) process: $u_t = \rho_u u_{t-1} + \epsilon_t^u$ such that $0 < \rho_u < 1$ and $\epsilon_t^u \sim IID(0, \sigma_u^2)$.

APPENDIX B: LINEAR DSGE (L) MODEL

Keeping all the structural parameters constant, the log-linearized first-order conditions lead to the following system of equations

$$\hat{\pi}_t = \gamma\hat{\pi}_{t-1} + \beta\mathbb{E}_t(\hat{\pi}_{t+1} - \gamma\hat{\pi}_t) + \\ + \psi\left[\frac{\xi+1}{\alpha}\hat{y}_t + \frac{b}{b+\lambda}(\hat{y}_t - \hat{y}_{t-1})\right] + \psi\hat{v}_t, \tag{22a}$$

$$\hat{y}_t = \left(\frac{\lambda}{\lambda+b}\right)\mathbb{E}_t(\hat{y}_{t+1}) + \left(\frac{b}{\lambda+b}\right)\hat{y}_{t-1} + \left(\frac{b-\lambda}{\lambda+b}\right)(\hat{r}_t - \mathbb{E}_t(\hat{\pi}_{t+1})) + \\ + \left(\frac{\lambda\rho_v - b}{\lambda+b}\right)\hat{v}_t + + \left[\frac{(\lambda-b)(1-\rho_a)}{\lambda+b}\right]\hat{a}_t, \tag{22b}$$

$$\hat{r}_t = \phi_p\hat{\pi}_t + \phi_y\hat{y}_t + \hat{u}_t, \tag{22c}$$

where

$$\psi = \frac{1}{1+\theta(1-\alpha)/\alpha}\left(\frac{(1-\beta\zeta)(1-\zeta)}{\zeta}\right),$$

where $\hat{\pi}_t$ denotes a deviation of the inflation rate from the (non-stochastic) steady state value, $\hat{y}_t$ denotes the output gap, and $\hat{r}_t$ denotes a deviation of the short-term interest rate from its (non-stochastic) steady state value, $\hat{v}_t$, $\hat{a}_t$, $\hat{u}_t$ denote technology, preference, and monetary policy shocks. All shocks are assumed to follow an AR(1) model, where ρ_v, ρ_a, and ρ_u denote the persistence (i.e. AR(1)) parameters of shock variables, and σ_v^2, σ_a^2, and σ_u^2 denote variances of shock innovations. The deep (structural) parameters, collected in a vector $\boldsymbol{\omega}$, have the following meaning: β, represents the discount factor, b the habit formation of households, ξ the inverse Frish elasticity of labour supply, λ steady state of technology progress, ζ probability that a firm cannot reset its price, γ a fraction of firms following a simple indexation of prices, α production function parameter, θ the elasticity of substitution among differentiated intermediate goods.

Uhlig (1995) shows that the model in (22) can be written into the following matrix form

$$\mathbf{y}_t = \mathbf{A}(\boldsymbol{\omega})\mathbb{E}_t(\mathbf{y}_{t+1}) + \mathbf{B}(\boldsymbol{\omega})\mathbf{y}_{t-1} + \mathbf{C}(\boldsymbol{\omega})\mathbf{x}_t, \\ \mathbf{x}_t = \mathbf{R}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t,$$

where $\mathbf{y}_t = (\hat{\pi}_t, \hat{y}_t, \hat{r}_t)'$ is a (3×1) vector of dependent variables, $\mathbf{x}_t = (\hat{v}_t, \hat{a}_t, \hat{u}_t)'$ a vector of shocks, and $\mathbf{A}(\boldsymbol{\omega})$, $\mathbf{B}(\boldsymbol{\omega})$, $\mathbf{C}(\boldsymbol{\omega})$, $\mathbf{R}$ are (3×3) matrices of structural (deep) parameters. Since the above system is exactly identified, the state-space model can be further simplified into the form of a VAR(2) model given by

$$\mathbf{y}_t = \boldsymbol{\Phi}_1(\boldsymbol{\omega})\mathbf{y}_{t-1} + \boldsymbol{\Phi}_2(\boldsymbol{\omega})\mathbf{y}_{t-2} + \boldsymbol{\Theta}(\boldsymbol{\omega})\boldsymbol{\epsilon}_t. \tag{23}$$

