Government Spending and the Term Structure of Interest Rates in a DSGE Model

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Abstract

We explore asset pricing implications of productive, wasteful and utility enhancing government expenditures in a New Keynesian macro-finance model with Epstein-Zin preferences. We decompose the pricing kernel into four underlying macroeconomic factors (consumption growth, inflation, time preference shocks, long run risks for consumption and leisure) and design novel method to quantify the contribution of each factor to bond prices. Our methodology extends the performance attribution analysis typically used in finance literature on portfolio analysis. Using this framework, we show that bonds can serve as an insurance vehicle against the fluctuations in investors wealth induced by government spending. Increase in uncertainty surrounding government spending rises the demand for bonds leading to decrease in yields over the whole maturity profile. Bonds insure investors by $i)$ providing buffer against bad times, $ii)$ hedging inflation risk and $iii)$ hedging real risks by putting current consumption gains against future losses. In a special case where the central bank does not respond to changes in output bonds leverage inflation risk. Spending reversals strongly reduce the sensitivity of bond prices to changes in government spending.

Non-technical summary

We contribute to the public debate on implications of fiscal policy by analyzing the impact of productive, utility enhancing and wasteful government spending on bond prices. The strong link between bond prices and real economy has been documented both empirically and theoretically (for a survey see Grkaynak and Wright, 2012) and became prominent after the financial crisis. Term structure of interest rates is recently at a focal point of the monetary policy transmission (see Brunnermeier, Eisenbach, and Sannikov, 2012 or Brazdik, Hlavacek, and Marsal, 2012 for a survey). On the other hand, the effects of government spending on the real economy have been studied extensively at least since Keynes (among many others see Barro, 1990 or Turnovsky, 1996). Macroeconomic models analyzing the impact of fiscal policies on real economy nevertheless usually neglect the impact on bond prices and thus omit an arguably important transmission channel on the real economy. The main focus of this paper is on decomposing the mechanism transmitting government spending to bond prices.

From the policy perspective, understanding how changes in government spending affect bond prices is important for designing fiscal stimulus or austerity policies and it also has direct effect on the cost of government debt. Stimulus packages are most often financed by issuing debt. If the increase in government spending leads to an increase in the level and slope of the term structure of interest rates it has direct adverse consequences for the current and future costs of government debt financing. Traditional cost benefit analysis of the stimulus packages may thus underestimate the true costs. We show that the government can decrease the cost of its debt by rising the uncertainty about its future government spending. This is because bonds serve as an insurance against potential risk generated by the uncertainty about government spending. Government thus creates risk for which it at the same time sells the only available insurance product.

We build our analysis on a variant of macro-finance DSGE model (similar to Rudebusch and Swanson, 2012, Andreasen, 2012a, Ferman, 2011, Li and Palomino, 2014) which we augment by wasteful, utility enhancing and productive government expenditures and reversals in government spending . This class of models have been argued to be able to match both macro and finance stylized facts and thus allows us to explore asset pricing implications of government expenditures in full fledged dynamic stochastic general equilibrium model.

From the methodology point of view, the primary contribution of the paper is the proposal of novel method, which allows us to link the term structure of interest rates with specific macroeconomic factors such as consumption growth, inflation, changes in investors' preferences and long-run risks. Our framework sheds new light on how the policy uncertainty translates into bond market risk premium. Consequently, it allows us to construct a policy immunizing the effect of government spending uncertainty on the term structure of interest rates.

When we study bond prices we ask two questions. First, in a spirit of Krishnamurthy and

Vissing-Jorgensen (2012) we ask why are investors willing to pay lower or higher price for government bond compared to the bond risk free price. Further, we follow the literature studying nominal term premium in production economies (i.e. Rudebusch and Swanson, 2012) and ask why the longer maturity bonds are on average cheaper than short maturity bonds. In other words, why is the term structure upward sloping and long term bonds bear term premium. The answer to the first question is that the model world is risky and bonds help investors with diversifying their income against future uncertain outcomes. Nominal term premium is on the other hand a compensation for forming wrong expectations about future short rates because long term bonds are compounded risk adjusted expectations about future short bond prices. Long maturity bonds are cheaper compared to short maturity bonds because future short term interest rates are uncertain which makes them a risky form of investment and thus investors ask for a discount to hold bonds with longer maturity. The ability of investors to predict uncertain outcomes of economy is therefore at the heart of determining bond prices.

The predictability of future government spending policy is important for both the term and risk premium. We assume such preference structure (Epstein Zin preferences) which makes investors dislike risks which last for long (see Bansal and Yaron, 2004). If the volatility of government spending is large and thus the size of future spending is uncertain, investors will increase their positions in bond portfolios to ensure a stable level of consumption over time. They are willing to pay an insurance premium in the form of a higher price for bonds. If the unexpected changes in government spending have high persistence the investors want to hedge their level of consumption for the whole period of the above or under average government spending. In other words, the duration of the portfolio matters. Thus, investors are willing to hold high duration bond portfolio only with substantial compensation (discount). For this reason, long-run risks for future consumption and leisure are important factors determining the size of the nominal term premium. Furthermore, we argue that a fiscal policy that becomes more predictable in terms of the evolution of debt and taxes (e.g. a commitment to spending reversals) mitigates the impact of uncertainty on bond prices.

The risk premium in bond prices is driven by two incentives, *i)* the precautionary savings motives and *ii)* hedging property of bonds against the fluctuations in investor real value of wealth. We focus on how these two incentives are affected by government spending. Government spending directly affects investors' (households') consumption through inter-temporal budget constraint as a rise in government expenditures decreases disposable income of investors due to higher taxes. For consumption smoothing investors are even moderate fluctuations in consumption costly in terms of utility. Bonds represent a useful instrument providing buffer compensating the unexpected drops in consumption. The rise in uncertainty about the size of future government spending decreases the predictability of future consumption. Investors thus buy bonds for precautionary reasons. In addition, bonds provide insurance against nominal and real risk. Bonds hedge inflation risk if the real value of bonds rises (inflation drops) in bad times (when consumption drops). We show that when a central bank sets the nominal interest

rates in response to changes in output, bonds act as a hedging instrument against inflation risk because of the negative correlation of consumption with inflation (mechanism promoted by Rudebusch and Swanson, 2012). On the other hand, bonds leverage inflation risks for investors wealth if the central bank targets strictly inflation. Furthermore, bonds help to transfer wealth in time, they enable using current consumption against future losses and the other way around. Bonds thus may hedge real risks if the future consumption is negatively correlated with the current consumption[3] (see also Kaltenbrunner and Lochstoer, 2010 for an analogy in production economy). The character of government expenditures affects both term and risk premia. Bonds carry large insurance premium if government expenditures are wasteful (e.g. defense expenditure). Bonds serve as poor hedging instrument against productive (infrastructure expenditures) and utility enhancing (health, education expenditures) risk in government expenditures. Spending reversals reduce the insurance premium in bond prices if government expenditures are wasteful by improving the predictability of future path of government expenditures.

More generally, there are four factors that directly determine the bond prices in our model: consumption growth, inflation, time preference and long-run risks for consumption and leisure. We confirm the findings of empirical literature stressing the importance of inflation risk for bond prices. However, we emphasize, that the size of inflation risks is highly sensitive to monetary policy conduct. When monetary policy responds more to inflation (i.e. coefficient on inflation in the Taylor rule is larger), inflation risks are smaller. The higher is the coefficient on output gap in the Taylor rule the lower are real risks but the higher are inflation risks. This is due to the tradeoff between inflation and output gap stabilization.

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[3]For the simplicity of the exposition we abstract here from the labor smoothing channel.

1. INTRODUCTION

How does the term structure of interest rates respond to a rise in government expenditures? Is the uncertainty about the size of government spending important for bond prices? Does the impact of uncertainty depends on monetary policy conduct? Can fiscal policy immunize its impact on the term structure of interest rates? The literature is mostly silence. The impact of government expenditures on real economy has been for long studied in literature on fiscal multipliers.[4] Impact on the term structure of interest rates has been however mostly neglected. Recent development in financial friction literature[5] stressed the impact of bond prices on real economy and the term structure of interest rates become one of the instrument of monetary policy. We contribute to the debate on implications of fiscal policy by studying the link between government expenditures (productive, wasteful and substitutable and non-substitutable utility enhancing) and bond prices. We build our analysis on a variant of macro-finance DSGE model (similar to Rudebusch and Swanson (2012), Andreasen (2012a), Ferman (2011) and Li and Palomino (2014)) matching both macro and finance stylized facts. We show that investors use bonds to protect their wealth against economic uncertainty caused by unpredictability of future government spending. As increasing uncertainty about future wasteful government spending rises the price of issued Treasuries, fiscal authority can decrease the cost of its debt by increasing the unpredictability of future expenditures. This is because bonds serves as the only insurance vehicle[6] for investors against the fluctuations in their wealth.

We extend the existing literature in several directions. First, we augment the method of Hordahl, Tristani, and Vestin (2007) and show how to derive the second order approximation of the pricing kernel in the model with Epstein Zin (EZ) preferences. Deriving the pricing kernel in terms of conditional second moments of underlying macro variables provides us with an analytical explanation of how exogenous shocks translate into bond prices. It allows us to interpret the bond risk premium in terms of macroeconomic factors and therefore we can provide richer structural decomposition than Andreasen (2012a) who decomposes term premia only into real and nominal part.

Second, we propose a new method to explicitly calculate the model implied conditional second moments of underlying macro variables which we refer to as macroeconomic factors. The idea rests on a simple factor model used in asset management for performance attribution.[7] The linear property of the approximated pricing kernel allows us recursively calculate the conditional

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[4]See for a survey Christiano, Eichenbaum, and Rebelo, 2011.

[5]See Brunnermeier, Eisenbach, and Sannikov, 2012 or Brazdik, Hlavacek, and Marsal, 2012 for a survey.

[6]The insurance property of bonds has been empirically documented by Barsky (1989), Fama and French (1989), Ilmanen (2003), Shiller and Beltratti (1992) among others. Gulko (2002) calls this phenomenon decoupling, meaning that U.S. Treasury bonds offer effective diversification during financial crises, at the time diversification is needed most.

[7]Performance attribution often called as the Brinson model (Brinson and Fachler, 1985). The model is widely used by the investment management community to attribute portfolio returns by using a simple sector-based investment process of sector allocation and stock selection.

moments in terms of consumption growth, inflation, time preference shocks and long-run consumption and leisure risks. To our knowledge this is the first attempt to quantify the conditional second moments considering explicitly the underlying macroeconomics variable.

Third, the existing macro finance literature studies exclusively the determinants of the nominal term premium. Motivated by Krishnamurthy and Vissing-Jorgensen (2012) we extend the analysis to stochastic steady state of bond prices. Krishnamurthy and Vissing-Jorgensen (2012) shows empirically that liquidity and safety drive investors' hight valuation of US Treasuries. They show that Treasury yields are reduced by 73 basis point, on average, compared to "riskless" rate. By studying the spread between stochastic and deterministic ("riskless") steady state of bond prices we can ask, as Krishnamurthy and Vissing-Jorgensen (2012), what determines the valuation of bonds. We show that bonds contain (apart from the safety and liquidity[8]) also premium for its diversification attributes against business cycle risks. We calculate that bond yields are reduced by 96 basis point, on average[9], because they contain premium for $i)$ providing buffer against bad times, $ii)$ hedging inflation risk and $iii)$ hedging real risks by putting current consumption gains against future losses. The importance of these three determinants of risk premia were stressed separately in the literature before. The precautionary saving was studied for one period bond price in NK DSGE model by Paoli and Zabczyk (2012a). The inflation risk was shown by Rudebusch and Swanson (2012) to be a way how to generate high nominal term premium in NK DSGE model. Kaltenbrunner and Lochstoer (2010) shows in RBC model how the hedging property of bonds against real risk contributes to low nominal term premium. Our novel decomposition allows us to decouple and quantify the effect of this three channels on both risk and term premia in a single model.

Finally, when describing the yield curve we study separately the impact of transitory changes in government spending and the impact of changes in volatility of government spending.

We find that the character of government expenditures affects both term and risk premia. Bonds carry large insurance premium if government expenditures are wasteful (defense expenditure). Bonds serve as poor hedging instrument against productive (infrastructure expenditures) and utility enhancing (health, education expenditures) risk in government expenditures. Spending reversals reduce the insurance premium in bond prices if government expenditures are wasteful by improving the predictability of future path of government expenditures. Further, using the structural decomposition into macroeconomic factors we show that the ability of bonds to hedge investors against inflation risk is the most important driver for high valuation of bonds. However, we emphasize, that the size of inflation risks is highly sensitive to monetary policy conduct and type of exogenous shock.

Wasteful government expenditures increase nominal term premium by 2.2 basis point and only about 1 basis point if the monetary policy puts zero weight on output gap. Productive expen-

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[8]We approximate safety and liquidity by preference shocks as suggested by Fisher (2015)

[9]Average over the year 1969 to 2009 in our baseline calibration and also averaged over the maturities.

ditures increase term premium only for high production elasticity of this expenditures. Both substitutable and non-substitutable to consumption utility enhancing expenditures decrease significantly the term premium.

The term structure rises on the impact of government spending shocks due to higher expected future short term interest rates. The uncertainty related to wasteful government spending on the other hand shifts the whole term structure down. This can be attributed partly to precautionary saving motives and partly to the hedging property of bonds[10]. High volatility in government spending motivates households to insure themselves against a fluctuations in their wealth. The precautionary saving motive grows with the size of uncertainty. The decomposition of the pricing equation further shows that a rise in fiscal uncertainty amplifies the hedging property of bonds against consumption risks given by the negative covariance between expected future consumption and leisure with realized consumption growth and is independent of monetary policy.

We illustrate the intuition on a negative shock to wastful government purchases. The representative household associates lower government spending with lower current and future taxes which have a positive wealth effect stimulating current private consumption and implying higher realized consumption growth. Due to the transitory nature of the shock the government spending reverts back to its long-run mean. Hence, a negative transitory shock to government spending generates a positive shock for realized consumption growth (crowding-in) but negative shocks to future consumption as government spending increases back to its long-run mean. This is salient feature for models with EZ preferences and transitory shocks (see Kaltenbrunner and Lochstoer, 2010). The impact on short bonds is stronger however, so the influence of the hedging property decays over the maturity profile. This fact allows to generate a sizable nominal term premium within our New Keynesian DSGE model.

The response of monetary policy to government spending determines the degree of diversification of bonds to inflation risks. If monetary policy responds to output gap in the Taylor rule (given the response to current inflation), inflation works as a hedge against the long-run consumption and leisure risks and bonds serve as a form of insurance. This is in striking contrast to the way productivity (TFP) shocks impact the risk premium. In case of TPF shocks, when monetary policy responds more to inflation (coefficient on inflation in the Taylor rule is larger) inflation risks are smaller. The higher is the coefficient on output gap in the Taylor rule the lower are real risks but the higher are inflation risks. This is due to the tradeoff between inflation and output gap stabilization.

We also find that spending reversals break the link between the uncertainty about government spending and risk premium. The increase in predictability of the evolution of debt and taxes mitigates the impact of uncertainty (through second order terms) on macroeconomic variables.

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[10]The hedging property comes from negative correlation between macro variables, thus it can be understood as gains from diversification.

This fact helps the investor to form a more accurate expectation. Larger time t conditional information set decreases the risk of bond mispricing, therefore the risk premiums are lower.

The remainder of the paper is structured as follows. We provide literature review in section 2. The model is lay out in section 3. In section 4 we discuss our results where we also detail the decomposition of the pricing kernel. Section 5 concludes. Appendix A provides numerous sensitivity checks.

2. LITERATURE REVIEW

The standard dynamic stochastic general equilibrium (DSGE) models face difficulties in matching jointly macro and finance empirical moments in data e.g. Rudenbush and Swanson (2008), Hordahl, Tristani, and Vestin (2007), Jermann (1998). The inability of DSGE models to match the term premium is known in literature as the Backus, Gregory, and Zin (1989) puzzle. For this reason, most of the literature in the field focuses on adding new modeling features to fit additional moments in data. A number of models have been developed to improve the poor performance of DSGE models in pricing assets. Hordahl, Tristani, and Vestin (2007) shows that after modifying the standard New Keynesian DSGE model with nominal rigidities and internal habits, the model delivers sizable term premia. At the same time the model fits relatively well moments of consumption growth and inflation, although the results are sensitive to the specific calibration.

De Paoli, Scott, and Weeken (2010) demonstrate in a similar model that implications of composition of preferences for asset prices depend on the source of the shock. Paoli and Zabczyk (2012b) argue in the model with external habits that neglecting the precautionary saving motive in the model may have considerable consequences for the design of monetary policy.

Probably the most successful recent contribution comes from Rudebusch and Swanson (2012). They introduce Epstein and Zin (EZ) preferences into a basic New Keynesian DSGE model with fixed capital and solve it by the third order. The model fits the level and volatility of term premium reasonably well without compromising the fit of macro data. This is because EZ preferences have only second order effects on macro variables but first order effects on risk premia. The model good performance largely stands on the type of Taylor rule they consider and ability to generate large inflation risks.

Gallmeyer, Hollifield, Palomino, and Zin (2009) demonstrate in the endowment economy the high sensitivity of term premia to the formulation of Taylor rule. Ferman (2011) documents that introducing the regime switching to model with EZ preferences helps to account for the response of term premium to the different regimes of monetary policy. Caldara, Fernandez-Villaverde, Rubio-Ramirez, and Yao (2012) shows that the model solution by third order perturbation is competitive in terms of accuracy with most global methods. Diercks (2015) studies the welfare implications of monetary policy when the model incorporates the latest advances in

literature on matching the equity and term premium. Christoffel, Jaccard, and Kilponen (2013) in the model with habits looks at how the pro-cyclical fiscal policy affects welfare and risk premia. Lopez, Lopez-Salido, and Vazquez-Grande (2015) use Campbell and Cochrane (1995) type habits to match both macro and asset pricing stylized facts.

Similarly to De Paoli, Scott, and Weeken (2010) we do not attempt to solve asset pricing puzzles. Instead, we explore asset pricing implications of fiscal policy in what becomes a paradigm in macro-finance literature. Bansal and Shaliastovich (2012), Segal, Shaliastovich, and Yaron (2014) and Wright (2011) document in the empirical investigation the link between macroeconomic uncertainty and asset prices. Similarly to these studies, we evaluate how the quantity of risk coming from macroeconomic uncertainty connects to the risk premia. We show that our model can replicate the results of Bansal and Shaliastovich (2012). They show that increase in the uncertainty about expected inflation raises bond premia whereas uncertainty related to expected consumption growth decreases the bond premia.

Andreasen (2012a) studies the link between stochastic volatility in productivity shocks and term premia. He defines uncertainty shocks as stochastic changes in the conditional standard deviation of the innovations. Distinct from this, we explicitly define the size of the uncertainty shocks consistently with the data. Pastor and Veronesi (2012) finds in the general equilibrium model that uncertainty about government policy increases risk premia. Compared to our paper, Pastor and Veronesi (2012) use a different modeling framework and assume that fiscal policy uncertainty affects directly the firms profitability. The main difference in the story compared to our results comes however from the definition of fiscal uncertainty as they link it to the effectiveness of government spending. They focus on what is the impact of announcements about policy changes on the stock prices. In one closely related study, Croce, Kung, Nguyen, and Schmid (2012) examine the effect of the uncertainty about government spending and tax policy on asset prices. They however omit monetary policy and the interactions of monetary policy with fiscal policy. Our model encompasses the hedging property of bonds emphasized by Kaltenbrunner and Lochstoer (2010). They examine the asset pricing implication of long-run consumption risk in the standard production economy model. They document that the long-run risk component acts as a hedge for shocks to realized consumption. A transitory shock generating a drop in realized consumption growth implies high future expected consumption growth as the shock reverts back to the long-run trend and thus the long-run risk counterbalances the realized economic slowdown. The important source of risk in our model is that investors dislike when inflation is the carrier of bad news for the future economic outlook. This feature was empirically examined in the representative agent asset pricing model by Piazzesi and Schneider (2007). Positive correlation between real value of bonds and news about future consumption growth leverages the risk implying a high risk premium.

3. THE MODEL

We rely on a general equilibrium model similar to Rudebusch and Swanson (2012), Andreasen (2012a) and Ferman (2011) to quantitatively examine the links between government spending and dynamics of the term structure of interest rates. Our economy is populated by: i) a representative household who has recursive preferences, supplies labor and buys public bonds, ii) firms operating on the final and intermediate goods market with the latter facing nominal rigidities, iii) a monetary policy following a Taylor rule and iv) a government which funds its expenditures by by lump-sum taxes and by issuing government bonds. Government expenditures can be either wasteful, productive (G_t^p) or utility enhancing (G_t^u). The utility enhancing expenditures are divided into non-separable, G_t^n, from private consumption government expenditures (we assume that G_t^n are perfectly substitutable to C_t), and separable, G_t^s.

3.1 HOUSEHOLDS

The economy is inhabited by a continuum of households. The representative household chooses paths for consumption C_t and leisure, L_t to maximize expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, G_t^u) \tag{1}$$

where β is the subjective discount factor of future stream of utilities, subject to a budget constraint:

$$P_t C_t + E_t Q_{t,t+1} B_{t+1} \leq B_t + W_t N_t + T_t + \Pi_t \tag{2}$$

where $E_t Q_{t,t+1} B_{t+1}$ is the present value of a portfolio of risk free bonds. $Q_{t,t+1}$ is the stochastic discount factor, $W_t N_t$ is the household labor income, time constrain is normalized to one, $N_t + L_t = 1$ and P_t is the aggregate price level. T_t summarizes all lump-sum transfers to the household and Π_t are firms' profits.

The objective function in equation (1) can be written in recursive form as

$$V_t = u(C_t, L_t) + \beta E_t V_{t+1} \tag{3}$$

We follow Rudebusch and Swanson (2012) and use the following transform of Epstein and Zin (1989) preferences:

$$V_t = u(C_t, L_t) + \beta (E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{4}$$

when $u(C_t, L_t) > 0$.

If $u(C_t, L_t) < 0$, as in our benchmark calibration, the recursion takes the form:

$$V_t = u(C_t, L_t) - \beta(E_t[-V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{5}$$

The period utility is represented by

$$u(C_t, L_t) = e^{b_t}\left(\frac{(C_t + G_t^n)^{1-\gamma}}{1-\gamma} - \chi\frac{N_t^{1+\eta}}{1+\eta} + \frac{(G_t^s)^{\eta_g}}{1-\eta_g}\right) \tag{6}$$

where η is the inverse of Frisch elasticity. Notice, that with recursive preferences G_t^n enters the first order conditions (through V_t) even if it is separable from leisure and consumption. b_t is the preference shock which follows the autoregressive process:

$$b_t = \rho_b b_{t-1} + \sigma_b \epsilon_t^b \tag{7}$$

where $\epsilon_t^b \in N(0,1)$, σ_b controls the volatility of the preference shocks and ρ_b sets the persistence. Fisher (2015) shows that preference shocks can be understood as shocks to demand for safe assets. Taking advantage of this interpretation and complying with the estimates of Andreasen (2012b) we introduce exogenous changes in the preferences even if this means that part of the nominal term premium is explained exogenously.

Swanson (2012) shows the relationship of parameter α, intertemporal elasticity of substitution (γ) and inverse Frisch elasticity to the relative risk-aversion[11].

The household optimization exercise delivers an Euler equation which allows us to price a bond of any maturity:

$$Q_{t,t+1} = e^{b_{t+1}-b_t}\beta\left(\frac{C_t + G_t^n}{C_{t+1} + G_{t+1}^n}\right)^{\gamma}\frac{P_t}{P_{t+1}}\left[\frac{V_{t+1}}{[E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right] \tag{8}$$

and labor supply:

$$W_t = \chi\frac{N_t^\eta}{C_t^{-\gamma}} \tag{9}$$

Letting $\Pi_{t+1} \equiv P_{t+1}/P_t$ denote inflation the price of a τ-period nominal bond can be written as:

$$P_{\tau,t}^{bond} = E_t\left[Q_{t,t+1}P_{\tau-1,t+1}^{bond}\Pi_{t+1}^{-1}\right] \tag{10}$$

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[11]The connection between the coefficient of relative risk-aversion (CRRA) and parameter α in the recursive formulation for the particular form of the period utility in equation (6) is given by $CRRA = \frac{\gamma}{1+\frac{\gamma}{\eta}} + \frac{\alpha(1-\gamma)}{1+\frac{\gamma-1}{1+\eta}}$.

3.2 FIRMS

Final good firms operate under perfect competition with the objective to minimize expenditures subject to the aggregate price level $P_t = \left(\int_0^1 P_t(i)^{\frac{-1}{\lambda_t}} di \right)^{-\lambda_t}$, where $P_t(i)$ is the price of intermediate good produced by firm i, using the technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_t}} di \right)^{1+\lambda_t}$. The final good firms aggregate the continuum of intermediate goods i on the interval $i \in [0,1]$ into a single final good. Here λ_t means that the markup is time-varying (see equation (17)).

The cost-minimisation problem of final good firms deliver demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t}{P_t(i)} \right)^{\frac{1+\lambda_t}{\lambda_t}} Y_t \tag{11}$$

A continuum of intermediate firms operates in the economy. Intermediate firm i uses the Cobb-Douglas technology

$$Y_t(i) = A_t \bar{K}^\theta N_t(i)^{1-\theta} (G_t^p)^{\theta_g} \tag{12}$$

where $\bar{K}$ refers to the fact that firms have fixed capital[12] and $N_t(i)$ is the amount of labor employed. The capital share of output is controlled by θ. The output elasticity with respect to productive government expenditures is determined by θ_g. In equation (12) technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_t^A \tag{13}$$

where ϵ_t^A is an independently and identically distributed (iid) shock with zero mean and constant variance.

Intermediate firms face quadratic adjustment costs as in Rotemberg (1982):
$PAC_t(i) = \frac{\zeta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} \frac{1}{\bar{\pi}} - 1 \right)^2 P_t Y_t$ where ζ stands for the adjustment cost parameter. Intermediate firm chooses price, $P_t(i)$, so as to maximize the expected discounted sum of future profits corrected by adjustments costs:

$$E_t \{ \sum_{j=0}^{\infty} Q_{t,t+j} \frac{P_t}{P_{t+j}} [D_{t,t+j}(i) - PAC_{t+j}(i)] \} \tag{14}$$

where $D_{t,t+j}(i) = P_{t+j}(i) Y_{t+j}(i) - W_{t+j} N_{t+j}(i)$ is the profit of firm i between time t and $t+j$ and $Q_{t,t+j}$ is the stochastic discount factor which is given by equation (8). The term $W_{t+j} N_{t+j}$ represents the cost of labor.

The profit maximization exercise delivers the New Keynesian Phillips curve,

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[12] Firm-specific capital can be interpreted as a model with endogenous investment that features high adjustment costs in investment. Further, Rudebusch and Swanson (2012) emphasize the importance of firm-specific fixed factors for generating a degree of inflation persistence which can be found in the actual data.

$$MC_t = \frac{1}{1+\lambda_t} + \frac{\lambda_t}{1+\lambda_t}\zeta\left(\frac{\pi_t}{\bar{\pi}}-1\right)\frac{\pi_t}{\bar{\pi}} - \frac{\lambda_t}{1+\lambda_t}E_tQ_{t,t+1}\zeta\left(\frac{\pi_{t+1}}{\bar{\pi}}-1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{Y_{t+1}}{Y_t} \quad (15)$$

where profits are discounted by $Q_{t,t+1}$ and the average real marginal cost is defined as

$$MC_t = \left[(G_t^p)^{\theta_g}\bar{K}^\theta\right]^{\frac{1}{\theta-1}}\frac{1}{1-\theta}\left(\frac{W_t}{A_t}\right)\left(\frac{Y_t}{A_t}\right)^{\frac{\theta}{1-\theta}} \quad (16)$$

In equation (15) the markup (or cost-push) shock is given by:

$$\log(1+\lambda_t) = (1-\rho_\lambda)\log(1+\bar{\lambda}) + \rho_\lambda\log(1+\lambda_{t-1}) + \sigma_\lambda\epsilon_t^\lambda \quad (17)$$

In case of flexible prices, $\zeta = 0$ and in the absence of cost-push shocks the marginal cost is constant and is equal to the inverse of the gross markup ($\frac{1}{1+\bar{\lambda}}$).

3.3 MONETARY POLICY

The model is closed with a monetary policy rule assuming that monetary authority sets the short-term nominal interest rate i_t based on a simple Taylor rule.

$$i_t = \bar{i} + \phi_\pi\hat{\pi}_t + \phi_y\hat{\mathbf{y}}_t \quad (18)$$

where $\hat{\pi}_t$ and $\hat{\mathbf{y}}_t$ denote the percentage deviations of inflation and aggregate output from their corresponding deterministic steady states, $\bar{i}$ is the deterministic steady state of one period nominal interest rate. Parameters ϕ_π and ϕ_y determine the weight monetary policy authority puts on stabilizing the deviations of inflation and output from their steady state values.

3.4 FISCAL POLICY

The total government expenditures are shared as follows: $G_t^p = \tau_p G_t$, $G_t^u = \tau_n G_t + (1-\tau_p-\tau_n)G_t$ with $0 \leq \tau_p + \tau_n \leq 1$. Total government spending, G_t, is assumed to follow an exogenous autoregressive process of the form:

$$\log G_t = (1-\rho_G)\log\bar{G} + \rho_G\log G_{t-1} + \sigma_G\epsilon_t^G \quad (19)$$

where ϵ_t^G is an iid shock with zero mean and unit variance. Parameter σ_G scales the standard deviation of the shock. We assume in our benchmark model that government runs a balanced budget financed through lump-sum taxes obtained from the household sector. The fixed nature of capital implies fixed investment that is used to replace depreciated capital: $I_t = \bar{I} = \delta\bar{K}$, where δ is the depreciation rate.

We consider two fiscal scenarios. The first one is the simple fiscal setup assuming that government spending has stochastic variation and is covered by lump-sum taxes in each period (the case of balanced budget). The second fiscal arrangement allows for deficit, government debt and spending reversals as in Corsetti, Meier, and Müller (2009). With spending reversal the reduction in debt is aided by restraint on government purchases in the future. Corsetti, Meier, and Müller (2009) show that spending reversals and, hence, higher savings of the government in the future generate crowding-in effects of government spending.

3.5 EXTENSION–SPENDING REVERSALS

We utilize the framework introduced by Corsetti, Meier, and Müller (2009) to study the effects of fiscal consolidation on the term structure. Government consumption is financed through either lump-sum taxes, T_t (taxes are in nominal terms) or the issuance of nominal debt, D_t, real government expenditures are denoted G_t.

$$T_t + Q_{t,t+1} D_{t+1} = D_t + P_t G_t \tag{20}$$

which can alternatively be expressed in real terms after dividing by the price level:

$$T_{Rt} + Q_{t,t+1} D_{Rt+1} = \frac{D_{Rt}}{\pi_t} + G_t \tag{21}$$

where $T_{Rt} = \frac{T_t}{P_t}$ are taxes in real terms and $D_{Rt} = \frac{D_t}{P_t}$ is a measure for real beginning-of-period debt.

Corsetti, Meier, and Müller (2009) use a fiscal rule of the following form:

$$T_{Rt} = \Psi_t D_{Rt} \tag{22}$$

Spending reversals are captured by the following process for government purchases:

$$\log G_t = (1 - \rho) \log \bar{G} + \rho \log G_{t-1} - \Psi_G \log D_{Rt} + \eta_t \tag{23}$$

where $\bar{G}$ is the steady state level of government spending and ρ controls the persistence. The Ψ-parameters capture a systematic feedback effect of public debt on government spending (negative) and taxes (positive).

Researchers typically assume that the government spending today leads eventually to an increase in taxes. The idea of Corsetti, Meier, and Müller (2009) is that it is not necessary to increase taxes in response to higher government debt because government expenditures can

be reduced to help settle debt. How does this work in our theoretical model? Spending reversals alter the short-run effects of the government spending innovations through a financial channel that captures the combined effect of fiscal and monetary policy on long-term interest rates. Households expect that the public spending will go down in the future. Monetary policy will increase short-term interest rates but long-term interest rates will decrease because of the expected lower future short term interest rates which will boost contemporaneous consumption.

In other words, an increase in government spending will subsequently cause spending to fall below trend level for some time. The anticipated spending reversal does not crowd out private consumption and boosts the expansionary effect of G on output at the impact.

3.6 MARKET CLEARING

In equilibrium firms and households optimally choose prices with respect to their constraints and each market clears. The market clearing in the goods market requires that the aggregate demand equals to aggregate output in the economy:

$$Y_t = C_t + G_t + \bar{I} \tag{24}$$

where investment is fixed, $\bar{I}$ due to fixed capital.

3.7 CALIBRATION AND SOLUTION METHOD

To assign values to the parameters in our model we follow what has become standard calibration in the literature for small closed economy models. Our calibration is similar to Rudebusch and Swanson (2012), Smets and Wouters (2007), Paoli and Zabczyk (2012b) or Ferman (2011). The parameter values are summarized in Table 1. Under Rotemberg price setting $\zeta = \frac{\varphi\left(1-\theta+\theta\frac{1+\bar{\lambda}}{\lambda}\right)}{(1-\varphi)(1-\varphi\beta)(1-\theta)}$ is set such that the slope of New Keynesian Phillips curve corresponds to the Calvo case with an average duration of price stickiness equal to $\frac{1}{1-\varphi} = 4$ quarters. An important portion of the nominal term premium in the model is driven by the calibration of preference shock, elasticity of intertemporal substitution and Frisch elasticity. Whereas Rudebusch and Swanson (2012) pick lower than usual values of IES and Frisch elasticity we use a somewhat higher persistence of preference shock and keep IES and Frisch elasticity at values standard in the literature. This is motivated by Fisher (2015) who provides structural interpretation to preference shock and identifies its increased importance since 2008.

The quantity of fiscal risk is represented by the volatility in the innovations to government spending, σ_g, from the range of 4 bps to 6 percent. To determine this range we build on the argument forcefully put forward in Ramey (2011). She argues that defense spending is consistent with the specification of government spending in most VAR and DSGE models. The Figure 28 in the

Monetary Policy Rule				Exogenous processes			
ϕ_π	2.19	ϕ_y	0.075	ρ_b	0.83	σ_b	0.020
	Fiscal Policy			ρ_A	0.98	σ_A	0.005
τ_n	0.18			ρ_λ	0.18	σ_λ	0.051
θ_g	0.2	τ_p	0.35	ρ_G	0.94	σ_G	0.008
	Structural Parameters				The Steady-State		
β	0.99	θ	0.33	$\bar{\Pi}$	1.004		
γ	2	$\bar{\lambda}$	0.2	$\bar{K}/(4\bar{Y})$	2.5		
η	0.40	ζ	233	$\bar{G}/\bar{Y}$	0.2		
α	-108	δ	0.02				

Table 1: Calibration of the model

Appendix shows that defense spending accounts for most of the volatility of total government spending. The major movements in defense spending are associated with the military build up distributed around the war dates. Ramey (2011) shows that since the state and local (non-defense) spending are driven in large part by cyclical fluctuations in state revenues, aggregate VARs are not very good at capturing shocks to this type of spending. In most of the DSGE literature government spending enters exogenously the model. Defence spending are orthogonal to business cycle and thus corresponds well to our model specification. We use the HP filter to assess the trend in the series of defense spending and use the filtered series to calculate the standard deviation of innovations.

Table 2 shows the volatility of the model consistent innovations and government spending for various sub-samples. The volatility of innovations in our data sub-samples ranges from 0.49 to 5.83 and justifies the wide range of σ_g we use to evaluate the model. Our baseline calibration matches the long run average period between 1969 and 2009[13]. We identify productive government spending with infrastructure spending and set $\tau_p = 0.35$. As public health care or public education services reduce the need for private schools we calibrate $\tau_n = 0.18$ based on the health and education component of government spending. The remainder, τ_s are the separable government expenditures and we link them to defense spending. As this expenditures do not enter first order conditions they are usually labeled as wasteful government expenditures. This however not the case with recursive preferences where both separable G_t^s and non-separable G_t^n enter first order conditions through the value function V_t.

The model is approximated to the second-order using Dynare routines. The second-order approximation is necessary to break the certainty equivalence of linearized models.

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[13]To calculate long run average we exclude the Korean and Vietnam war military build up as it is often done in the empirical literature - some argue that the Korean and Vietnam War were unusually large.

Period	Defense Spending		Total spending	
	σ_g	std(G)	σ_g	std(G)
1947 - 1957	5.83	17	3.5	8.1
1957 - 1967	1.55	4.53	0.8	2
1967 - 1977	1.61	4.71	0.94	2.15
1977 - 1987	0.49	1.43	0.65	1.48
1987 - 1997	0.61	1.79	0.4	0.91
1997 - 2007	0.9	2.63	0.4	0.99
1969 - 2009	0.8	2.43	0.5	1.27

Table 2: Standard deviation of defense spending and implied innovations. Results are in % deviations from the HP trend. When estimating the volatility in subperiods we hold fixed the whole sample estimate of persistence at $\rho_g = 0.94$ and $\rho_g = 0.9$ for total government spending

4. TRANSMISSION MECHANISM: INSIGHTS FROM THE MODEL

In this section, we first provide a summary of the core results and discuss some of the policy implications. Consequently we derive the second order approximation of the pricing kernel in terms of conditional moments of the underlying macro variables to identify the fundamental drivers behind our results. To quantitatively evaluate the conditional moments we modify what is known in portfolio management as performance attribution (Brinson factor model).

We discuss separately the impact of $i)$ transitory changes and $ii)$ the changes in volatility of government spending on the term structure of interest rates. Specific realized transitory shock impacts only first order terms in the model and do not have effect on risk premia. Changes in volatility of government spending on the other hand impact second order terms. In other words, shifts in volatility changes the stochastic steady state which determines the riskiness of bonds. We define the n -period nominal term premium as $NTP_t^n = i_t^n - \sum E_t[i_{t+j}]$. The deviation of the bond yield to maturity from the yields consistent with expectations hypothesis represent compensation for term risk[14]. Term premium is positive when it is riskier to invest in the long term bond than to invest in a sequence of short-term bonds for n periods[15]. Under risk premium we understand the difference between the stochastic and deterministic steady state of the term structure of interest rates[16]. The risk premium in bond prices are driven by two incentives, *i)* the precautionary savings motives and *ii)* hedging property of bonds against the fluctuations in investor real value of wealth. Thus, if the stochastic steady state of term structure of interest rates lies bellow the deterministic one, $i)$ bonds serve as a precautionary

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[14]Keynes referred to term premium as a liquidity premium, because investors have tided up capital for n-periods.

[15]Level of NTP and slope of the yield curve are equal in out model but not their volatility. See for instance Ferman (2011) or Rudebusch and Swanson (2012).

[16]In particular the second order correction terms $\sigma\sigma$ in the policy function, see Schmitt-Grohe and Uribe (2004) or Andreasen (2012b).
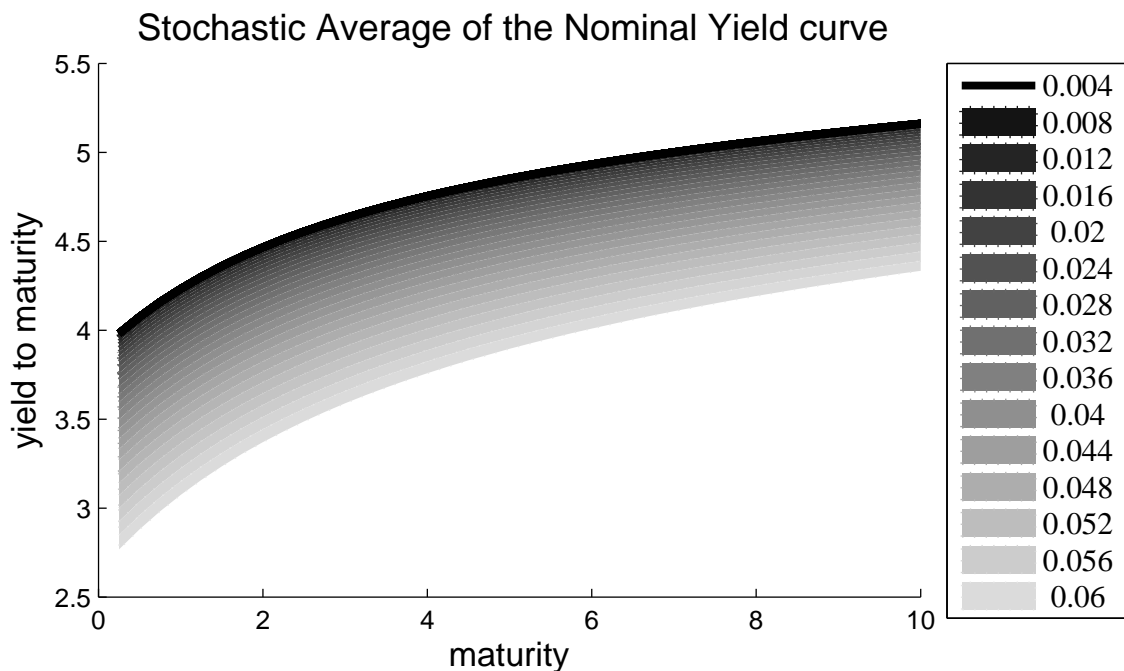
Stochastic Average of the Nominal Yield curve

Figure 1: Term structure and varying volatility of G shocks. The legend shows the volatility of the government spending innovation.

buffer against future uncertainty, and/or $ii)$ bonds provide a hedge against macroeconomic risk. Bond prices contain form of insurance premium. If stochastic steady state of term structure of interest rates lies above the deterministic one bond prices contain risk premium because bonds leverage the exposure to uncertainty.