The benchmark parameter configuration of deep parameters is as follows: $\beta = 0.995$, $\lambda = 1.005$, $\alpha = 0.7$, $\theta = 10$, $\xi = 2$, $b = 0.5$, $\rho_v = 0.95$, $\rho_r = 0.80$, $\rho_a = 0.70$, $\phi_p = 1.8$, $\phi_y = 0.4$, $\zeta = 0.7$, $\gamma = 0.7$, $\sigma_v^2 = \sigma_a^2 = \sigma_u^2 = 0.1$. The resulting reduced-form matrices of a linear DSGE model are given by

$$\mathbf{\Phi}_1(\boldsymbol{\omega}) = \begin{bmatrix} 1.60 & -0.09 & -0.01 \\ 0.03 & 0.93 & -0.04 \\ 1.45 & -0.11 & 0.77 \end{bmatrix}, \mathbf{\Phi}_2(\boldsymbol{\omega}) = \begin{bmatrix} -0.62 & 0.01 & 0.00 \\ 0.02 & -0.21 & 0.00 \\ -1.11 & -0.06 & 0.00 \end{bmatrix}, \mathbf{\Theta}(\boldsymbol{\omega}) = \begin{bmatrix} -0.27 & 0.06 & 0.36 \\ -0.61 & 0.17 & 0.42 \\ 0.27 & 0.17 & 0.81 \end{bmatrix}.$$

APPENDIX C: MARKOV-SWITCHING DSGE (MS) MODEL

It is important to point out that since some structural (deep) parameters of a DSGE model are allowed to follow a Markov-switching process, the above derived first-order conditions are no longer valid. Specifically, we replace the constant monetary policy parameters ϕ_p and ϕ_y in (21) by the regime-dependent parameters $\phi_p(S_t)$ and $\phi_y(S_t)$, and the Phillips curve parameters γ and ζ in (18) and (19) by $\gamma(S_{t-1})$ and $\zeta(S_{t-1})$. A new set of first-order conditions is given by

$$\begin{aligned} \hat{\pi}_t &= \gamma(S_t)\hat{\pi}_{t-1} + \beta\psi_1(S_t, S_{t-1})\mathbb{E}_t(\hat{\pi}_{t+1} - \gamma(S_t)\hat{\pi}_t) + \\ &+ \psi_2(S_{t-1})\left[\frac{\xi+1}{\alpha}\hat{y}_t + \frac{b}{b+\lambda}(\hat{y}_t - \hat{y}_{t-1})\right] + \psi_2(S_{t-1})\hat{v}_t, \end{aligned} \tag{24a}$$

$$\begin{aligned} \hat{y}_t &= \left(\frac{\lambda}{\lambda+b}\right)\mathbb{E}_t(\hat{y}_{t+1}) + \left(\frac{b}{\lambda+b}\right)\hat{y}_{t-1} + \left(\frac{b-\lambda}{\lambda+b}\right)(\hat{r}_t - \mathbb{E}_t(\hat{\pi}_{t+1})) + \\ &+ \left(\frac{\lambda\rho_v - b}{\lambda+b}\right)\hat{v}_t + + \left[\frac{(\lambda-b)(1-\rho_a)}{\lambda+b}\right]\hat{a}_t, \end{aligned} \tag{24b}$$

$$\hat{r}_t = \phi_p(S_t)\hat{\pi}_t + \phi_y(S_t)\hat{y}_t + \hat{u}_t, \tag{24c}$$

where

$$\begin{aligned} \psi_1(S_t, S_{t-1}) &= \frac{\bar{\zeta}}{\zeta(S_{t-1})}\left(\frac{1-\bar{\zeta}}{1-\zeta(S_t)}\right), \\ \psi_2(S_{t-1}) &= \frac{1}{1+\theta(1-\alpha)/\alpha}\left(\frac{(1-\beta\bar{\zeta})(1-\zeta(S_{t-1}))}{\zeta(S_{t-1})}\right). \end{aligned}$$

where $\bar{\zeta}$ denotes the ergodic mean of the random variable $\zeta(S_t)$, $\hat{\pi}_t$ denotes a deviation of the inflation rate from the (non-stochastic) steady state value, $\hat{y}_t$ denotes the output gap, and $\hat{r}_t$ denotes a deviation of the short-term interest rate from its (non-stochastic) steady state value,$\hat{v}_t$, $\hat{a}_t$, $\hat{u}_t$ denote technology, preference, and monetary policy shocks. All shocks are assumed to follow an AR(1) model. The deep (structural) parameters have the following meaning: β, represents the discount factor, b the habit formation of households, ξ the inverse Frish elasticity of labour supply, λ steady state of technology progress, ζ probability that a firm cannot reset its price, γ a fraction of firms following a simple indexation of prices, α production function parameter, θ the elasticity of substitution among differentiated intermediate goods. S_t represents the first-order time-homogenous hidden Markov chain defined on a discrete space $\{1,2\}$ with constant (irreducible and aperiodic) transition probabilities $p_{11} = \mathbb{P}(S_t = 1|S_{t-1} = 1)$ and $p_{22} = \mathbb{P}(S_t = 2|S_{t-1} = 2)$.