4.1 RESULTS

We show that: $i)$ a transitory increase in wasteful government spending raises at impact the current and future short term nominal interest rates and thus the whole term structure goes up $ii)$ the higher the uncertainty related to wasteful government spending the lower the level of the yield curve. $iii)$ the response of monetary policy to wasteful government spending determines the degree of diversification of bonds to inflation risks. $iv)$ fiscal authority committed to fiscal consolidation immunizes the effect of its spending on the term structure, $v)$ the effect of productive government expenditures depends on the production elasticity of government spending, $vi)$ utility enhancing government expenditures rise the risk premium in bond prices.

4.1.1 Transitory response

The response of the bond yields to the impact of government spending shock is depicted in figure 5 for $\phi_y = 0.075$ and $\phi_y = 0$. The transitory response of the term structure of interest rates to government spending shock is driven in our benchmark model solely by the expectation hy-

pothesis. Yield curve rises at impact with increase in government spending because monetary policy authority sets higher nominal interest rates in response to a positive output gap. The expectations about future higher nominal interest rates shift the yield curve up. The transitory character of the shock implies that as the nominal interest rate returns back to its steady state value. Thus, the expectations about future short term yields decrease as well. For this reason the response of yields with longer maturity is weaker at the impact of the shock. The price of bonds with higher maturity responds one to one to the expected path of nominal interest rates laid down by central bank[17].

4.1.2 Fiscal Policy Uncertainty -Wasteful Government Spending

Figure 1 demonstrates that the rise in uncertainty related to government spending in our benchmark model decreases the level of the term structure of interest rates. The darker line corresponds to term structure of interest rates with lower volatility. The lowest volatility of government spending innovations we consider is 40 bps and the highest 6 percentage points.

The driving force behind the drop is the insurance property of bonds against uncertain future realization of fiscal policy. High volatility in government spending motivates consumption smoothing households to insure themselves against a drop in their wealth. The precautionary savings motive grows with the volatility. Paoli and Zabczyk (2012b) argues that productivity shocks in the log-linearized models abstracting from precautionary saving may give significantly biased policy implications. We extend in this sense the Paoli and Zabczyk (2012b) argument to government spending shocks. High volatility of fiscal policy increases the importance of the households risk aversion for the evolution of the interest rates throughout the whole maturity structure. In addition to the precautionary motive the drop in yields is driven by the hedging property of bonds. The value of bonds is negatively correlated with the long-run consumption and leisure risks.

Policy making authorities in a certainty equivalent world will underestimate the growth in the demand for government bonds and thus the consequent drop in steady state consumption. The increase in uncertainty will make financing of the government debt cheaper in the default free world but at the cost of causing large demand shifts away from consumption to government bonds. By not taking these effects into account fiscal and monetary policy mix delivers suboptimal results from a welfare point of view.

Figure 15 in appendix shows the impact of fiscal uncertainty on the term structure when monetary policy authority doesn't responds fluctuations in aggregate output. The drop in the level of the yield curve is lower for the equal size of the uncertainty as in figure 1. This is driven by

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[17]The benchmark model is approximated to the second order thus risk premiums are constant in time and therefore do not play any role in explaining the economy dynamic response to the realized shocks. To check robustness of the result we solve the model up to 3th order to conclude that changes in risk premiums (the third order terms) do not affect the direction of yield curve response to realized shock.

the leverage property of real value of bonds against the long-run consumption and leisure risks and partly by lower precautionary savings against inflation volatility. This is because inflation is less volatile in a regime where monetary policy responds also to widening output gap.

Figure 15 also shows that yields of short maturity bonds decrease more than long maturity bonds. The slope of the term structure increases. In general, due to recursive preferences the timing of the resolution of uncertainty matters for households. The price of a 10-year bond is determined by a set of expectations conditional on time t, $E_t[P_t^{10}] = Q_{t,t+1}Q_{t+1,t+2}Q_{t+2,t+3} \cdots Q_{t+9,t+10}$. The nominal term premium comes from the stochastic character of $Q_{t,t+n}$, the volatility of underlying macro series increases the chance of forming wrong expectations and ex-post wrong bond prices. The nominal term premium represents the compensation for revaluation in expectations as the new information arrives. The predictability (persistence) in consumption and leisure means that the arrival of new information creates an impact lasting for several quarters. Agents with Epstein Zin preferences dislike surprises with long-run effects which makes the reevaluation in expectations especially costly.

To get a required pay off in ten years agents have two options: $i)$ to take a position in a long-term bond, $ii)$ to buy a one year bond and roll it forward each period.[18] Which variant the household chooses depends on the intensity of households' risk aversion and inter-temporal smoothing motives. Households with low inter-temporal elasticity of substitution (IES) have a strong desire to smooth utility over time and will choose to buy bonds with long maturity. Risk-averse households smoothing utility across the state of the world will prefer to roll over one year bonds. To purchase long-term bonds risk averse agents require a discount to compensate them for the uncertainty related to future shock realization. In our model the conditional volatility of the shocks is constant over time thus the precautionary saving motive plays a negligible role as the risks are the same for all maturities. What determines the NTP is the difference in how much the macro variables co-vary (leverage and hedging property of bonds) across the maturity. Long-run risk has little impact on short maturity bonds. It has nevertheless significant impact on the long maturity bonds because of the persistence. If the central bank does not accommodate output gap shocks, the increase in uncertainty about future government spending generates large inflation risks which are positively correlated with the long-run consumption and leisure risks. This creates significant risks for pricing the bonds correctly, thus agents require an extra premium for holding long maturity bonds.

Next we discuss the conditions under which positive government spending shocks generate inflation risks. The specification and the size of the coefficients on inflation and output gap in the interest rate rule have large impact on the effects of shocks in the model. In a framework similar to this paper Horvath, Kaszab and Marsal (2016) find that the form of the interest rate rule has large impact on the ability of the model to generate inflation risks. Here we briefly explain

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[18]In the model households can buy only one period bonds. The decision between taking position in 10-year bonds and rolling forward one quarter bonds is only illustrative. The set of expectations households formed at time t is the analogy of being locked in bonds with long maturity.

that persistent and large government spending shocks are less inflationary with an a positive output-gap coefficient of $\phi_y > 0$ (rather than zero) because a positive output-gap coefficient raises the real interest rate even more in response to positive spending shocks discouraging households from further spending in the present. Furthermore, they point out that the general equilibrium outcome of a positive government spending shock financed by lump-sum taxes is a fall in inflation and short-term nominal interest rate when $\phi_y > 0$. A rise in government purchases leads to higher future taxes (a negative wealth effect) inducing households to cut consumption expenditures and to have less leisure as long as both are normal goods. With a given time frame less leisure translates into higher hours worked (an outward shift in labor supply). The shrinkage in household spending causes firms to produce less and, therefore, demand less labor. The leftward movement of the labor demand curve pushes the real wages down which has downward pressure on inflation through the New Keynesian Phillips curve (see also Linnemann and Schabert (2003)).

4.1.3   Fiscal Policy Uncertainty - Productive Government Spending

Figure 2 shows the response of the stochastic steady state of the term structure of interest rates to the rise in uncertainty related to productive government spending. The effect of σ_g on the term structure depends on the production elasticity of government expenditures. For $\theta_g = 0.2$ term structure does not respond to size of σ_g. For $\theta_g < 0.2$ short maturity bonds carry significant insurance premium. For $\theta_g > 0.2$ long maturity bonds carry significant insurance premium. The sensitivity on θ_g of the economy response is also documented in Getachew and Turnovsky (2015).

The size of insurance property of bonds rises with the production elasticity of government expenditures.

4.1.4   Fiscal Policy Uncertainty - Utility Enhancing Government Spending

Figure 3 shows the response of the stochastic steady state of the term structure of interest rates to the rise in uncertainty related to productive government spending. Uncertainty about government expenditures directed to goods which are perfect substitutes to private consumption (i.e. health care, education) rises the term structure of interest rates. Bonds leverage the risks which this type of government expenditures carry for investors wealth. The leverage property increases when monetary policy puts zero weight on output gap in Taylor rule, $\phi_y = 0$.

Figure 4 shows the utility enhancing government expenditure which are not substitutable to private consumption. The insurance property of bonds vanishes for $\eta_g > 1$. For $\eta_g < 1$ we get back strong insurance property of bonds. There is no agreement in the literature what this type of government spending represents. Baxter and King (1993) motivate defense expenditures by putting them into utility function in separable form. They argue that defense spending are not
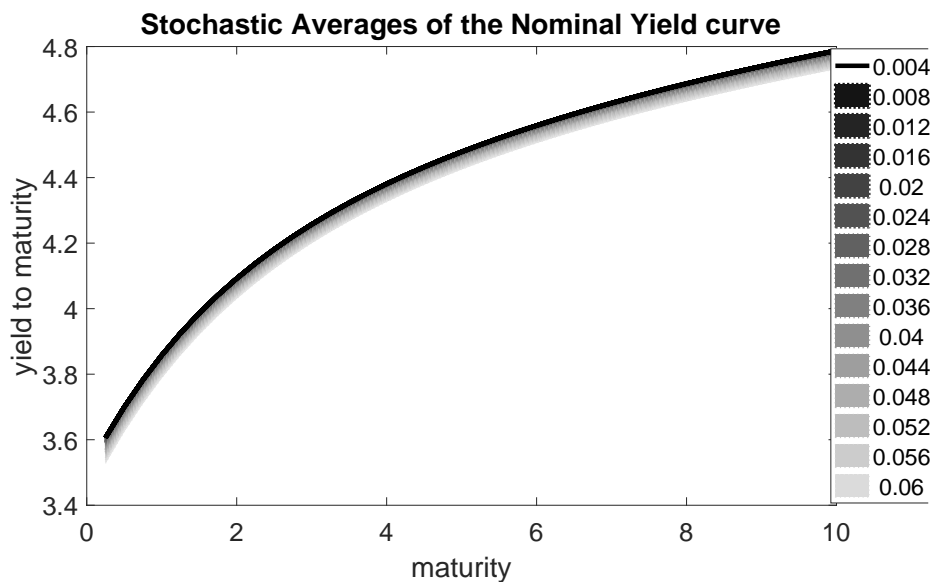
Figure 2: Term structure and the volatility of productive government spending. The legend shows the volatility of the total government spending innovation.
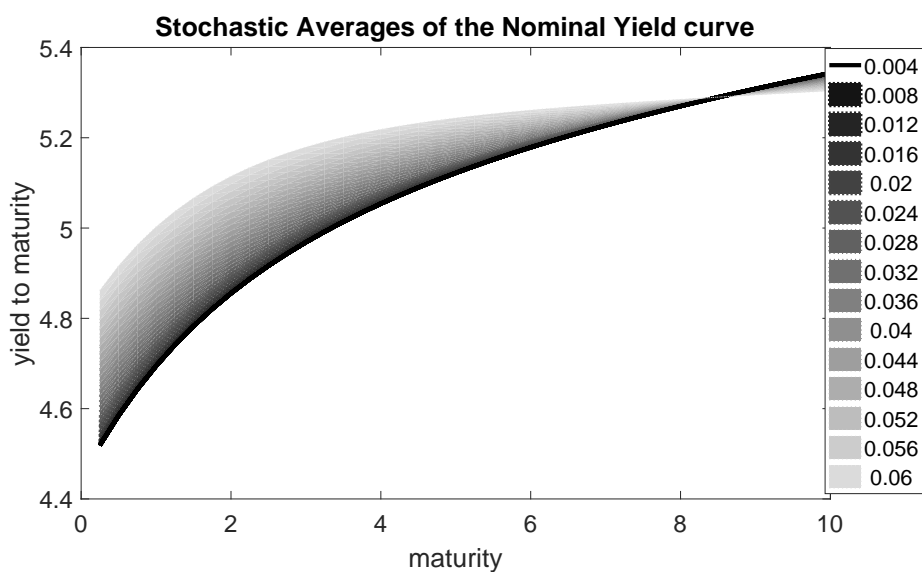

Figure 3: Term structure and the volatility of utility enhancing government spending - the case of perfect substitutes. The legend shows the volatility of the total government spending innovation.
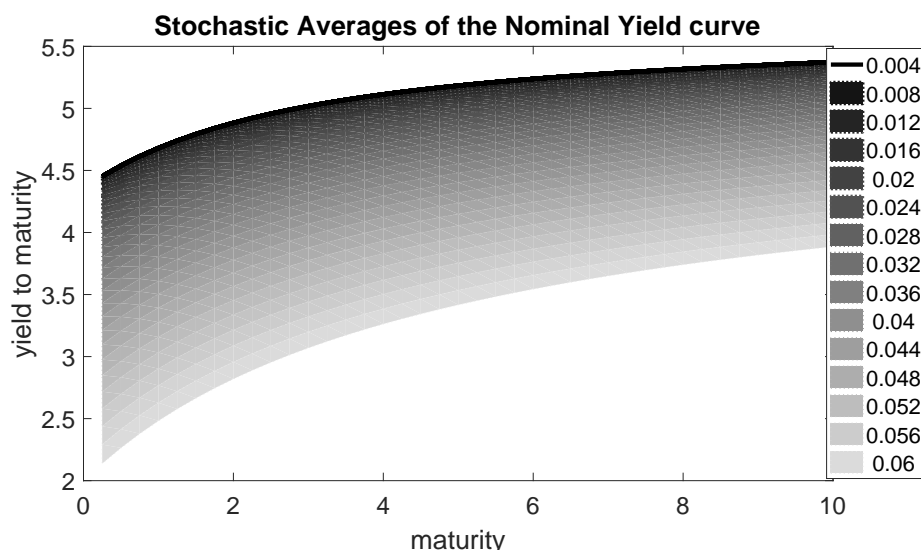
Figure 4: Term structure and the volatility of utility enhancing government expenditure which are not substitutable to private consumption. The legend shows the volatility of the total government spending innovation. We assume that $\eta_g = 0.5$

productive but they increase utility of households without affecting their consumption leisure decisions as G falls our from equilibrium conditions due to its exogeneity. Azevedo and Ercolani (2012) or Albertini, Poirier, and Roulleau-Pasdeloup (2014) consider defense spending as a part of production function by arguing that defense spending increases the productivity of private capital and leisure. Barro (1990) and Turnovsky (1996) associate utility enhancing government expenditures with for example cultural and recreational public services such as museums, public parks or public social events like fireworks. The interpretation of government expenditures not substitutable to private consumption is in our case affected by the fact that we use recursive preferences. In this case, government spending enters equilibrium conditions through the continuation value, V_t and thus affects long run consumption and leisure decisions of households. We leave the discussion without clear interpretation and just report the results here.

4.1.5 Implications for Monetary Policy

Figure 5 shows the impulse responses to a positive government spending shock on impact for the whole term structure of interest rates. Each panel represents the impact of the shock with around 40 bps for the maturity of one quarter and 6 percentage points for bond with a maturity of 10 years. The red dashed line with dots is the stochastic steady state (unconditional mean) of the term structure assuming that the monetary policy authority adjusts its interest rate solely in response to inflation and there is no response to the output gap ($\phi_y = 0$). The red line is the term structure one period after the economy is hit by an increase in government spending. The blue line constitutes the analogy when the weight on output stabilization is positive

$(\phi_y = 0.075)$. The degree of uncertainty about government spending has important consequences for setting the monetary policy. In an economy with a low degree of uncertainty about government spending monetary policy targeting only inflation reduces the inflation risk premia more than monetary policy which also smooths deviations from the output gap. Nevertheless, when the degree of uncertainty about government spending increases above 3 percentage points, the inflation risk generated by fiscal policy overweighs the stabilizing effect of strictly inflation targeting policy towards productivity shocks[19]. In a strictly inflation targeting regime the inflation risks generated by fiscal uncertainty are very costly. Building on the argument by Linnemann and Schabert (2003) discussed above, the monetary policy reacting to output mitigates inflation and reinforces the hedging property of real bonds to long-run consumption and leisure risks.

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[19]Positive TFP shock pushes inflation down. Inflation drops more if $\phi_y > 0$ because the drop in marginal costs is accompanied by rise in interest rates from Taylor rule. Therefore, the correlation between consumption and inflation is stronger if $\phi_y > 0$. Bonds loose their real value more in bad times. Investors ask higher premium for holding bonds.
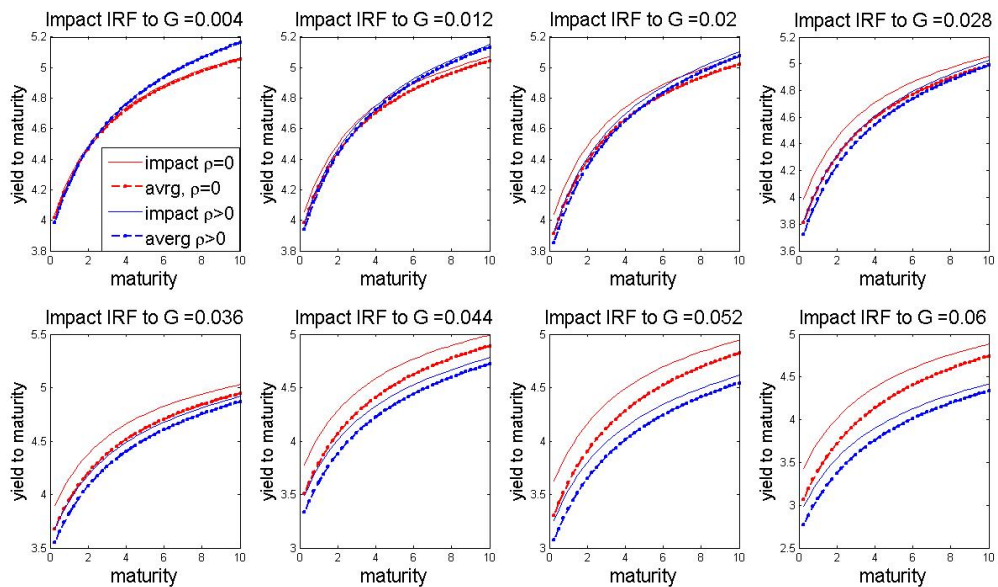
Figure 5: **Wasteful Government Spending and The Term Structure of Nominal Interest Rates: The Role of Monetary Policy.** The stochastic steady state of the term structure and the impact of increase in government spending on the yield curve. The red lines are the case of zero weight on output stabilization in the Taylor rule. The blue line correspond to the case of $\phi_y = 0.075$.
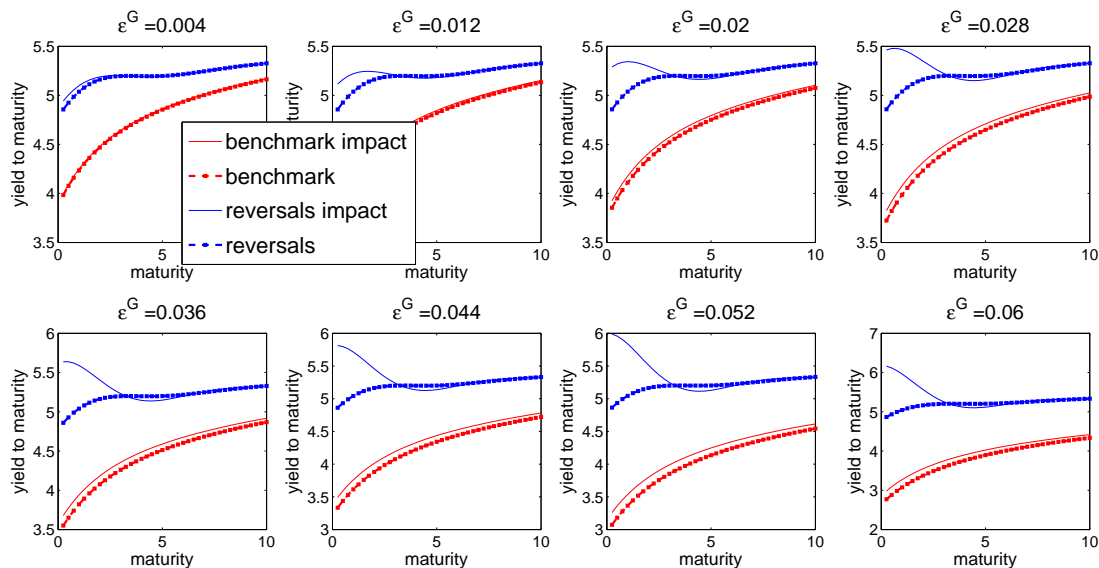
Figure 6: Impact impulse response function for different sizes of the shock. The stochastic steady state of the term structure and the impact of increase in government spending on the yield curve. The red lines are the benchmark case. The blue lines represent the commitment to spending consolidation.

4.1.6 Spending Reversals

We introduce credible commitment of fiscal policy into the model such that government reduces its expenditure when government debt increases. The government spending is therefore an endogenous function of the government debt and fiscal policy decisions about government spending are history dependent. The benchmark model augmented by spending reversals predicts that there will be no crowding out of private investment by government. Households work more in a response to increases in aggregate demand. Higher government spending is financed through extra taxes and government debt. The price of debt rises to encourage additional savings. Expectations about future lower than steady state government spending imply lower future taxes and debt pushing the future expected interest rates down. Higher future disposable income makes households form expectations about future higher consumption. The intertemporaral smoothing assumption raises the current level of consumption and discourages savings.

Figure 6 depicts the impact impulse response functions of the term structure of interest rates for different sizes of the government spending shock. The red line represents the yield curve implied by the benchmark model and blue line shows the term structure of interest rates when fiscal policy commits to spending reversals. In an economy with spending reversals $i)$ the yield curve is immune to the degree of fiscal uncertainty, $ii)$ the insurance and nominal term premium are both lower than in the benchmark case.

The impact response of term structure of interest rates is driven by the intertemporal substitu-

tion effect (expectation hypothesis). The impact on yield curve stochastic steady state is negligible. The precautionary saving component of the term structure is neutralized in the presence of spending reversals. The transitory increase in government spending shows up at the short tail and is driven by the rise in debt and intertemporal smoothing motives from households. Fiscal policy commitment to finance temporarily higher spending by future austerity significantly decreases the price of risk related to uncertainty about government spending. Figure 16 and figure 17 demonstrate that the risk premiums are negligible for both monetary policy regimes considered. The history dependence in otherwise stochastic evolution of debt introduces into the model economy a new source of information. The increase in predictability of the evolution of debt and taxes mitigates the impact of uncertainty (second order terms) on macroeconomic variables. This fact helps the investor to form a more accurate expectation. Larger time t conditional information set decreases the risk of bond miss-pricing, therefore the risk premia are lower.

4.2 THE THEORETICAL DECOMPOSITION

The principal issue of the discussion in Section 4.1 was concentrated on the effect of government spending on the term structure of interest rates. Yet, the impact of government spending on the yield curve is not direct but propagates through the macroeconomic fundamentals. We are interested in disentangling the transmission and quantitatively evaluating the importance of specific channels. We motivate the decomposition and show how to quantitatively evaluate the channels of the transmission mechanism.

As emphasized by Kreps and Porteus (1978) and Backus, Routledge, and Zin (2004) the recursive preferences in the form considered in this paper cannot be reduced by simply integrating out future information about the consumption process. Instead the timing of information has a direct impact on preferences and the intertemporal composition of risk matters. To illustrate how macroeconomic risk factors enter the pricing equation we analytically derive the second order approximation to the pricing kernel in terms of conditional second moments of macroeconomic fundamentals. The unconditional mean of the price of bond with maturity n can be written as [20]

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[20] Detailed derivation can be found in appendix.

$$
\begin{aligned}
E[\widehat{ytm}^n] &= -\frac{1}{2n}E\left[\mathrm{Var}_t\sum_{j=1}^{n}\left(\hat{\zeta}_{t+j}\right)\right] - \frac{\gamma^2}{2n}E\left[\mathrm{Var}_t\left(\Delta^n\hat{\mathbf{c}}_{t+n}\right)\right] - \frac{1}{2n}E\left[\mathrm{Var}_t\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right)\right] \\
&+ -\frac{\alpha^2}{2n}E\left[\mathrm{Var}_t S_{t+n}\left(\cdot\right)\right] + \frac{\gamma}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j}, \Delta^n\hat{\mathbf{c}}_{t+n}\right)\right] \\
&+ \frac{1}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right)\right] - \frac{\gamma}{n}E\left[\mathrm{Cov}_t\left(\Delta^n\hat{\mathbf{c}}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right)\right] \\
&+ \frac{\alpha}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j}, S_{t+n}\left(\cdot\right)\right)\right] - \frac{\gamma\alpha}{n}E\left[\mathrm{Cov}_t\left(\Delta^n\hat{\mathbf{c}}_{t+n}, S_{t+n}\right)\right] \\
&- \frac{\alpha}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+n}, S_{t+n}\right)\right]
\end{aligned}
\tag{25}
$$

where $S_{t+n}\left(\sum_{j=0}^{\infty}\beta^j\left[a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j} - b\hat{\mathbf{n}}_{t+j}\right]\right)$ can be interpreted as the revaluation in the expectations or long-run risk or news (surprise) about the future path of consumption and leisure. The long-run risk is directly related to the predictability in consumption and leisure. Therefore understanding the source of the long risk in our model economy is equivalent to understanding how the dynamic behavior of consumption and leisure is determined. The parameter α determines if agents prefer early or late resolution of uncertainty. Early resolution of uncertainty means that agents wish to smooth consumption over the state of nature rather than over time[21]. Equation (25) shows how the quantity of macroeconomic risk translates into the bond prices. There are two sources of risk: $i)$ precautionary savings represented by the variance terms $ii)$ leverage/hedging effect demonstrated by the covariance terms.

The precautionary saving motive highlights the risk aversion of investors to macroeconomic volatility. Risk averse consumption smoothing investors buy bonds as a form of insurance against unpredictable stochastic shocks. The cost of the shocks depends on the context, negative news is more costly in a recession than in an economic boom. In what follows we focus on detailed discussion of the risk premia generated by the covariance terms (context).

The term $\mathrm{Cov}_t\left(\Delta^n\hat{\mathbf{c}}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right)$ captures the risk that shock is bad news for realized consumption growth and at the same time inflation undermines the real value of savings. High inflation in periods of low consumption is especially hurtful for bond holders because bonds lose their value exactly when the consumption smoothing households need their savings most. This risk component was highlighted especially in models using habit formation (e.g. Hordahl, Tristani, and Vestin (2007).

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[21]Note that $\alpha = -108$, model is calibrated to feature preference for early resolution of uncertainty. The intuition and calibration behind α is discussed in appendix B.3 .

The $\mathrm{Cov}_t \left(\sum_{j=1}^{n} \hat{\pi}_{t+n}, S_{t+n} \right)$ premium compensates investors for the risks inflation presents for long-run consumption and leisure. This term embodies the aversion of investors to the drop in consumption and leisure lasting for extended periods of time. They are willing to pay a high price for bonds which pay well in real terms throughout the whole period of low consumption and leisure. In other words, investors require a high risk premium for holding risk affecting the bond portfolio real value for many periods. This risk attribute plays a crucial role in explaining the differences of yield curve response to government spending in different monetary policy setups. Inflation as a carrier of bad news for future consumption growth was empirically documented for example by Piazzesi and Schneider (2007).

Fisher (2015) shows that there exists a mapping between preference shock and the demand for safe and liquid assets. The term $\mathrm{Cov}_t \left(\sum_{j=1}^{n} \hat{\zeta}_{t,t+j}, \sum_{j=1}^{n} \hat{\pi}_{t+j} \right)$ thus represents the risk that rise in demand for safe assets (negative preference shock) will be accompanied by growth in inflation. For instance, the flight to safety as a consequence of market crashes leads to reallocation from stocks to bonds. If the increase in demand for safe assets creates inflationary pressures, bond holders will ask for the risk premium to compensate them for the loss in the real value of bonds[22].

The term $\mathrm{Cov}_t \left(\Delta^n \hat{c}_{t+n}, S_{t+n} \right)$ shows that it depends what realized consumption growth predicts for the expected consumption and leisure. Kaltenbrunner and Lochstoer (2010) discuss the implication of this term in the model without labor-leisure choice and with technology shocks only. They argue that investors with Epstein Zin preferences demand a premium for holding bonds when shock to realized consumption growth is positively correlated with shock to expected consumption growth. In the case of negative correlation the expected consumption and leisure works as a hedge against the realized drop in consumption. A similar transition works in the case of government spending shocks. A transitory shock to government spending is expected to revert back to its long-run trend. Thus, while the shock to realized consumption growth is negative (positive), the shock to expected future long-run consumption growth is positive (negative) as consumption reverts to the long-run trend. If agents have a preference for early resolution of uncertainty as suggested by the empirical literature, and thus dislike shocks to both realized and expected consumption growth, the long-run risk component acts as a hedge for shocks to realized consumption growth and the real term premium is lower. For this reason Kaltenbrunner and Lochstoer (2010) argue in their paper that investors need to form preferences for late resolution of uncertainty to match the high price of risk found in data. The previous line of logic does not necessary hold in the model with labor-leisure choice where the negative shock to realized consumption growth is followed by upward reevaluation in expected consumption growth but downward revision in expected leisure time. If the adjustment in leisure is strong enough the implication for timing of uncertainty resolution might be reversed. Households' future con-

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[22]Taking advantage of this interpretation and complying with the estimates of Andreasen (2012b) we introduce exogenous changes in the preferences even if that large part of the nominal term premium is in this way explained exogenously. To show that our key results do not depend on the preference shock we perform our analysis also in the Rudebusch and Swanson (2012) model.

sumption growth is driven by an even higher increase in hours worked turning the covariance term to positive[23]. Agents with a preference for early resolution of uncertainty will ask an extra premium for holding bonds in such a case.

The term $\text{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j}, S_{t+n}\left(\cdot\right)\right)$ indicates that when long-run consumption and leisure respond positively to increases in the preferences for safe assets then the long-run effects work as a hedge against the shifts in preferences. Negative covariance between preference shock and long-run consumption and leisure risks on the other hand implies that preference shocks will tend to increase the real term premium.

Finally, the term $\text{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j}, \Delta^n\hat{\mathbf{c}}_{t+n}\right)$ represents the fact that bond holders ask for a premium when the increase demand for current consumption (positive preference shock) is positively correlated with realized consumption growth. Note that a preference shock increases bond prices directly through the pricing kernel. When the positive preference shock associated with increase in price of bonds generates drop in realized consumption growth the price of bond portfolio increases exactly when agents need it the most (in times of low consumption) and thus pushes the risk premium down.

Although the preference shock is important for the nominal term premium, it does not affect the shift in the yield curve induced by government spending shock as we assume shocks are not correlated. To separate the quantity of macroeconomic risk from the price of risk, we rewrite the covariance terms in the equation (25) using correlations. Correlations represent the price of risk which is independent of the volatility of exogenous shocks and depends solely on the model lay out. Transmission of change in quantity of risk coming from government spending to the change in risk premium can be expressed by[24]:

$$
\begin{aligned}
E\left[\frac{\partial \widehat{ytm}_t^n}{\partial \sigma_g}\right] &= -\frac{1}{2n}\left\{\gamma^2 \frac{\partial \sigma_{\Delta\hat{\mathbf{c}}}^2}{\partial \sigma_g} + \frac{\partial \sigma_{\hat{\pi}}^2}{\partial \sigma_g} + \alpha^2 \frac{\partial \sigma_S^2}{\partial \sigma_g}\right\} \\
&\quad - \frac{\gamma}{n}\frac{\partial(\sigma_{\hat{\mathbf{c}}}\sigma_{\hat{\pi}})}{\partial \sigma_g}\rho_{\Delta\hat{\mathbf{c}},\hat{\pi}} - \frac{\gamma\alpha}{n}\frac{\partial(\sigma_{\hat{\mathbf{c}}}\sigma_S)}{\partial \sigma_g}\rho_{\Delta\hat{\mathbf{c}},S} - \frac{\alpha}{n}\frac{\partial(\sigma_{\hat{\pi}}\sigma_S)}{\partial \sigma_g}\rho_{\hat{\pi},S} \qquad (26)
\end{aligned}
$$

where $\left[\frac{\partial \widehat{ytm}_t^n}{\partial \sigma_g}\right]$ is the change in the stochastic steady state of the nominal term structure of interest rates induced by change in government spending uncertainty, σ_x^2 stands for the conditional volatility of variable x. The correlation between variable x and y is denoted by $\rho_{x,y}$ The correlations among variables are not affected by the size of the shock[25].

The issue here is that in general $E_t Var_{t+j-1}x_{t+j} \neq Var_t x_{t+j}$ for $j \geq 1$, thus one cannot quantitatively evaluate the above decomposition of the pricing kernel based on the ex-post simulated

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[23]Nevertheless, as Gali (1999) argues the covariance between hours and productivity is negative or near zero in the data.

[24]Derivations are in the appendix.

[25]This can be seen for example also from impulse response functions, the direction of response of variables to government spending shock is independent of the shock size.

data.[26] The price of uncertainty arises through the second order terms in the conditional expectations about the risk factors. Unconditional moments of model generated data don't price the risk as the uncertainty has already materialized. We thus turn to what we call attribution analysis and explain how to quantitatively evaluate the conditional moments.

4.3 ATTRIBUTION

In this section we propose a method to quantitatively evaluate the specific channels of the transmission mechanism as discussed in the section 4.2. The price of bonds contains compensation for risk related to macroeconomic fundamentals.

The second order approximation of the benchmark model pricing kernel points to four risk factors driving term structure, i) consumption growth, ii) inflation, iii) long-run risk [27], iv) preference shock. To track the propagation of exogenous shock to yields through macroeconomic factors is complicated by the fact that the effects of the factors are cross correlated. For instance, in the case of two factors, consumption growth and inflation, the term structure of interest rates can be written as a composite function $ytm(c(G, \pi(G)), \pi(G, c(G)))$. The yield curve moves in response to G shock because consumption directly adjusts to the new level of government expenditure and because consumption responds to the new inflation rate. Taking the derivative with respect to G delivers $\frac{\partial ytm}{\partial c} \left[\frac{\partial c}{\partial G} + \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial G} \right] + \frac{\partial ytm}{\partial \pi} \left[\frac{\partial \pi}{\partial G} + \frac{\partial \pi}{\partial c} \frac{\partial c}{\partial G} \right]$. In the following analysis we quantify the change in yields driven separately by factor stand alone effects, consumption growth $\frac{\partial ytm}{\partial c} \frac{\partial c}{\partial G}$ and inflation $\frac{\partial ytm}{\partial \pi} \frac{\partial \pi}{\partial G}$ and the interaction effect coming from the factor cross derivatives, $\frac{\partial ytm}{\partial c} \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial G} + \frac{\partial ytm}{\partial c} \frac{\partial \pi}{\partial G} \frac{\partial c}{\partial G}$. For n factors the derivative of composite function can be written

$$\frac{\partial ytm}{\partial G} = \sum_{i=1}^{n} \sum_{j=1}^{n-1} \frac{\partial ytm}{\partial F_i} \left[\frac{\partial F_i}{\partial G} + \frac{\partial F_i}{\partial F_j} \frac{\partial F_j}{\partial G} \right] \quad \text{for } i \neq j \tag{27}$$

where F stands for the macroeconomic factor driving the yield curve dynamics. To decompose the effects of changes in government spending on the yield curve we use the idea of Brinson multi-factor model[28] (Brinson and Fachler (1985)). Figure 7 illustrates the idea behind the decomposition. Without loss of generality let us abstract from the preference shock for now and consider only the remaining three factors. We start the analysis at the deterministic steady state where the term structure is just a flat line at $\frac{1}{\beta}$. Adding the stand alone risk factor increases the level of yield curve to the factor specific node. In terms of equation (27) we quantify the first term after multiplying the bracket. However, factors interactions contribute to the change

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[26]Note that even if we average over the ergodic distribution of the yield curve to calculate the unconditional mean of the term structure the variance and covariance in the decomposition are still conditional on the information in time t.

[27]The long-run risk may be interpreted in several ways as highlighted in Epstein and Zin (1989). The crucial point is that time to resolve uncertainty matters thus shocks to continuation value matters.

[28]this version of factor model is widely use in portfolio management for return attribution analysis
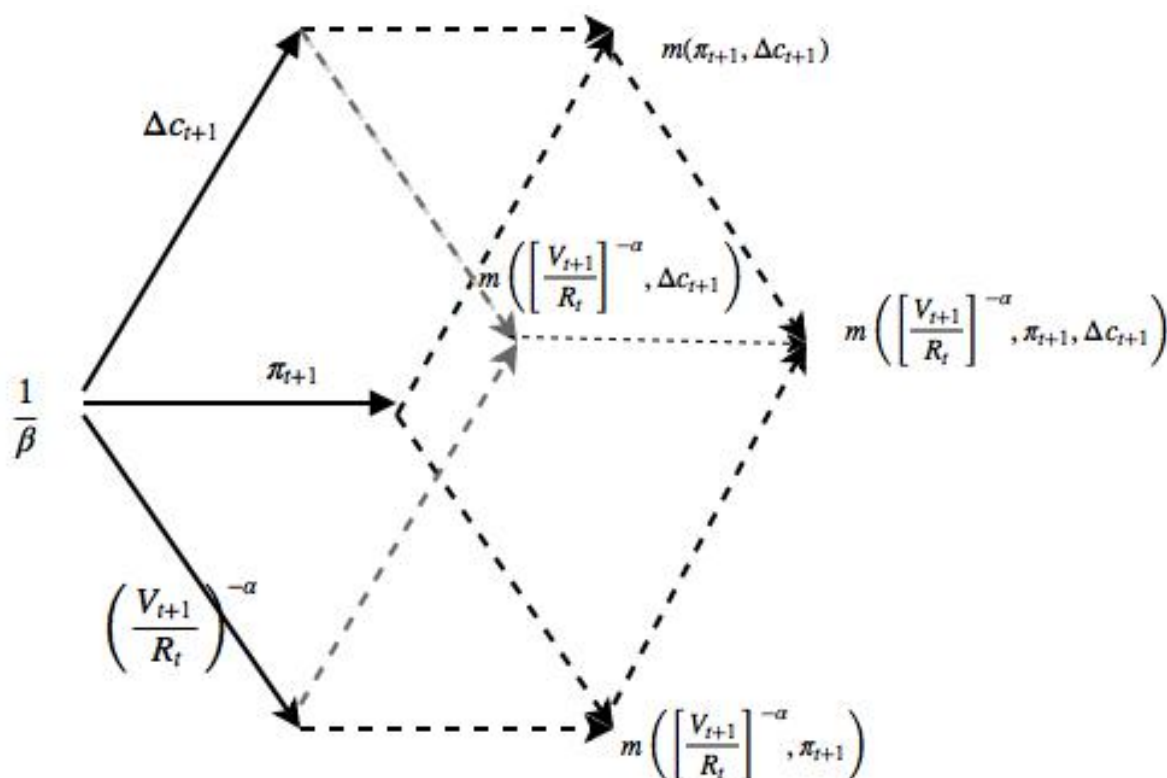
Figure 7: Intuition behind the decomposition.

in the yield curve as well. Thus, we need to calculate the factor cross derivatives as well. In figure 7 this is represented by the nodes at the dashed lines intersection. For example, the total effect of changes in consumption growth and inflation on the yield curve is the sum of the stand alone impacts, Δc_{t+1}, π_{t+1}, and their interaction, $m(\Delta c_{t+1}, \pi_{t+1})$. Considering all three factors in figure 7, the total change is the sum of risks attributed to the stand alone factors, interaction of two factors and interaction of all three factors together. In general, the total effect in the n-factor pricing equation can be decomposed into n groups of factor interactions and stand alone factor risks.