Cho (2011) shows that the MS-DSGE model in (24) can be written as follows

$$\mathbf{y}_t = \mathbf{A}(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbb{E}_t(\mathbf{y}_{t+1}) + \mathbf{B}(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{y}_{t-1} + \mathbf{C}(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{x}_t,$$
$$\mathbf{x}_t = \mathbf{R}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t,$$

where $\mathbf{y}_t = (\hat{\pi}_t, \hat{y}_t, \hat{r}_t)'$ is a (3×1) vector of dependent variables, $\mathbf{x}_t = (\hat{v}_t, \hat{a}_t, \hat{u}_t)'$ a vector of shocks, and $\mathbf{A}(\boldsymbol{\omega}_{S_t,S_{t-1}})$, $\mathbf{B}(\boldsymbol{\omega}_{S_t,S_{t-1}})$, $\mathbf{C}(\boldsymbol{\omega}_{S_t,S_{t-1}})$, are (3×3) matrices of structural (deep) parameters subject to Markov-switching, whereas $\mathbf{R}$ is a (3×3) matrix of fixed parameters. Cho (2011) also points out that the closed-form solution of the model, if it exists, can be written into the form of a state-space representation. Since the above system is exactly identified, the state-space model can be further simplified into the form of a MS-VAR(2) model given by

$$\mathbf{y}_t = \boldsymbol{\Phi}_1(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{y}_{t-1} + \boldsymbol{\Phi}_2(\boldsymbol{\omega}_{S_t,S_{t-1}})\mathbf{y}_{t-2} + \boldsymbol{\Theta}(\boldsymbol{\omega}_{S_t,S_{t-1}})\boldsymbol{\epsilon}_t. \tag{25}$$

Appendix C1: MS-DSGE (MS1) model

In this particular case, only the monetary policy rule parameters are allowed to change over time. The benchmark parameter configuration of deep parameters is as follows: $\beta = 0.995$, $\lambda = 1.005$, $\alpha = 0.7$, $\theta = 10$, $\xi = 2$, $b = 0.5$, $\zeta = 0.7$, $\gamma = 0.7$, $\rho_v = 0.95$, $\rho_r = 0.80$, $\rho_a = 0.70$, $\phi_p = 1.8$, $\gamma = 0.7$, $\sigma_v^2 = \sigma_a^2 = \sigma_u^2 = 0.1$, $\phi_p(S_t) \in \{2.0, 1.0)$, $\phi_y(S_t) \in \{0.5, 0.1)$, for $S_t \in \{1,2\}$. The transition matrix of the S_t variable is set as follows: $p_{11} = 0.95$ and $p_{22} = 0.80$, $S_t = 1$ represents the expansion (active) regime, whereas $S_t = 2$ the recession (passive) regime. The calibrated parameter are based on MS-DSGE models in Liu et al. (2009) and Liu and Mumtaz (2011). It is important to emphasize that due to the fact that only the monetary policy rule parameters are subject to a change (without any effect of intertemporal optimization), the reduced form MS-DSGE parameters take the following functional form: $\boldsymbol{\Phi}_1(\boldsymbol{\omega}_{S_t,S_{t-1}}) \equiv \boldsymbol{\Phi}_1(\boldsymbol{\omega}_{S_t})$, $\boldsymbol{\Phi}_2(\boldsymbol{\omega}_{S_t,S_{t-1}}) \equiv \boldsymbol{\Phi}_2(\boldsymbol{\omega}_{S_t})$, and $\boldsymbol{\Theta}(\boldsymbol{\omega}_{S_t,S_{t-1}}) \equiv \boldsymbol{\Theta}(\boldsymbol{\omega}_{S_t})$, for $S_t \in \{1,2\}$. The resulting reduced-form matrices are given by