Figure 7 demonstrates how to calculate the risk groups within our macro model. Let's again focus only on two factors, consumption growth and inflation. First, calculate the yield curve within the macro model where the pricing equation contains only consumption growth or inflation. Second, subtract the determinist steady state. In this way we can isolate the individual contribution of inflation and consumption growth as a risk factor in pricing equation. Third, evaluate the model with both risk factors and subtract the stand alone risks factors calculated in the previous step and subtract again the determinist steady state to find the attribution of the factors interaction. More formally,

$$R_1 = \sum_i^n (M(F_i) - M(st.st.)) \tag{28}$$

$$R_{2,i} = \sum_i^n \sum_j^n M(F_i, F_j) - R_1 - M(st.st.)) \tag{29}$$

$$R_g = \sum_g^n \sum_i^n \sum_j^n (M(F_i, F_j, \dots F_g) - R_{n-1} - R_{n-2} \dots R_1) \tag{30}$$

where $M(F_i)$ is the model with the risk factor i and $M(st.st.)$ is the model at the steady state.

Up to the second order approximation the interaction terms of two factors correspond to the covariance terms in the equation (25) and stand alone factors represent the variances. Higher order interactions are non-zero only for higher order approximation. This can be very clearly seen from the second order approximation of the term structure. We define the purely risk-free rate as $ytm_t^{rf,n} = \frac{\beta}{\bar{\pi}}$ which serves as a benchmark. The pricing kernel reflecting the consumption growth risk is $ytm_t^{c,n} = \frac{\beta}{\bar{\pi}} - \frac{\gamma^2}{2n}\text{Var}_t(\Delta^n \hat{\mathbf{c}}_{t+n})$. The risk premium attributed to the consumption growth, rp_c is then

$$E[ytm_t^{c,n} - ytm_t^{rf,n}] = -\frac{\gamma^2}{2n}E\text{Var}_t(\Delta^n \hat{\mathbf{c}}_{t+n}) \tag{31}$$

Up to the second order, the pricing kernel accounting both for nominal and real risk can be written as

$$Eytm_t^{c,\pi,n} = \frac{\beta}{\bar{\pi}} - \frac{\gamma^2}{2n}E\text{Var}_t(\Delta^n \hat{\mathbf{c}}_{t+n}) + \frac{1}{2n}E\text{Var}_t \sum_{j=1}^n (\hat{\pi}_{t,t+j}) - \frac{\gamma}{n}E\text{Cov}_t\left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^n \hat{\pi}_{t+j}\right) \tag{32}$$

thus we can calculate the covariance term as the difference between the total risk premium, $E[ytm_t^{c,\pi,n} - ytm_t^{rf,n}]$, and the risk premia of individual factors, rp_c, rp_π.

$$E[ytm_t^{c,\pi,n} - ytm_t^{rf,n} - RP_i] = \frac{\gamma}{n}E\left[\text{Cov}_t\left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^n \hat{\pi}_{t+j}\right)\right] \tag{33}$$

where the sum of stand alone risk premiums is $RP_i = rp_c + rp_\pi$. Adding other factors and calculating the risk premiums follows the same pattern.

4.4 PRECAUTIONARY SAVING EFFECT

In the first step we decompose the level of the yield curve into the deterministic and stochastic part[29]. We can use the equation (71) to write one period yield to maturity as

$$-ytm_t = \underbrace{E_t q_{t,t+1}}_{\text{Intertemporal substitution}} + \frac{1}{2} \underbrace{\text{Var}_t(q_{t,t+1})}_{\text{Compensation for risk}} \tag{34}$$

where $q_{t,t+1}$ is the log deviation of one period stochastic discount factor from its deterministic steady state. To discourage agents from savings the interest rate that clears the market is affected by the intertemporal smoothing and compensation for risk. The unconditional mean of the intertemporal substitution part corresponds to the deterministic steady state. The variance term determines how uncertainty affects interest rates. To analyze how the compensation for risk affects the transmission mechanism of shocks, we need to understand the determinants of $\text{Var}_t(\hat{q}_{t,t+1})$. In our benchmark model, the average deterministic level of the yield curve is 5.72 percentage points, and it is constant over the maturity, the average yield to maturity of the stochastic component is -0.96 and is increasing with maturity. Figure 25 illustrates the difference between the stochastic steady, deterministic steady state, insurance premium, nominal term premium and yield curve consistent with the expectations hypothesis. In what follows we focus on explaining the bond's insurance premium generated by uncertainty about government spending.

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[29]precisely, we decompose the mean of the ergodic distribution of the stochastic system (stochastic steady state) to its deterministic and stochastic component

Stand alone factors	Benchmark		Gov spend $\phi_y = 0.075$		Gov spend $\phi_y = 0.0$		TFP		Mark up		Preference	
	Levels	%	L*100	%	L*100	%	Level	%	Level	%	Level	%
$-\frac{\gamma^2}{2n}Var_t(\Delta C_{t+n})$	-0.012	(1.3)	-0.016	(0.9)	-0.012	(1.3)	-0.008	(1.4)	0	(0.2)	-0.003	(1.17)
$-\frac{1}{2n}Var_t(\sum_{j=1}^n \pi_{t+n})$	-0.821	(85.4)	-1.04	(59.7)	-0.77	(78.5)	-0.36	(63.3)	-0.078	(100.1)	-0.37	(126.4)
$-\frac{\alpha^2}{2n}Var_t(S_{t+n})$	0.077	(-8.1)	0.08	(-5.1)	0.087	(-8.8)	0.05	(-8.8)	0	(0)	0.026	(-8.9)
$-\frac{1}{2n}\mathrm{Var}_t \sum_{j=1}^n (\zeta_{t,t+j})$	-0.02	(2.1)	0	(0)	0	(0)	0	(0)	0	(0)	-0.02	(6.84)
Factor interactions												
$-\frac{\gamma}{n}Cov_t(\Delta C_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.001	(-0.2)	0	(0.1)	0.009	(-0.9)	0.005	(-0.1)	0	(-0.2)	-0.004	(1.43)
$-\frac{\gamma\alpha}{n}Cov_t(\Delta C_{t+n}, S_{t+n})$	-0.283	(29.4)	-0.72	(41.8)	-0.63	(64.6)	-0.41	(73.18)	0	(0)	0.144	(-49.57)
$-\frac{\alpha}{n}Cov_t(S_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.418	(-43.5)	-0.004	(2.6)	0.34	(-34.7)	0.16	(-28.2)	0	(-0.1)	0.25	(-87.9)
$+\frac{\alpha}{n}Cov_t(S_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	-0.349	(36.4)	0	(0)	0	(0)	0	(0)	0	(0)	-0.34	(120)
$+\frac{\gamma}{n}Cov_t(\Delta C_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	0.016	(-1.7)	0	(0)	0	(0)	0	(0)	0	(0)	0.016	(-5.76)
$+\frac{1}{n}Cov_t(\sum_{j=1}^n (\zeta_{t,t+j}), \sum_{j=1}^n \pi_{t+n})$	0.01	(-1)	0	(0)	0	(0)	0	(0)	0	(0)	0.01	(-3.44)
Total												
$E[\frac{1}{n}\sum_{n=1}^{40} \hat{\mathbf{ytm}}^n]$	-0.96	(100)	-0.017	(100)	-0.099	(100)	-0.58	(100)	-0.078	(100)	-0.29	(100)

Table 3: Shows the factors attribution to the stochastic component of the yield curve. The reported numbers are averages over the maturity profile $n = 40$, each column shows the contribution of the attribute to the mean of ergodic distribution of the yield curve in levels and percentages ($\frac{f_i}{\sum_i f_i}$). The higher order interactions are zero up to the 2nd order approximation. The benchmark model in the second column is decomposed to contribution of single shocks to overall risk. $L * 100$ means that numbers in the column are multiplied by 100 for better readability. Note that parameters $\alpha = -108$, $\gamma = 2$.

Table 3 summarizes the results of the factor attribution. The variance of the pricing kernel is broken down to conditional moments of macroeconomic factors attributed to the risk premium and thus quantifies equation (25).

The variance of the macro variables triggers the precautionary savings behavior. Covariance terms determine the diversification property of macroeconomic fundamentals. By re-writing covariances in terms of correlation and standard deviation we can separate quantity of risk from the price of risk. The sign of correlation (price of risk) determines if the specific macro risk factor functions as a leverage or hedge against other macro factors. The standard deviation of the risk factor together with the coefficient of risk aversion, ψ, and intertemporal elasticity of substitution, $1/\gamma$, defines the intensity with which the risk is reflected in the bond prices. The second column of the table 3 represents our benchmark model using a full set of exogenous variables. The third column isolates the impact of government spending when monetary policy accommodates shocks to deviations in output gap ($\phi_y = 0.075$), the fourth column decomposes the fiscal risk when the monetary policy lets the money supply freely adjust ($\phi_y = 0$). The rest of the columns separate the effects of individual shocks on the yield curve average maturity stochastic steady state. Our main focus is on fiscal policy, therefore we concentrate in the following discussion mainly on the third and fourth column. Fiscal policy explains only a negligible part of the total risk in the economy for the benchmark calibration. Nevertheless, we have documented in the Section 4, that increase in the uncertainty about government spending strongly reflects in bond prices and uncertainty about fiscal policy dominates the compensation for risk. The relative importance of the transmission channel is independent of the degree of fiscal uncertainty. For example, when we double the volatility of government spending in the benchmark model, 59.7% of the increase in the risk still attributes to the volatility of inflation. In other words, the effect of government spending uncertainty on the risk compensations is proportionate.

In both monetary policy scenarios the compensation for precautionary saving effect (variance terms in table 3) comes from inflation risk and generates more than half of the bond insurance premium. The uncertainty associated with the size of government spending translates through the aggregate supply[30] into the uncertainty about inflation. The uncertainty about the level of inflation motivates agents to build up a precautionary saving buffer. Consequent higher demand for bonds pushes the yields down. The precautionary saving channel is dominant for other shocks as well.

We now turn to discussion of factor interactions (covariances) in table 3. Positive numbers in terms of % contributions represent the hedging property of bonds while the negative numbers constitute the leverage. To emphasize the intuition behind the hedging property of bonds against long-run risk generated by fiscal uncertainty consider the period impact of exogenous increase in government spending, $\epsilon^G > 0$ on the average deviation of bond yield to maturity

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[30]in response to positive government spending shock households compensate for higher current or future taxation by raising hours worked. The production function ensures rise in aggregate output which puts downward pressure on prices

from the deterministic steady state,

$$E[ytm_t^n] - \overline{ytm_t} \propto Cov\left(\frac{\partial \Delta C_{t+n}}{\partial \epsilon^G} < 0, \frac{\partial S_{t+n}}{\partial \epsilon^G}(C_{t+n}, L_{t+n}) > 0\right) < 0 \qquad (35)$$

Positive innovation in government spending lowers realized consumption growth through the wealth effect. Nevertheless, the transitory character of shocks implies positive reevaluation in expectations about the future path of consumption relative to after shock consumption. The positive news about future consumption thus provides a hedge against the realized drop in consumption growth. The hedging character of long-run risk is strengthened by the update in expectations about the future path of hours worked. Investors expect that they will work less in the future and at the same time their consumption grows. The predictability of the future consumption and leisure together with preference for early resolution of uncertainty means that the short run consumption risks are compensated by long-run consumption and leisure risk. This mechanism is in detail discussed by Kaltenbrunner and Lochstoer (2010) for productivity shocks. In general, this holds for most transitory shocks apart from preference shocks which generates a rise in consumption growth, a consequent sharp drop and then again a rise creating positive covariance between realized consumption growth and long-run consumption growth.

Figure 20 reports the comparison of impulse response functions for the main macroeconomic variables in the regime with $\phi_y = 0.075$ and $\phi_y = 0$. The core difference between the regimes lies in the behavior of inflation which rises in response to positive government spending shock in the regime where monetary policy authority puts zero weight on output stabilization. The long-run risk drives the differences in response of risk premia to changes in the fiscal uncertainty. We consider again the case that the economy is hit by $\epsilon^G > 0$ and that monetary policy sets $\phi_y = 0$. The average deviation of bond yield to maturity from the deterministic steady is proportionate to,

$$E[ytm_t^n] - \overline{ytm_t} \propto Cov\left(\frac{\partial \sum_{j=1}^n \pi_{t+n}}{\partial \epsilon^G} > 0, \frac{\partial S_{t+n}}{\partial \epsilon^G}(C_{t+n}, L_{t+n}) > 0\right) > 0 \qquad (36)$$

If the monetary authority lets the money supply freely adjust in response to government spending shock the inflation risk is leveraged by the long-run consumption and leisure risk. Agents in the model know the impulse response functions of consumption and leisure to government spending shock which results in the degree of predictability of consumption and leisure after the shock is realized. Agents know that after a negative shock their consumption will be lower in the long-run relative to the pre-shock period and at the same time the real value of their bond portfolio drops due to the rise in inflation. Deterioration of the investor's wealth after a positive government spending shock is positively correlated with losses of his bond portfolio.

Whereas in case of monetary policy with non-zero weight on output gap, $\phi_y = 0.075$, shocks to

investor's wealth are hedged by the real value of his bond portfolio.

$$E_t[ytm_t^n] - \overline{ytm_t} \propto Cov\left(\frac{\partial \sum_{j=1}^{n} \pi_{t+n}}{\partial \epsilon^G} < 0, \frac{\partial S_{t+n}}{\partial \epsilon^G}(C_{t+n}, L_{t+n}) > 0\right) < 0 \qquad (37)$$

Monetary policy response to government spending shock is the key element determining the ability of bonds to provide diversification.

Interestingly, bonds do not offer any hedging or leverage against macroeconomic risk in case of mark up shocks. This is because the change in consumption is almost perfectly offset by the adjustment of hours worked (see figure **??**).

4.5 VARIATIONS IN UNCERTAINTY

Figures 1, 16 and 17 document that a rise in fiscal uncertainty is followed by a drop in the level[31] of the yield curve. The change in the yield curve is driven solely by the quantity of risk. The correlation between macro factors is not affected by the changes in the volatility. Note that we assume that exogenous shocks in our model are not cross-correlated, thus the interaction of macroeconomic factors with the preference shock does not play any role thus we can omit the shocks to demand for safe assets in the pricing equation. To explain which factors drive the yield curve decrease as a response to the rise in fiscal uncertainty we rewrite equation (25) in the following form,

$$
\begin{aligned}
E_t\left[\frac{\widehat{\partial ytm_t^n}}{\partial \sigma_g}\right] &= -\frac{1}{2n}\left\{\gamma^2 \frac{\partial}{\partial \sigma_g}\text{Var}\left(\Delta^n \hat{\mathbf{c}}_{t+n}\right) + \frac{\partial}{\partial \sigma_g}\text{Var}\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right) + \alpha^2 \frac{\partial}{\partial \sigma_g}\text{Var}S_{t+n}\left(\cdot\right)\right\} \\
&\quad - \frac{\gamma}{n}\frac{\partial(\sigma_{\hat{\mathbf{c}}}\sigma_{\hat{\pi}})}{\partial \sigma_g}\text{Corr}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right) - \frac{\gamma\alpha}{n}\frac{\partial(\sigma_{\hat{\mathbf{c}}}\sigma_S)}{\partial \sigma_g}\text{Corr}\left(\Delta^n \hat{\mathbf{c}}_{t+n}, S_{t+n}\right) \\
&\quad - \frac{\alpha}{n}\frac{\partial(\sigma_{\hat{\pi}}\sigma_S)}{\partial \sigma_g}\text{Corr}\left(\sum_{j=1}^{n}\hat{\pi}_{t+n}, S_{t+n}\right)
\end{aligned}
\qquad (38)
$$

Equation (38) highlights the fact that since the parameters and correlations are constant the change in the level of the yield curve is coming from the change in volatility of macroeconomic factors.

Table 8 shows that the relative contribution of the macro factors to the change in yields is proportionate to the rise in shock volatility and these proportions are constant in the size of uncertainty.

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[31]change in the mean of ergodic distribution of the stochastic process averaged over the maturity profile implied by the change in the volatility of the innovations in government spending shock
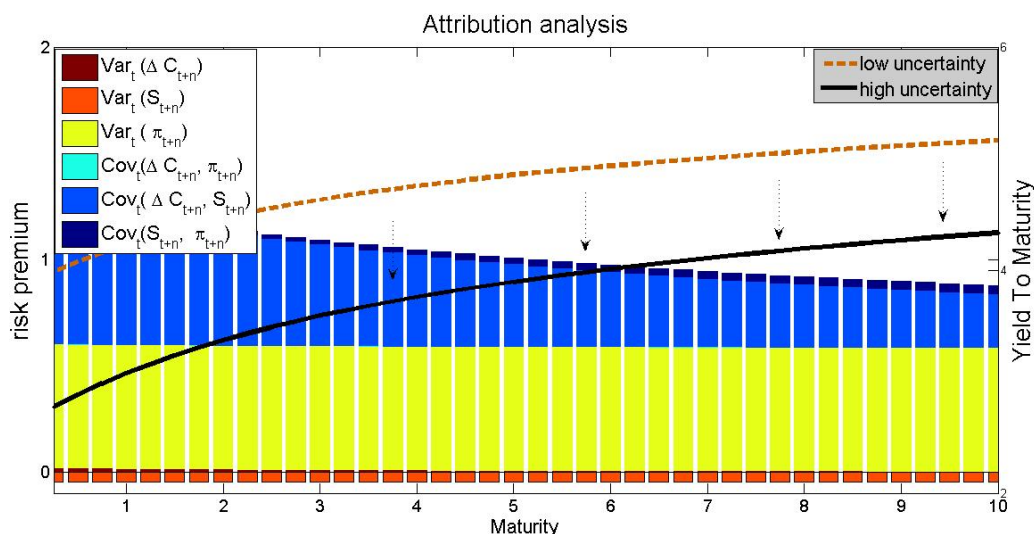
Figure 8: The drop in the yield curve due to the increase in the fiscal uncertainty (right hand axes) in monetary policy regime with ($\phi_y = 0.075$) is decomposed into the factors attributed to the shift in the risk premium (left hand axes).

The uncertainty associated with higher volatility of government spending increases the overall macroeconomic quantity of risk in the economy. This is especially hurtful in case of inflation. The increase in volatility of inflation directly translates into the volatility of the real value of bond portfolio. The uncertainty about the future inflation motivates agents to build up a precautionary saving buffer. Higher macroeconomic volatility increases the total amount paid for hedging/leveraging the risk.

4.6 FACTOR ATTRIBUTION OVER THE MATURITY PROFILE

The figure (8) shows how macroeconomic factors attribute to the drop of the term structure of interest rates when there is an increase in uncertainty and monetary policy authority sets $\phi_y = 0.075$. The hedging property of long-run risks against the realized consumption growth decreases with maturity as the predictability of consumption and leisure worsens in the long horizon.

The figure (9) shows the decomposition of the drop in the yield curve in response to increase in fiscal uncertainty when monetary policy authority sets $\phi_y = 0$.

The hedging property of long-run risks to counterbalance the drop in realized consumption risks decreases with maturity as in the case when $\phi_y = 0.075$. However, the positive covariance of inflation with long-run risks increases over the maturity profile. A persistent increase in inflation means that a real bond portfolio looses its real value more over the longer horizon than at the impact period. Bonds with longer maturity face by default higher inflation risks in the model.
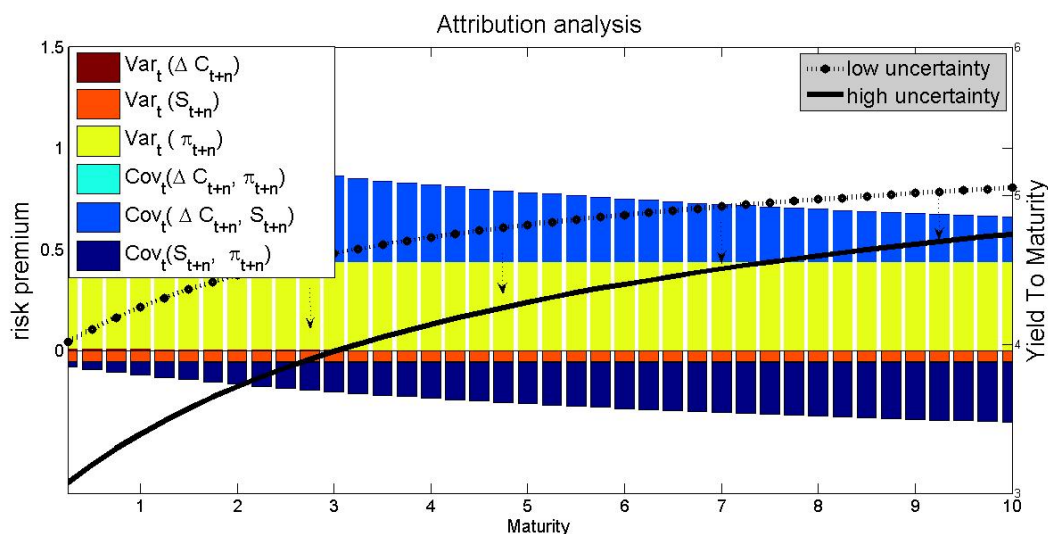
Figure 9: The drop in the yield curve due to the increase in the fiscal uncertainty (right hand axes) when monetary policy regime sets ($\phi_y = 0$) is decomposed into the factors attributed to the shift in the risk premium (left hand axes).

4.7 NOMINAL TERM PREMIUM

Table 4 shows the attribution of macroeconomic factors to nominal term premium[32].

A large part, 63%, of the nominal term premium is explained exogenously by preference shocks. Andreasen (2012b) estimates the term structure and shows that 59% of $10Y$ NTP is explained by preference shocks.

The covariance of long-run risk with inflation is an important part of the overall NTP confirming the results of Rudebusch and Swanson (2012), Andreasen (2012b) or Piazzesi and Schneider (2007). When the monetary policy regime responds directly to the fluctuations in the output gap after government spending shock, bonds provide good protection for their holders against the nominal risks and thus contribute to the lower NTP.

The transitory character of shocks means that the hedging property of bonds implied by the transitory character of shocks gets weaker as the shock returns back to its steady state and therefore is relatively less important for long maturity bonds. This is why $Cov_t(\Delta C_{t+n}, S_{t+n})$ increases the real term premium but decreases the risk premium. This is the reason why this class of models can generate a sizable term premium despite the Kaltenbrunner and Lochstoer (2010) result.

To show that our key results related to fiscal policy don't rely on the preference shock we perform our analysis also in the Rudebusch and Swanson (2012) model. See appendix for the robustness checks.

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[32]The figure 25 demonstrates how the nominal term premium differs from the insurance property of bonds. NTP is the same as the slope of the term structure in our benchmark model.

Stand alone factors	Benchmark		Gov spend $\phi_y = 0.075$		Gov spend $\phi_y = 0.0$		TFP		Mark up		Preference	
	Levels	%	L*100	%	L*100	%	Level	%	Level*100	%	Level	%
$-\frac{\gamma^2}{2n}\Delta^n Var_t(\Delta C_{t+n})$	0.02	(1.7)	0.03	(4.1)	-0.022	(2)	0.005	(1.3)	0.167	(446)	-0.012	(1.7)
$-\frac{1}{2n}\Delta^n Var_t(\sum_{j=1}^n \pi_{t+n})$	-0.014	(-1.2)	0	(0)	0	(0)	-0.004	(-1.1)	-0.014	(-37.6)	-0.009	(-1.3)
$-\frac{\alpha^2}{2n} Var_t(S_{t+n})$	0.074	(0)	0	(0)	0	(0)	0	(0)	0	(0)	0	(0)
$-\frac{1}{2n}\Delta^n Var_t \sum_{j=1}^n (\zeta_{t,t+j})$	-0.074	(6.2)	0	(0)	0	(0)	0	(0)	0	(0)	0.073	(9.8)
Factor interactions												
$-\frac{\gamma}{n}\Delta^n Cov_t(\Delta C_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.008	(0.7)	0	(0.1)	0	(0)	0.005	(1.7)	0.139	(-372)	-0.002	(0.3)
$-\frac{\gamma\alpha}{n}\Delta^n Cov_t(\Delta C_{t+n}, S_{t+n})$	-0.173	(-14.6)	0.73	(105.2)	0.637	(56)	0.15	(36)	0.022	(58.4)	-0.33	(-44.8)
$-\frac{\alpha}{n}\Delta^n Cov_t(S_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.528	(44.5)	-0.06	(-9.2)	0.479	(42)	0.266	(62)	0	(5.3)	0.26	(35)
$+\frac{\alpha}{n}\Delta^n Cov_t(S_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	0.81	(68.3)	0	(0)	0	(0)	0	(0)	0	(0)	0.81	(108.1)
$+\frac{\gamma}{n}\Delta^n Cov_t(\Delta C_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	-0.061	(0)	0	(0)	0	(0)	0	(0)	0	(0)	-0.06	(-8.1)
$+\frac{1}{n}\Delta^n Cov_t(\sum_{j=1}^n (\zeta_{t,t+j}), \sum_{j=1}^n \pi_{t+n})$	-0.006	(0)	0	(0)	0	(0)	0	(0)	0	(0)	-0.005	(-0.7)
Total												
NTP^{10Y}	1.185	(100)	-0.0069	(100)	0.0136	(100)	0.429	(100)	0.00037	(100)	0.74	(100)

Table 4: Shows the factors attribution to nominal term premium contained in a 10 year bond. Each column shows the contribution of the attributes to NTP levels and percentages ($\frac{f_i}{\sum_i f_i}$). The higher order interactions are zero up to the 2nd order approximation. The benchmark model in the second column is decomposed to contribution of single shocks to overall risk. $L*100$ means that numbers in the column are multiplied by 100 for better readability. $\alpha = -108$, $\gamma = 2$.

5. CONCLUDING REMARKS

We study the impact of wasteful, productive and utility enhancing government expenditures on the term structure of interest rates. We decompose the pricing kernel into the underlying macroeconomic factors that directly determine the bond prices in our model: consumption growth, inflation, time preference and long-run risks for consumption and leisure. We take advantage of the linear character of perturbation solution and design new method to recursively quantify these factors.

The character of government expenditures affects both term and risk premia. Bonds carry large insurance premium if government expenditures are wasteful (defense expenditure). Bonds serve as poor hedging instrument against productive (infrastructure expenditures) and utility enhancing (health, education expenditures) risk in government expenditures. Spending reversals limit the insurance property of bonds against wasteful government expenditures.

Further, using the structural decomposition into macroeconomic factors we confirm the findings of empirical literature stressing the importance of inflation risk for bond prices. However, we emphasize, that the size of inflation risks is highly sensitive to monetary policy conduct and type of exogenous shock. We show that when central bank sets the nominal interest rates in response to changes in output bonds act as a hedging instrument against inflation risk because of the negative correlation of consumption with inflation. On the other hand, bonds leverage inflation risks for investors wealth if central bank targets strictly inflation. Furthermore, bonds help to transfer wealth in time, they enable to use current consumption against future losses and the other way around. Bonds thus may hedge real risks if the future consumption is negatively correlated with the current consumption.

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A. ROBUSTNESS CHECKS

We test robustness of our results with respect to model and parameter specification. First, we identify the parameters driving the asset prices and vary these over the grid of admissible values. Second, we look if our results hold in the Rudebusch and Swanson (2012) model which has became very popular in the field.

A.1 MODEL EVALUATION

We report the model implied macro and finance moments along with the empirical moments for quarterly US data from 1961 to 2007. The table 5 demonstrates that the model is able to replicate the core macro-finance features reasonably well and comparably with the state of the art literature, e.i. Rudebusch and Swanson (2012), van Binsbergen, Fernndez-Villaverde, Koijen, and Rubio-Ramrez (2012).

The resulting model set up is a compromise between complexity and clarity. We focus on matching the factors driving the nominal term premium and fiscal policy to be as close to the data as possible. The somewhat poorer match of the variables not contained in the pricing kernel goes on the costs of keeping the model simple and tractable. Further in the paper, we also argue that the results are robust to a wide range of model specification and we analyze the sensitivity of the results to large grid of the underlying parameters values.

Moments	1961 - 2007	B	C	D	E	F	G
$SD(\Delta C_t)$	2.9*	2.42	3.48	2.58	2.51	2.49	3.22
$SD(C_t)$	0.83	2.07	2.25	2.19	2.16	2.16	2.61
$SD(N_t)$	1.71	2.22	2.61	2.15	2.13	2.08	2.46
$SD(\pi_t)$	2.52	1.20	1.48	1.20	1.27	1.27	1.27
$SD(i_t)$	2.71	2.61	3.16	2.62	2.72	2.72	2.72
$SD(w_t)$	0.82	3.50	4.42	3.78	3.72	3.72	3.96
$SD(r_t)$	2.30	1.93	2.34	1.93	1.96	1.95	2.08
$E(NTP_t)$	1.06	1.19	0.40	1.21	-0.01	1.62	1.07

Table 5: Empirical and model based unconditional moments. B is the benchmark model, C is the model with the spending reversals extension, D $\phi_y = 0$ is the benchmark model with zero weight on output gap in Taylor rule, E benchmark model with standard CRRA preferences, F represents benchmark model with $\sigma_G = 0.004$, G benchmark model with $\sigma_G = 0.06$, All variables are quarterly values expressed in percent. Inflation, interest rates are at an annual rate.

A.2 PARAMETER DEPENDENCE

First, we focus on the sensitivity of our results to parameters driving the monetary policy. As highlighted above, the conduct of monetary policy is an important determinant of the slope and

level of the term structure in response to government spending shock. For this reason, we test the robustness of our findings over the whole grid of Taylor rule estimates found in the data. Table 6 shows the estimated ranges in most influential recent studies quantifying the parameters' values in the Taylor rule.

Study	Period	ϕ_{pi}	ϕ_y
Taylor (1996)	1987 - 1997	1.53	0.77
Judd and Rudebush (1998)	1987 - 1997	1.54	0.99
Clarida, Gali and Gertler (2000)	1979 - 1996	2.15	0.93
Orphanides (2003)	1979 - 1995	1.89	0.18

Table 6: Taylor rule estimates for US.

We take the maximal boundary values for each parameter and plot the slope, level and their changes over the whole grid of parameter combinations. Figure 10 shows the average maturity stochastic steady state of the yield curve and its relationship between the weight monetary authority puts on inflation and output with respect to volatility of government spending shock. Figure 11 plots the same for the 10Y bond nominal term premium. We can see that when the volatility of the government spending shock is low it is the high weight on output gap stabilization which increases the stochastic steady state of the yield curve as well as the nominal term premium. On the other hand, when the volatility of government spending shock is high then it is the low weight on output gap stabilization increasing the level and NTP of the term structure. The economic story behind relates to the dichotomous situation of monetary policy authority when the high weight on inflation works well in accommodating productivity shocks but poorly with government spending shocks. In the economy with low fiscal volatility the relative importance of productivity shocks is much higher thus the low weight on stabilizing inflation generates big inflation long run risks and drives the level and NTP up.

Specifically we look what happens with the slope and level after the rise of volatility from $\sigma_G = 0.004$ to $\sigma_G = 0.06$. The upper part of figure 12 demonstrates that the drop in the term structure of interest rates after rise in fiscal uncertainty is independent of the choice of weights in the Taylor rule. In other words, the level of the yield curve decreases after the rise in volatility of government spending for any considered combination of weights on inflation and output. The bottom chart in Figure 12 shows that the nominal term premium of the term structure of interest rates rises with increase in volatility only if the weight on output gap stabilization in Taylor rule is very close to zero.

As documented elsewhere in the literature, see for example Rudebusch and Swanson (2012), Hordahl, Tristani, and Vestin (2007) or Kaszab and Marsal (2013), the size of the term premium is directly related to the coefficient of relative risk aversion. The micro and macro estimates of this parameters varies over the wide range of values. van Binsbergen, Fernndez-Villaverde, Koijen, and Rubio-Ramrez (2012) estimate risk aversion to be 79. In general, standard equilibrium models used in macroeconomics require rather high risk aversion to deliver the basic
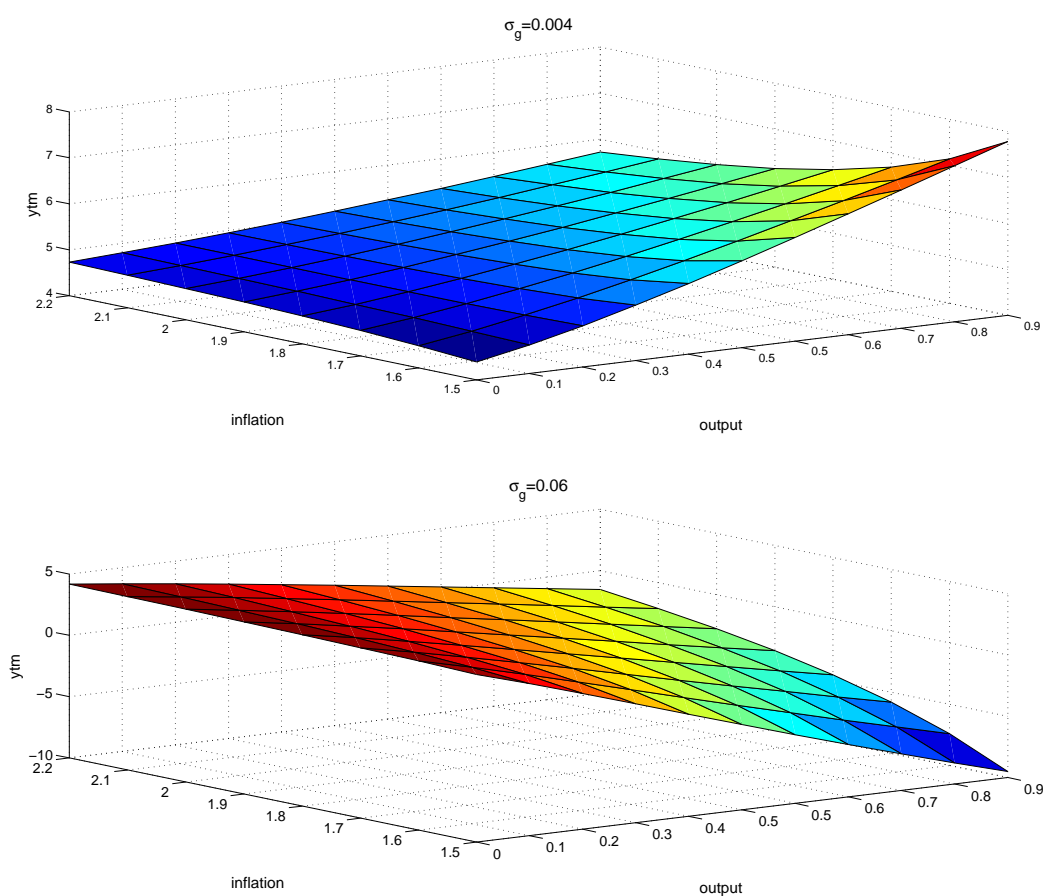
Figure 10: Average maturity stochastic steady state of the term structure of over the grid of Taylor rule regimes for volatility (upper one) $\sigma_G = 0.004$ and volatility $\sigma_G = 0.06$ (bottom).
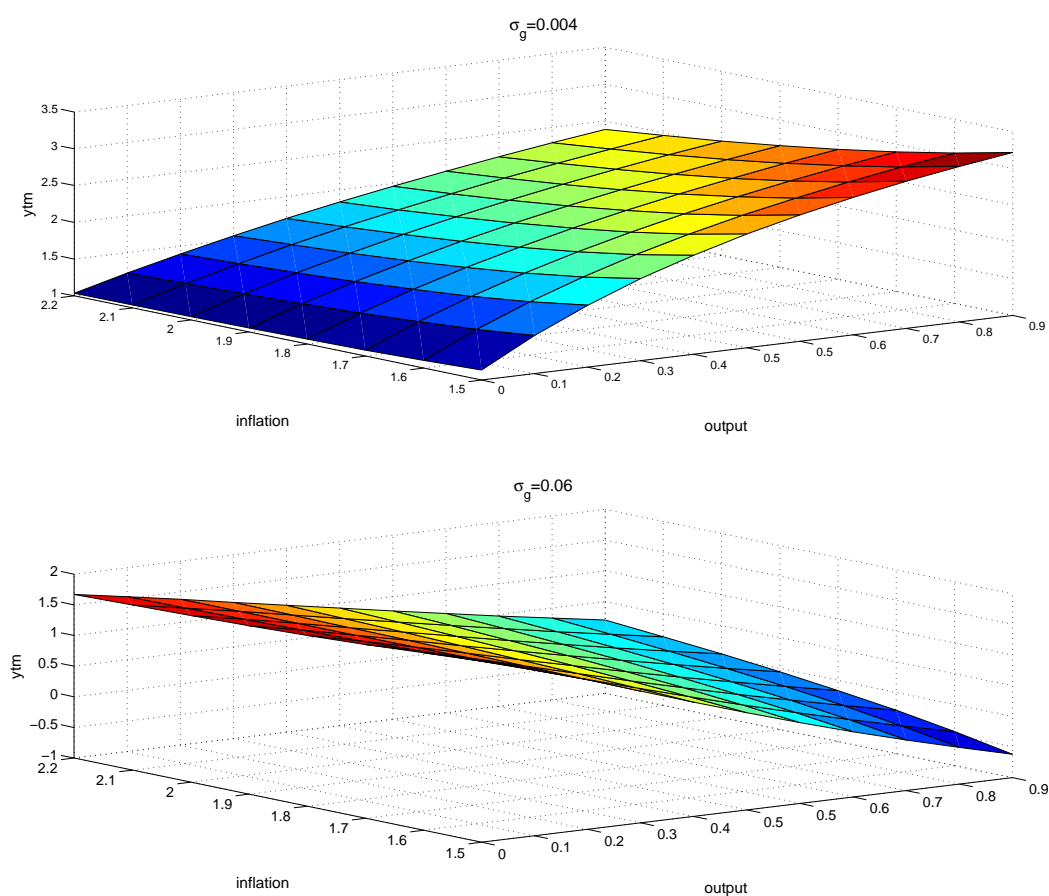
Figure 11: Average nominal term premium of 10Y bond over the grid of Taylor rule regimes for volatility (upper one) $\sigma_G = 0.004$ and volatility $\sigma_G = 0.06$ (bottom).
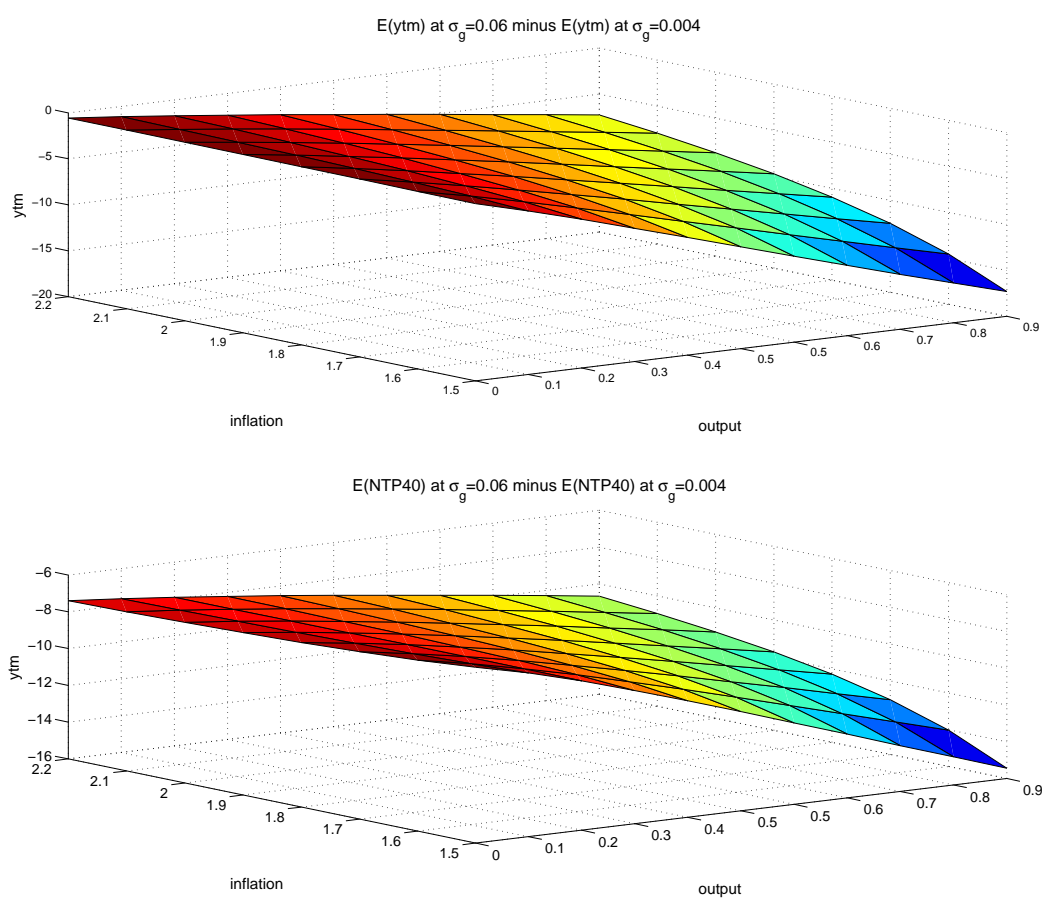
Figure 12: (upper one) E(ytm) with $\sigma_G = 0.06$ minus E(ytm) with $\sigma_G = 0.004$ and (bottom) NTP of 10Y bond at $\sigma_G = 0.06$ minus NTP of 10Y bond at $\sigma_G = 0.004$.

asset pricing stylized facts. Here we check robustness of our results to the range of sensible values of risk aversion. Further, we reproduce the chart in Rudebusch and Swanson (2012) and Kaszab and Marsal (2013) to directly compare relationship between the nominal term premium and coefficient of relative risk aversion.