$$\mathbf{\Phi}_1(\boldsymbol{\omega}_1) = \begin{bmatrix} 1.57 & -0.08 & -0.01 \\ -0.08 & 0.94 & -0.03 \\ 1.50 & -0.10 & 0.77 \end{bmatrix}, \mathbf{\Phi}_1(\boldsymbol{\omega}_2) = \begin{bmatrix} 1.74 & -0.11 & -0.01 \\ 0.48 & 0.88 & -0.05 \\ 0.98 & -0.10 & 0.79 \end{bmatrix},$$

$$\mathbf{\Phi}_2(\boldsymbol{\omega}_1) = \begin{bmatrix} -0.60 & 0.01 & 0.00 \\ 0.10 & -0.21 & 0.00 \\ -1.16 & -0.08 & 0.00 \end{bmatrix}, \mathbf{\Phi}_2(\boldsymbol{\omega}_2) = \begin{bmatrix} -0.71 & 0.02 & 0.00 \\ -0.29 & -0.22 & 0.00 \\ -0.74 & 0.00 & 0.00 \end{bmatrix},$$

$$\mathbf{\Theta}(\boldsymbol{\omega}_1) = \begin{bmatrix} -0.26 & 0.06 & 0.36 \\ -0.55 & 0.16 & 0.36 \\ 0.21 & 0.19 & 0.89 \end{bmatrix}, \mathbf{\Theta}(\boldsymbol{\omega}_2) = \begin{bmatrix} -0.45 & 0.09 & 0.62 \\ -1.08 & 0.27 & 0.99 \\ 0.44 & 0.12 & 0.72 \end{bmatrix}.$$

Appendix C2: MS-DSGE (MS2) model

In the case of the MS2 model, the monetary policy rule parameters and the New Keynesian Phillips curve parameters are allowed to change over time. The benchmark parameter configuration of deep parameters is as follows: $\beta = 0.995$, $\lambda = 1.005$, $\alpha = 0.7$, $\theta = 10$, $\xi = 2$, $b = 0.5$, $\rho_v = 0.95$, $\rho_r = 0.80$, $\rho_a = 0.70$, $\phi_p = 1.75$, $\sigma_v^2 = \sigma_a^2 = \sigma_u^2 = 0.1$, $\phi_p(S_t) \in \{2.0, 1.0\}$, $\phi_y(S_t) \in \{0.5, 0.1\}$, $\zeta(S_t) \in \{0.75, 0.25\}$, $\gamma(S_t) \in \{0.75, 0.25\}$ for $S_t \in \{1, 2\}$. The transition matrix of the S_t variable is set as follows: $p_{11} = 0.95$ and $p_{22} = 0.80$, where $S_t = 1$ represents the expansion (active) regime, whereas $S_t = 2$ the recession (passive) regime. The calibrated parameter are based on MS-DSGE models in Liu et al. (2009) and Liu and Mumtaz (2011). Only for simplicity, the reduced-form parameter matrices are reported for two most important combinations of states: $(S_t, S_{t-1}) = (1, 1)$ and $(S_t, S_{t-1}) = (2, 2)$. The resulting reduced-form matrices are given by

$$\mathbf{\Phi}_1(\boldsymbol{\omega}_{1,1}) = \begin{bmatrix} 1.65 & -0.09 & 0.01 \\ -0.31 & 0.77 & 0.20 \\ 3.16 & 0.20 & 0.12 \end{bmatrix}, \mathbf{\Phi}_1(\boldsymbol{\omega}_{2,2}) = \begin{bmatrix} 1.38 & -0.29 & -0.01 \\ -0.26 & 0.93 & 0.22 \\ 1.36 & 0.20 & 0.04 \end{bmatrix},$$

$$\mathbf{\Phi}_2(\boldsymbol{\omega}_{1,1}) = \begin{bmatrix} -0.70 & 0.01 & 0.00 \\ -0.09 & -0.19 & 0.00 \\ -1.45 & -0.07 & 0.00 \end{bmatrix}, \mathbf{\Phi}_2(\boldsymbol{\omega}_{2,2}) = \begin{bmatrix} -0.42 & 0.05 & 0.00 \\ 0.04 & -0.23 & 0.00 \\ -0.42 & 0.02 & 0.00 \end{bmatrix},$$

$$\mathbf{\Theta}(\boldsymbol{\omega}_{1,1}) = \begin{bmatrix} -0.04 & 0.05 & 0.28 \\ -0.32 & 0.17 & 0.45 \\ 0.75 & 0.18 & 0.79 \end{bmatrix}, \mathbf{\Theta}(\boldsymbol{\omega}_{2,2}) = \begin{bmatrix} -0.15 & 0.24 & 1.65 \\ -0.38 & 0.21 & 0.61 \\ 0.81 & 0.26 & 1.71 \end{bmatrix}.$$