Figure 13 shows that the maturity average of the stochastic steady state of the yield curve decreases with the increase in risk aversion. This is driven by the precautionary saving motive. The second part of the picture shows the nominal term premium. It is remarkable to find that in the economy with dominant fiscal shock the rise in risk aversion actually decreases the nominal term premium. This is because bonds provide hedge against rise in inflation (not just nominal but also real bond price increases when inflation is high).

The figure14 illustrates the relationship between risk aversion and term premium across different models. We compare our benchmark model with the model by Rudebusch and Swanson (2012) and various extensions of the model which are part of Kaszab and Marsal (2013). We can get higher nominal term premium with lower risk aversion in our model but this result is driven by the preference shock which is exogenous.

A.3 MODEL DEPENDENCE

Figure 27 shows the decomposition of the pricing kernel for Rudebusch and Swanson (2012) model. One can see that the success of Rudebusch and Swanson (2012) is driven by the ability of the model to generate large inflation risks for the long run consumption and leisure. The risk premia turn negative with higher risk aversion because the insurance property of bonds overweighs the leverage effects.

The transmission of the shocks is qualitatively in line with our benchmark results. We pursue this exercise also in the model with richer fiscal sector as in Kaszab and Marsal (2013) to find that our conclusions hold[33].
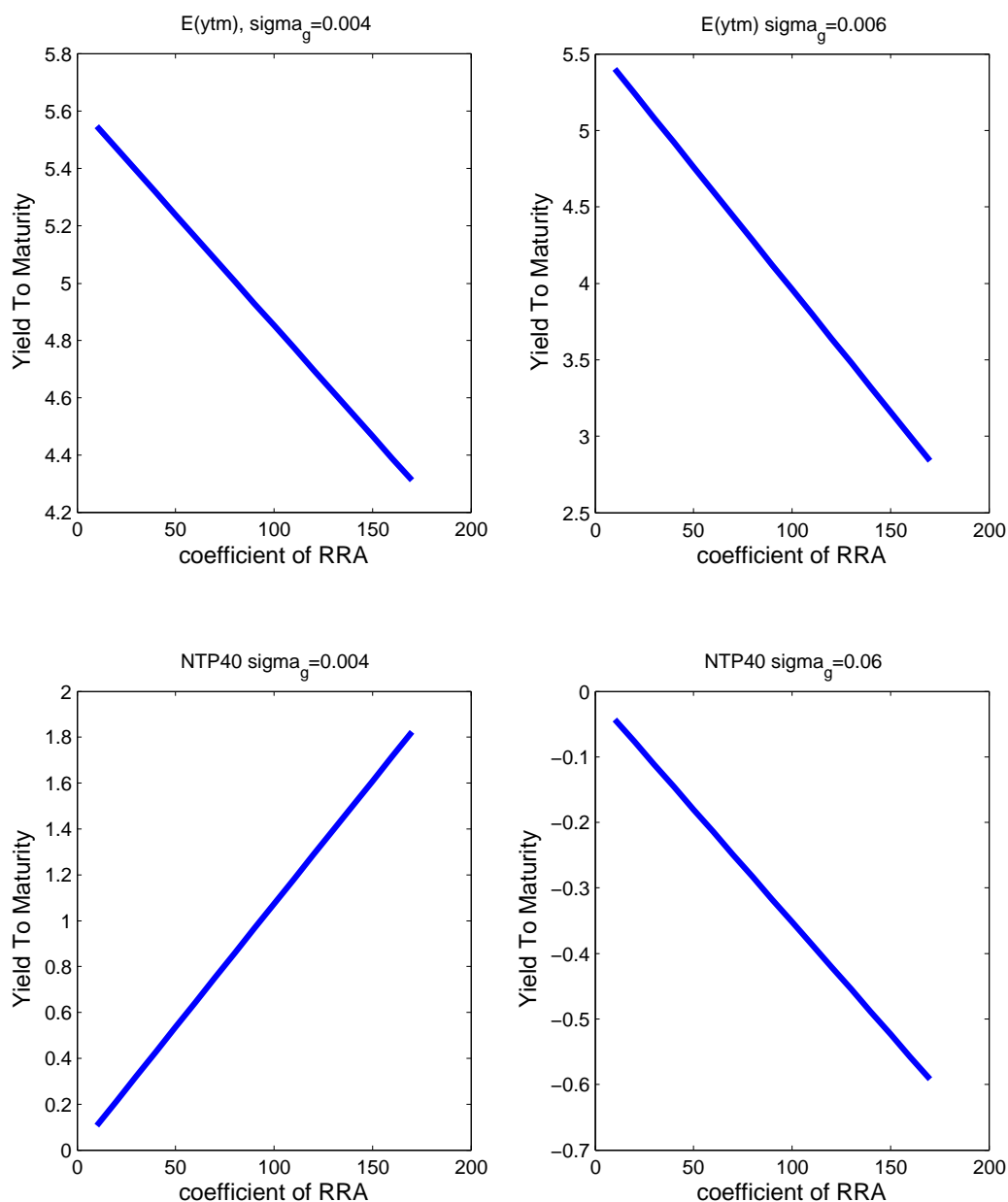
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[33]Numbers are not reported here.

Figure 13: Upper block shows how the maturity average of the stochastic steady state of the yield curve varies with relative risk aversion. Bottom block shows the sensitivity of NTP to risk aversion. Both for low and high fiscal volatility. The second picture in the bottom is the case when only wasteful government expenditures move the economy.
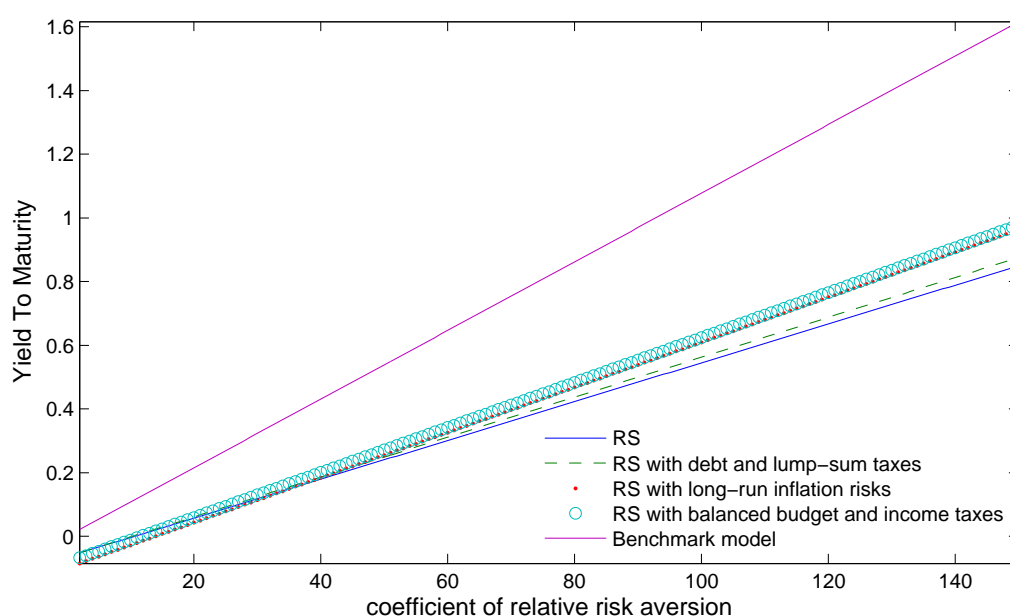
Figure 14: The relationship between the coefficient of relative risk-aversion and the mean of the nominal term premium using our benchmark model and variants of Rudebush and Swanson (2012) model .

Stand alone factors	RS RP $CRRA = 75$		RS NTP $CRRA = 75$		RS BF RP $CRRA = 110$		RS BF NTP $CRRA = 110$	
	Levels	%	Levels	%	Levels	%	Level	%
$\Delta^n Var_t(\Delta C_{t+n})$	-0.004	(1)	0	(4.1)	-0.036	(4)	0.01	(1.3)
$\Delta^n Var_t(\sum_{j=1}^n \pi_{t+n})$	0.104	(34)	-0.043	(-10)	-1.1	(108)	-0.09	(-7)
$Var_t(S_{t+n})$	0.013	(4)	0	(0)	0.12	(-11)	0	(0)
Factor interactions								
$Cov_t(\Delta C_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.015	(5)	0.014	(3)	0.06	(06)	0.005	(4)
$Cov_t(\Delta C_{t+n}, S_{t+n})$	-0.17	(-55)	-0.009	(-2)	-0.87	(86)	0.17	(13)
$Cov_t(S_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.352	(114)	0.466	(109)	0.82	(-81)	1.15	(89)
Total								
premium	0.31	(100)	0.428	(100)	-1.01	(100)	1.295	(100)

Table 7: RS stands for Rudebusch and Swanson (2012) model with benchmark calibration, RP stands for risk premium, BF for best fit calibration of Rudebusch and Swanson (2012). Each column shows the contribution of the attributes to NTP and risk premium in levels and percents ($\frac{f_i}{\sum_i f_i}$). The higher order interactions are zero up to the 2nd order approximation.

B. Note on Recursive preferences

The functional form of preferences is the crucial element driving large part of the results. Recursive preferences has been utilized increasingly in the asset pricing literature. Nevertheless, in macroeconomic literature Epstein Zin preferences belongs, yet, to group a of so called exotic preferences (see Backus (2014)). We provide detailed solution of the bond pricing equation and its second order approximation. The explicit second order solution to bond prices is useful because it helps us to better develop the intuition about the drivers of dynamics of the term structure of interest rates and relate them to macroeconomic fundamentals.

We lay out the recursive preferences as in Weil (1990). First, we use the utility transformation as in Rudebusch and Swanson (2012) that simplifies the work with utility kernels containing leisure. Next, we derive and log-linearize the stochastic discount factor (SDF). To substitute out the recursive element and to get SDF just as a function of macroeconomic fundamentals we log-linearize the value function and introduce the surprise operator as in Uhlig (2010). Consequently, using the method developed by Sutherland (2002) we derive the general form of second order approximation to the bond pricing equation. Finally, we merge the results to highlight the drivers of the yield curve dynamics.

B.1 Value function transformation

In the asset pricing literature, the recursive preferences are usually formulated in the following form (see Weil (1990), Epstein and Zin (1989), Bansal and Yaron (2004), Uhlig (2010), Guvenen (2009)),

$$\tilde{V} = \left\{ u(c_t, N_t)^{1-\gamma} + \beta [E_t \tilde{V}_{t+1}^{1-\psi}]^{\frac{1-\gamma}{1-\psi}} \right\}^{\frac{1}{1-\gamma}} \tag{39}$$

where ψ stands for the risk aversion and γ is the inverse of inter-temporal elasticity of substitution. In this paper we follow Rudebusch and Swanson (2012) and use slightly different form of value function

$$\tilde{V} = \left\{ u(C_t, N_t) + \beta [E_t \tilde{V}_{t+1}^{1-\psi}]^{\frac{1-\gamma}{1-\psi}} \right\}^{\frac{1}{1-\gamma}} \tag{40}$$

when using the additively separable period utility function it is useful to transform the value function as in Rudebusch and Swanson (2012) . We set $\frac{1-\psi}{1-\gamma} = 1 - \alpha$ and define $V_t = \tilde{V}_t^{1-\gamma}$

$$V_t = u(C_t, N_t) + \beta (E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{41}$$

when $u(C_t, L_t) > 0$. If $u(C_t, L_t) < 0$, as in our benchmark calibration [34], the recursion takes the form:

$$V_t = u(C_t, L_t) - \beta(E_t[-V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{42}$$

To obtain the first order conditions, we solve for the constrain optimization problem.

B.2 SOLVING FOR SDF

There are several ways how to find optimal size of savings (bond purchases).

1. The social planner's problem formulation allows us to find the pricing kernel of the economy $m_{t+1} = \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t}$ (see Caldara, Fernandez-Villaverde, Rubio-Ramirez, and Yao (2012))

2. use Bellman equation - dynamic programming approach (see for example Ferman (2011), Andreasen (2008), Tristani and Amisano (2010))

3. formulate the problem as lagrangian (see Rudebusch and Swanson (2012), Andreasen (2012a)

Here we use the dynamic programming approach and define

$$V_t = \max_{B_{t+1}, C_t, N_t} \{u(C_t, L_t) + \beta(E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} + \lambda_t[P_t C_t + E_t Q_{t,t+1} B_{t+1} - B_t - P_t W_t N_t - T_t]\} \tag{43}$$

where

$$u(C_t, L_t) = \left[\frac{C^{1-\gamma}}{1-\gamma} - \chi\frac{N^{1+\eta}}{1+\eta}\right] e^{b_t} \tag{44}$$

note that

$$\begin{aligned}
V_{t+1} &= \max_{B_{t+1}, C_t, N_t} \{u(C_{t+1}, N_{t+1}) + \beta(E_{t+1}[V_{t+2}^{1-\alpha}])^{\frac{1}{1-\alpha}} \\
&+ \lambda_{t+1}[P_{t+1}C_{t+1} + E_{t+1}Q_{t+1,t+2}B_{t+2} - B_{t+1} - P_{t+1}W_{t+1}N_{t+1} - T_{t+1}]\}
\end{aligned} \tag{45}$$

$\frac{\partial V_t}{\partial B_{t+1}}$:

$$\lambda_t Q_{t,t+1} = \beta\frac{1}{1-\alpha}[E_t V_{t+1}^{1-\alpha}]^{\frac{\alpha}{1-\alpha}} V_{t+1}^{-\alpha}(1-\alpha)\lambda_{t+1} \tag{46}$$

$\frac{\partial V_t}{\partial N_t}$:

$$e^{b_t}\chi N_t^\eta = -\lambda_t P_t W_t \tag{47}$$

---

[34] the first order conditions will be correct however in either way

$\frac{\partial V_t}{\partial C_t}$:

$$\lambda_t = -\frac{C_t^{-\gamma}}{P_t} e^{b_t} \tag{48}$$

Substituting λ from equation 48 to equation 47 gives labor supply equation:

$$\chi N_t^\eta = C_t^{-\gamma} W_t \tag{49}$$

Combining equation 46 with equation 48 delivers stochastic discount factor at time t for stochastic payoff at time $t + 1$.

$$Q_{t,t+1} = \zeta_t \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_t}{P_{t+1}} \left[\frac{V_{t+1}}{R_t}\right]^{-\alpha} \tag{50}$$

where $\zeta_t = e^{b_{t+1}-b_t}$ is the preferences shock, $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the inflation between period t and $t + 1$, and certainty equivalent value of future consumption and leisure R_t is given by:

$$R_t = [E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}} \tag{51}$$

Because of term $\left[\frac{V_{t+1}}{R_t}\right]^{-\alpha}$ [35], news at t + 1 about consumption in $c_{t+2}, c_{t+3} \ldots$ and leisure in $n_{t+2}, n_{t+3} \ldots$ affects marginal utility of c_{t+1} and n_{t+1} relative to marginal utility of c_t and n_t. Good news at $t + 1$ about future consumption and leisure is a positive shock to $R_{t+1}(V_{t+2})$, and therefore to $V_{t+1} = F(c_{t+1}, n_{t+1}; R_{t+1}(V_{t+2}))$. The more concave is the utility function and the more uncertain V_{t+1} is, the lower is the certainty equivalent R_t. Note that $R_t = V_{t+1}$ if there is no uncertainty on V_{t+1}.

There are two advantages to SDF of time-separable expected utility. First, it separates EIS from coefficient of relative risk aversion. Second, it is another source of risk premium, not just covariance with contemporaneous consumption growth, but also covariance with return to total wealth matters.

By chaining the stochastic discount factor we can price bond of any maturity:

$$Q_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t}\right)^{-\gamma} \prod_{j=0}^{n} \frac{\zeta_{t+j}}{\pi_{t+j+1}} \left[\frac{R_{t+j}}{V_{t+j+1}}\right]^\alpha \tag{52}$$

B.3 LOG-LINEARIZING SDF

First we log linearize equation (51)

---

[35]next periods value relative to its certainty equivalent

LHS:

$$\bar{R}^{1-\alpha}e^{(1-\alpha)\hat{\mathbf{r}}_t} \approx \bar{R}^{1-\alpha}(1+(1-\alpha)\hat{\mathbf{r}}_t) \tag{53}$$

RHS:

$$\bar{V}^{1-\alpha}E_t e^{(1-\alpha)\hat{\mathbf{v}}_{t+1}} \approx \bar{V}^{1-\alpha}(1+(1-\alpha)E_t\hat{\mathbf{v}}_{t+1}) \tag{54}$$

Canceling out steady state delivers:

$$\hat{\mathbf{r}}_t = E_t\hat{\mathbf{v}}_{t+1} \tag{55}$$

Next, we log linearizing equation (50). RHS after taking Taylor expansion

$$st.st. + st.st E_t[\zeta_t - \gamma\Delta\hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1} - \alpha(\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t) \tag{56}$$

Canceling out steady state and joining LHS with RHS we get log linearized price of one period bond:

$$q_{t,1} = \zeta_t - \gamma\Delta\hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1} - \alpha(\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t) \tag{57}$$

Next, we substitute equation 51 into equation (50) to highlight that $\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t$ is the next periods value relative to its certainty equivalent [36]

$$q_{t,1} = E_t\left\{\zeta_t - \gamma\Delta\hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1}\right\} - \alpha(E_{t+1}\hat{\mathbf{v}}_{t+1} - E_t v_{t+1}) \tag{58}$$

By chaining the stochastic discount factor we derive the price of bond with any maturity n:

$$q_{t+n} = \sum_{j=1}^{n}E_t\zeta_{t+j} - \gamma\Delta^n E_t\hat{\mathbf{c}}_{t+n} - E_t\sum_{j=1}^{n}\hat{\pi}_{t+j} - \alpha\left[\sum_{j=1}^{n}(\hat{\mathbf{v}}_{t+j+1} - \hat{\mathbf{r}}_{t+j})\right] \tag{59}$$

Note that risk aversion is denoted ψ and α is then:

$$\alpha = 1 - \frac{1-\psi}{1-\gamma} \tag{60}$$

so for the news to enter stochastic discount factor the risk aversion must be different of the inverse of intertemporal elasticity of substitution. γ and ψ are so called demands for smoothing parameters, intratemporally and intertemporally. Consider following hypothetical processes for consumption and leisure

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[36]note that the term $\hat{\mathbf{v}}_{t+1} - \hat{\mathbf{r}}_t$ in time t expectations equals to zero. This is given by the fact that the first order approximation eliminates uncertainty from the model and thus the $E_t v_{t+1} = R_t$. Agents expectations are up to the first order identical to certainty equivalent

1. coin flipped at $t = 0$ determines high or low consumption and leisure at all dates 1,2,3 …T

2. T coins flipped at $t = 0$ determine high or low consumption and leisure at all dates 1,2,3 …T

3. T coins flipped before each period to determine consumption and leisure that period

The first process implies intertemporally smooth path of consumption and leisure but is characterized by big time-zero volatility in V_t and thus dislike by risk averse agents. It will be preferred only if $1/\gamma$ is very small. In the second process all information is revealed at time $t = 0$ thus $E_t(V_{t+1})$ varies over time non-stochastically but the process features higher variation across time than the first one. The third process shares with the second one the volatility across time but differs in the timing of uncertainty resolution. When $\gamma < \psi$ agents prefer early resolution of uncertainty.