APPENDIX D: TABLES

Table 1: Average rejection frequency of the non-linearity tests: $T = 150$

			transformation 1				**transformation 2**				**transformation 3**			
DGP	tests	config.	TSAY	n	ARCH	n	TSAY	n	ARCH	n	TSAY	n	ARCH	n
L	multiv.	M(BIC)	0.097	3.0	0.060	3.0	0.049	3.0	0.065	3.0	0.062	3.0	0.056	3.0
		M(0.9)	0.092	3.0	0.057	3.3	0.047	4.1	0.064	3.4	0.061	4.9	0.054	3.3
		M(K)	0.099	3.7	0.058	3.5	0.059	5.1	0.065	3.6	0.062	4.8	0.050	3.5
	univ.	π	0.023	–	0.058	–	0.027	–	0.045	–	0.033	–	0.050	–
		y	0.037	–	0.051	–	0.046	–	0.051	–	0.040	–	0.048	–
		r	0.024	–	0.053	–	0.029	–	0.054	–	0.040	–	0.062	–
MS1	multiv.	M(BIC)	0.390	3.2	0.504	3.2	0.295	3.2	0.517	3.2	0.281	3.1	0.416	3.2
		M(0.9)	0.354	3.1	0.515	4.5	0.264	4.1	0.508	4.3	0.336	5.7	0.414	4.2
		M(K)	0.375	3.8	0.516	4.6	0.391	5.4	0.505	4.3	0.352	6.4	0.416	4.2
	univ.	π	0.216	–	0.423	–	0.210	–	0.430	–	0.167	–	0.398	–
		y	0.345	–	0.526	–	0.300	–	0.355	–	0.288	–	0.335	–
		r	0.261	–	0.615	–	0.263	–	0.606	–	0.413	–	0.643	–
MS2	multiv.	M(BIC)	0.691	5.0	0.762	4.0	0.772	5.3	0.860	4.1	0.791	4.3	0.738	3.9
		M(0.9)	0.577	3.2	0.753	4.2	0.663	4.1	0.865	4.1	0.850	5.7	0.737	4.1
		M(K)	0.634	4.1	0.758	4.5	0.806	5.8	0.870	4.3	0.869	6.8	0.749	4.3
	univ.	π	0.548	–	0.758	–	0.559	–	0.770	–	0.345	–	0.574	–
		y	0.397	–	0.661	–	0.602	–	0.819	–	0.608	–	0.834	–
		r	0.259	–	0.441	–	0.249	–	0.463	–	0.267	–	0.535	–

* Note that "M(·)" denotes a multivariate version of either the TSAY or ARCH test. "n" denotes the average number of principal components over all Monte Carlo replications. "BIC" denotes the test with the number of principal components selected using the BIC approach; "0.9" denotes the test with the number of principal components selected using the variance rule with the cutoff 0.9; "K" denotes the test with the number of principal components selected using the Kaiser (root) rule with the cutoff 1.0.

Table 2: Average rejection frequency of the non-linearity tests: $T = 300$

			transformation 1				transformation 2				transformation 3			
DGP	tests	config.	TSAY	n	ARCH	n	TSAY	n	ARCH	n	TSAY	n	ARCH	n
L	multiv.	M(BIC)	0.073	3.0	0.076	3.0	0.047	3.0	0.051	3.0	0.061	3.0	0.066	3.0
		M(0.9)	0.074	3.0	0.075	3.3	0.052	4.9	0.052	3.3	0.072	6.7	0.063	3.2
		M(K)	0.076	3.5	0.077	3.4	0.055	6.1	0.049	3.5	0.060	6.5	0.064	3.4
	univ.	π	0.036	–	0.061	–	0.025	–	0.064	–	0.039	–	0.053	–
		y	0.054	–	0.054	–	0.054	–	0.054	–	0.044	–	0.065	–
		r	0.041	–	0.067	–	0.034	–	0.056	–	0.048	–	0.049	–
MS1	multiv.	M(BIC)	0.371	3.1	0.687	3.3	0.280	3.1	0.697	3.3	0.307	3.0	0.523	3.2
		M(0.9)	0.344	3.1	0.699	4.5	0.319	4.6	0.703	4.5	0.418	7.3	0.543	4.3
		M(K)	0.373	3.7	0.700	4.5	0.438	6.2	0.696	4.5	0.429	7.6	0.543	4.2
	univ.	π	0.258	–	0.613	–	0.275	–	0.642	–	0.210	–	0.550	–
		y	0.464	–	0.684	–	0.455	–	0.511	–	0.391	–	0.476	–
		r	0.431	–	0.903	–	0.407	–	0.891	–	0.547	–	0.892	–
MS2	multiv.	M(BIC)	0.702	4.4	0.936	4.5	0.761	4.8	0.967	4.7	0.778	3.9	0.898	4.4
		M(0.9)	0.590	3.1	0.936	4.3	0.700	4.5	0.970	4.3	0.874	7.2	0.900	4.2
		M(K)	0.633	3.9	0.937	4.8	0.846	6.5	0.974	4.6	0.883	8.4	0.907	4.6
	univ.	π	0.616	–	0.934	–	0.626	–	0.951	–	0.411	–	0.743	–
		y	0.556	–	0.936	–	0.718	–	0.979	–	0.705	–	0.970	–
		r	0.297	–	0.675	–	0.348	–	0.660	–	0.365	–	0.704	–

* Note that "M(·)" denotes a multivariate version of either the TSAY or ARCH test. "n" denotes the average number of principal components over all Monte Carlo replications. "BIC" denotes the test with the number of principal components selected using the BIC approach; "0.9" denotes the test with the number of principal components selected using the variance rule with the cutoff 0.9; "K" denotes the test with the number of principal components selected using the Kaiser (root) rule with the cutoff 1.0.

Table 3: P-values of the non-linearity tests

	univariate tests							multivariate tests	
tests/variables	Y	P	R	W	L	C	I	$\mathbf{y}$ =(Y,P,R)	n
TSAY	0.760	0.028	0.000	–	–	–	–	–	–
MTSAY(BIC)	–	–	–	–	–	–	–	0.052	3
MTSAY(0.9)	–	–	–	–	–	–	–	0.000	8
MTSAY(K)	–	–	–	–	–	–	–	0.000	8
ARCH	0.318	0.908	0.000	–	–	–	–	–	–
MARCH(BIC)	–	–	–	–	–	–	–	0.199	3
MARCH(0.9)	–	–	–	–	–	–	–	0.001	6
MARCH(K)	–	–	–	–	–	–	–	0.305	4
tests/variables	Y	P	R	W	L	C	I	$\mathbf{y}$ =(Y,P,R,W,L)	n
TSAY	0.760	0.028	0.000	0.167	0.005	–	–	–	–
MTSAY(BIC)	–	–	–	–	–	–	–	0.002	5
MTSAY(0.9)	–	–	–	–	–	–	–	0.000	15
MTSAY(K)	–	–	–	–	–	–	–	0.000	21
ARCH	0.318	0.908	0.000	0.001	0.001	–	–	–	–
MARCH(BIC)	–	–	–	–	–	–	–	0.012	5
MARCH(0.9)	–	–	–	–	–	–	–	0.000	14
MARCH(K)	–	–	–	–	–	–	–	0.000	10
tests/variables	Y	P	R	W	L	C	I	$\mathbf{y}$ =(P,R,W,L,C,I)	n
TSAY	–	0.028	0.000	0.167	0.005	0.106	0.713	–	–
MTSAY(BIC)	–	–	–	–	–	–	–	0.000	6
MTSAY(0.9)	–	–	–	–	–	–	–	0.000	17
MTSAY(K)	–	–	–	–	–	–	–	0.000	21
ARCH	–	0.908	0.000	0.001	0.001	0.133	0.457	–	–
MARCH(BIC)	–	–	–	–	–	–	–	0.001	6
MARCH(0.9)	–	–	–	–	–	–	–	0.000	19
MARCH(K)	–	–	–	–	–	–	–	0.000	12

[a] "Y" denotes the growth rate of the real GDP series, "C" the growth rate of real consumption, "I" the growth rate of real investment, "P" the CPI inflation rate, "R" the 3M treasure bill rate, "W" the growth rate of nominal hourly wage, and "L" growth rate of hours worked.

[b] Note that "M(·)" denotes a multivariate version of either the TSAY or ARCH test. "n" denotes the number of principal components. "BIC" denotes the test with the number of principal components selected using the BIC approach; "0.9" denotes the test with the number of principal components selected using the variance rule with the cutoff 0.9; "K" denotes the test with the number of principal components selected using the Kaiser (root) rule with the cutoff 1.0.