B.4 LOG-LINEARIZING THE VALUE FUNCTION

The goal is to express bond prices as a function of macroeconomic fundamentals. Therefore, we need to eliminate the recursion. Lets assume that the period utility is additively separable CRRA.

$$V_t = \left[\frac{C^{1-\gamma}}{1-\gamma} - \chi\frac{N^{1+\eta}}{1+\eta}\right]\zeta_t + \beta(E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{61}$$

Remember that $R_t = [E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}}$ is the certainty equivalent of next period's utility. The log-linearized equation (61) around zero steady state is

$$\hat{\mathbf{v}}_t = \frac{\bar{\zeta}\bar{C}^{1-\gamma}}{\bar{V}}\hat{\zeta}_t + \frac{\bar{\zeta}\bar{C}^{1-\gamma}}{\bar{V}}\hat{\mathbf{c}}_t - \frac{\chi\bar{\zeta}\bar{N}^{1-\eta}}{\bar{V}}\hat{\mathbf{n}}_t + \beta\hat{\mathbf{r}}_t \tag{62}$$

simplifying the notation

$$\hat{\mathbf{v}}_t = a(\hat{\zeta}_t + \hat{\mathbf{c}}_t) - b\hat{\mathbf{n}}_t + \beta\hat{\mathbf{r}}_t \tag{63}$$

where $a = \frac{\bar{\zeta}\bar{C}^{1-\gamma}}{\bar{V}}$ and $b = \frac{\bar{\zeta}\bar{N}^{1-\eta}\chi}{\bar{V}}$.

Solving the equation (62) forward we get:

$$\hat{\mathbf{v}}_t = \sum_{j=0}^{\infty}\beta^j\left(a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j}\right) - \sum_{j=0}^{\infty}\beta^j b\hat{\mathbf{n}}_{t+j} \tag{64}$$

Next, it is convenient to follow Uhlig (2010) and introduce the "surprise" operator $S_{t+k|t}$ for any random variable x, given by

$$S_{t+n|t} = E_{t+k}(x) - E_t(x) \tag{65}$$

thus for the period $t + 1$, S_{t+1} is filtering out the surprise in conditional expectations and is defined

$$S_{t+1} = x_{t+1} - E_t[x_{t+1}] \tag{66}$$

Note that the surprise over n periods is simply

$$S_{t+n} = S_{t+n} + S_{t+n-1} + \ldots + S_{t+1} \tag{67}$$

Applying the filtering, using the equation (64) in the SDF, equation (57), we can show that the bond pricing equation is determined by the period consumption growth, inflation, exogenous preference shock and the surprise or news about the future consumption and labor.

$$q_{t,1} = \zeta_{t+1} - \gamma\Delta\hat{\mathbf{c}}_{t+1} - \hat{\pi}_{t+1} - \alpha S_{t+1}\left(\sum_{j=0}^{\infty}\beta^j\left[a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j} - b\hat{\mathbf{n}}_{t+j}\right]\right) \tag{68}$$

Notice that the labor enters the bond pricing equation which is usually the case only with non-separable preferences. Nevertheless, labor affects only higher order terms. Price of bond with maturity n is given by

$$q_{t+n} = \sum_{j=1}^{n}\zeta_{t+j} - \gamma\Delta^n\hat{\mathbf{c}}_{t+n} - \sum_{j=1}^{n}\hat{\pi}_{t+j} - \alpha S_{t+n}\left(\sum_{j=0}^{\infty}\beta^j\left[a\hat{\zeta}_{t+j} + a\hat{\mathbf{c}}_{t+j} - b\hat{\mathbf{n}}_{t+j}\right]\right) \tag{69}$$

The revaluation in the expectations can be understood as well as the news or surprise. Investors require compensation for the uncertainty underlying the surprise component. Net effect of good news about $c_{t+2}, c_{t+3} \ldots$ and $n_{t+2}, n_{t+3}, \ldots$ on marginal utility of c_{t+1} and n_{t+1} depends on α. If α is positive, news is a positive shock to SDF. Note also that news about c_{t+1} directly affect the consumption growth part of SDF but it also shows up in the second part of equation (69). If there is no news about c_{t+2}, c_{t+3} and n_{t+2}, n_{t+3} SDF reduces to $\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\pi^{-1}$. Each period agents make expectations about future consumption and hours worked for the remaining life of the bond and compare it with the previous period expectations. The difference between this two is the update in expectations reflected in price of bonds. The update of expectations is sum of all news (surprises) over the remaining maturity of the bond about the life time stream of consumption and leisure.

B.5 SECOND ORDER APPROX. TO THE TERM STRUCTURE

The derivations rely on Sutherland (2002) who argues that first order approximate solutions are sufficient to derive second order approximate solutions to second moments. Second order accurate solutions for second moments can be obtained by considering first-order accurate solutions to realized values because terms of order two and above in the behaviour of realized values become terms of order three and above in the squares and cross products of realized values. The first part of the derivations, which is not explicitly working with EZ preferences, is in line with Hordahl, Tristani, and Vestin (2007). The price of bond with maturity n is defined $P_t^{(n)} = E_t[Q_{t,t+n}]$; in the non-stochastic steady state $\bar{P} = \bar{Q}$. Lower case letters define logarithm of their upper case counterparts.

$$\begin{aligned}
\bar{p}(1 + \hat{\mathbf{p}}_{t,n} + \frac{1}{2}\hat{\mathbf{p}}_{t,i}^2) &= E_t\left[\bar{q}(1 + \hat{\mathbf{q}}_{t+n} + \frac{1}{2}\hat{\mathbf{q}}_{t+n}^2)\right] \\
&= \bar{q}_t E_t\left[1 + \hat{\mathbf{q}}_{t+n} + \frac{1}{2}\hat{\mathbf{q}}_{t+n}^2\right]
\end{aligned}$$

After canceling out steady state, we get:

$$\hat{\mathbf{p}}_{t,n} = E_t[\hat{\mathbf{q}}_{t,t+n} + \frac{1}{2}\hat{\mathbf{q}}_{t,t+n}^2] - \frac{1}{2}\hat{\mathbf{p}}_{t,n}^2$$

Up to the first order $\hat{\mathbf{p}}_{t,n} = E_t\{\hat{\mathbf{q}}_{t,t+n}\}$, thus we can substitute for the quadratic term $\hat{\mathbf{p}}_{t,n}^2 = (E_t\{\hat{\mathbf{q}}_{t,t+n}\})^2$. It follows that:

$$\hat{\mathbf{p}}_{t,n} = E_t\left[\hat{\mathbf{q}}_{t,t+n} + \frac{1}{2}\hat{\mathbf{q}}_{t,t+n}^2\right] - \frac{1}{2}(E_t\hat{\mathbf{q}}_{t,t+n})^2$$

From the last equation using the definition of variance [37] we can define price of one period bond.

$$\hat{\mathbf{p}}_{t,n} = E_t[\hat{\mathbf{q}}_{t,t+n}] + \frac{1}{2}\mathrm{Var}_t[\hat{\mathbf{q}}_{t,t+n}] \tag{70}$$

using the definition of yield to maturity, $\widehat{ytm_t} = -(1/n)\hat{\mathbf{q}}_{t,n}$ we can write equation (70)

$$\widehat{ytm}_t^n = -\frac{1}{n}E_t q_{t,t+n} - \frac{1}{2n}\mathrm{Var}_t(\hat{\mathbf{q}}_{t,t+n}) \tag{71}$$

and use equation (69) and plug it into 71 to get

---

[37] $Var(x) = E[x^2] - (E[x])^2$

$$\widehat{ytm}_t^n = -\frac{1}{n} \mathrm{E}_t \left\{ \sum_{j=1}^n \left[\hat{\zeta}_{t+n} \right] - \gamma \Delta^n \hat{\mathbf{c}}_{t+n} - \sum_{j=1}^n \left[\hat{\pi}_{t,t+n} \right] - \alpha S_{t+n} \left(\cdot \right) \right\} \tag{72}$$

$$-\frac{1}{2n} \mathrm{Var}_t \left(\sum_{j=1}^n \left[\hat{\zeta}_{t+n} \right] - \gamma \Delta^n \hat{\mathbf{c}}_{t+n} - \sum_{j=1}^n \left[\hat{\pi}_{t,t+n} \right] - \alpha S_{t+n} \left(\cdot \right) \right) \tag{73}$$

Unconditional mean of the term structure is then

$$
\begin{aligned}
E[\widehat{ytm}^n] &= -\frac{1}{2n} \left[\mathrm{Var}_t \sum_{j=1}^n \left(\hat{\zeta}_{t+j} \right) \right] - \frac{\gamma^2}{2n} \left[\mathrm{Var}_t \left(\Delta^n \hat{\mathbf{c}}_{t+n} \right) \right] - \frac{1}{2n} \left[\mathrm{Var}_t \sum_{j=1}^n \left(\hat{\pi}_{t,t+j} \right) \right] \\
&\quad + -\frac{\alpha^2}{2n} E \left[\mathrm{Var}_t S_{t+n} \left(\cdot \right) \right] + \frac{\gamma}{n} E \left[\mathrm{Cov}_t \left(\sum_{j=1}^n \hat{\zeta}_{t+j}, \Delta^n \hat{\mathbf{c}}_{t+n} \right) \right] \\
&\quad + \frac{1}{n} E \left[\mathrm{Cov}_t \left(\sum_{j=1}^n \hat{\zeta}_{t+j}, \sum_{j=1}^n \hat{\pi}_{t+j} \right) \right] - \frac{\gamma}{n} E \left[\mathrm{Cov}_t \left(\Delta^n \hat{\mathbf{c}}_{t+n}, \sum_{j=1}^n \hat{\pi}_{t+j} \right) \right] \\
&\quad + \frac{\alpha}{n} E \left[\mathrm{Cov}_t \left(\sum_{j=1}^n \hat{\zeta}_{t+j}, S_{t+n} \left(\cdot \right) \right) \right] - \frac{\gamma \alpha}{n} E \left[\mathrm{Cov}_t \left(\Delta^n \hat{\mathbf{c}}_{t+n}, S_{t+n} \right) \right] \\
&\quad - \frac{\alpha}{n} E \left[\mathrm{Cov}_t \left(\sum_{j=1}^n \hat{\pi}_{t+n}, S_{t+n} \right) \right]
\end{aligned}
\tag{74}
$$

S_{t+n} embodies the intensity of surprises from consumption, leisure and preference shocks over the maturity horizon. For a random variable ytm_t, the unconditional mean is simply the average of the realized yields. In contrast, the conditional mean of ytm_t is the expected value of ytm_t given a conditioning set of variables, Ω_t (shock realization). The term under the expectations in the equation (72) is on average zero. The term thus corresponds to the determinist steady state. The variance components represent the Jensen's inequality term and arise from the relative convexity of nominal bonds. Note also that even if we calculate mean of the stochastic steady state of the yield curve the variance and covariance is still conditional on the information in time t.

Next, we rewrite the covariance terms using the definition for correlation. To make the equation more compact we rewrite the equation (74) using different notation. Note that $\sigma_{\Delta \hat{c}} = [Var_t(\Delta^n \hat{\mathbf{c}}_{t+n})]^{\frac{1}{2}}$, $\rho_{\Delta \hat{c}, S} = \mathrm{Corr}_t (\Delta^n \hat{\mathbf{c}}_{t+n}, S_{t+n})$ and other variables in equation (74) are rewrit-

ten analogically.

$$
\begin{aligned}
E\widehat{ytm}_t^n &= -\frac{1}{2n}\left\{\sigma_{\hat{\zeta}}^2 + \gamma^2\sigma_{\Delta\hat{\mathbf{c}}}^2 + \sigma_{\hat{\pi}}^2 + \alpha^2\sigma_S^2\right\} + \frac{\gamma}{n}\sigma_{\hat{\zeta}}\sigma_{\Delta\hat{\mathbf{c}}}\rho_{\hat{\zeta},\Delta\hat{\mathbf{c}}} + \frac{1}{n}\sigma_{\hat{\zeta}}\sigma_{\hat{\pi}}\rho_{\hat{\zeta},\hat{\pi}} \\
&\quad - \frac{\gamma}{n}\sigma_{\Delta\hat{\mathbf{c}}}\sigma_{\hat{\pi}}\rho_{\Delta\hat{\mathbf{c}},\hat{\pi}} + \frac{\alpha}{n}\sigma_{\hat{\zeta}}\sigma_S\rho_{\hat{\zeta},S} - \frac{\gamma\alpha}{n}\sigma_{\Delta\hat{\mathbf{c}}}\sigma_S\rho_{\Delta\hat{\mathbf{c}},S} - \frac{\alpha}{n}\sigma_{\hat{\pi}}\sigma_S\rho_{\hat{\pi},S} \quad (75)
\end{aligned}
$$

C. Deriving the Model's Steady State

Labor supply in steady state

$$\frac{W}{C^\gamma} = \chi N^\eta \tag{76}$$

χ is calibrated in such way that steady state hours worked are $N = 1$.

$$N = \left[\frac{W}{C^\gamma \chi}\right]^{\frac{1}{\eta}} = 1 \tag{77}$$

Next, from the Philips curve I get

$$MC = \frac{1}{1+\lambda} \tag{78}$$

so then from the definition of aggregate marginal costs

$$\frac{1}{1+\lambda} = W \frac{1}{1-\theta} \left(\frac{Y}{K}\right)^{\frac{\theta}{1-\theta}} \tag{79}$$

so I can express W. Capital and government spending as s fraction of output are calibrated thus known.

$$W = \frac{1-\theta}{1+\lambda} \left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}} \tag{80}$$

Thus, we know steady state wage just as a function of parameters. Now I can get χ as a function of parameters.

Plugging equation 80 into 76 and using the market clearing condition $Y = C + G + \delta K$ and consequently $C = (1 - \frac{G}{Y} - \delta\frac{K}{Y})Y$

$$\left\{\frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[(1 - \frac{G}{Y} - \delta\frac{K}{Y})Y\right]^\gamma \chi}\right\}^{\frac{1}{\eta}} = N \tag{81}$$

So we search for χ making $N = 1$:

$$\left\{\frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[(1 - \frac{G}{Y} - \delta\frac{K}{Y})Y\right]^\gamma}\right\} = 1^\eta \chi \tag{82}$$

From production function:

$$N = \left(\frac{K^\theta}{Y}\right)^{\frac{-1}{1-\theta}} = \left(\frac{K}{Y}\right)^{-\frac{\theta}{1-\theta}} Y \tag{83}$$

So it should follow that if $N = 1$ then:

$$Y = \left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}} \tag{84}$$

and also,

$$K^\theta = Y \tag{85}$$

From equation 82,

$$\left\{ \frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[\left(1-\frac{G}{Y}-\delta\frac{K}{Y}\right)\right]^\gamma} \right\} = 1^\eta \chi Y^\gamma \tag{86}$$

Now using equation 84

$$\left\{ \frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[\left(1-\frac{G}{Y}-\delta\frac{K}{Y}\right)\right]^\gamma} \right\} = 1^\eta \chi \left(\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}} \right)^\gamma \tag{87}$$

simplifying we derive value for χ making the $N = 1$.

$$\chi = \frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta(1-\gamma)}{1-\theta}}}{\left[\left(1-\frac{G}{Y}-\delta\frac{K}{Y}\right)\right]^\gamma} \tag{88}$$

So using equation 81 and plugging in from production function I get:

$$\left\{ \frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta}{1-\theta}}}{\left[\left(1-\frac{G}{Y}-\delta\frac{K}{Y}\right)Y\right]^\gamma \chi} \right\}^{\frac{1}{\eta}} = \left(\frac{K}{Y}\right)^{-\frac{\theta}{1-\theta}} Y \tag{89}$$

getting out output in steady state:

$$Y = \left\{ \frac{\frac{1-\theta}{1+\lambda}\left(\frac{K}{Y}\right)^{\frac{\theta(1+\eta)}{1-\theta}}}{\left[\left(1-\frac{G}{Y}-\delta\frac{K}{Y}\right)\right]^\gamma \chi} \right\}^{\frac{1}{\eta+\gamma}} \tag{90}$$

Knowing steady state output I can back up steady state hours worked from equation 83. From the market clearing condition I can get steady state consumption $C = (1 - \frac{G}{Y} - \delta\frac{K}{Y})Y$.

Consequently, $\frac{K}{Y} = 10$ so $K = 10Y$. Capital is at its steady state value so the investment must exactly offset capital depreciation. Therefore, $I = \delta K$. Similarly, $G = 0.2Y$.

The last steady state value is the value function. Note that you can rewrite the value function as infinite geometric sum by iteration. In steady state

$$V = u(C, N) + \beta[u(C, N) + \beta\{u(C, N) + \beta u(C, N) \ldots\}] \tag{91}$$

taking the utility out of the bracket

$$V = u(C, N)[1 + \beta + \beta^2 + \beta^3 \ldots] \tag{92}$$

so the steady state is

$$V = u(C, N)\frac{1}{1 - \beta} \tag{93}$$

C.0.1   Firm Problem - Productive G

Firms care about the real value of the profit thus they discount the future steam of cashflow by nominal discount factor.

$$\max_{P_t(i), N_t(i)} E_t \sum_{j=0}^{\infty} Q_{t,t+j} \left[D_{t+j}(i) - \frac{\epsilon}{2} \left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} \frac{1}{\bar{\pi}} - 1 \right)^2 P_{t+j} Y_{t+j} \right] \tag{94}$$

such that

$$D_{t+j}(i) = P_{t+j}(i) Y_{t+j}(i) - P_{t+j} W_{t+j} N_{t+j}(i) \tag{95}$$

$$Y_{t+j}(i) = A_t \bar{K}^\theta (G_t^p)^{\theta_g} N_t(i)^{1-\theta} \tag{96}$$

where G_t are productive government expenditures and ι being elasticity of output to productive government spending.

$$Y_{t+j}(i) = \left(\frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\zeta} Y_{t+j} \tag{97}$$

$$TC_{t+j}(i) = P_{t+j} W_{t+j} N_{t+j}(i) = P_{t+j} W_{t+j} \left[\frac{Y_{t+j}(i)}{A_{t+j} \bar{K}^\theta (G_{t+j}^p)^{\theta_g}} \right]^{\frac{1}{1-\theta}} \tag{98}$$

marginal costs are

$$\frac{\partial TC_{t+j}(i)}{\partial Y_{t+j}(i)} = \frac{1}{1-\theta} P_{t+j} W_{t+j} N_{t+j}^{\theta} \frac{1}{A_{t+j}\bar{K}^{\theta}(G_{t+j}^{p})^{\theta_g}} \tag{99}$$

real marginal costs of the firm i are then

$$MC_{t+j}^{r}(i) = \frac{1}{1-\theta} \frac{W_{t+j}}{N_{t+j}^{-\theta} A_{t+j}\bar{K}^{\theta}(G_{t+j}^{p})^{\theta_g}} \tag{100}$$

this happens to be also the aggregate marginal cost by symmetric equilibrium.

$$\frac{\partial MC_{t+j}(i)}{\partial P_{t+j}(i)} = MC_{t+j}(i) P_{t+j}^{-\zeta-1}(i) P_{t+j}^{\zeta} Y_{t+j} \tag{101}$$

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} Q_{t,t+j} \left[P_{t+j}(i) Y_{t+j}(i) - TC_{t+j}(i) - \frac{\epsilon}{2} \left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} \frac{1}{\bar{\pi}} - 1 \right)^2 P_{t+j} Y_{t+j} \right] \tag{102}$$

$$\left[(1-\zeta) P_t(i)^{-\zeta} P_t^{\zeta} Y_t - MC_t(i)(-\zeta) P_t^{\zeta} P_t^{-\zeta-1}(i) Y_t - \epsilon \left(\frac{P_t(i)}{P_{t-1}(i)} \frac{1}{\bar{\pi}} - 1 \right) \frac{1}{\bar{\pi}} \frac{P_t}{P_{t-1}(i)} Y_t \right]$$
$$+ E_t \left\{ Q_{t,t+1} \left[\epsilon \left(\frac{P_{t+1}(i)}{P_t(i)} \frac{1}{\bar{\pi}} - 1 \right) \frac{1}{\bar{\pi}} \frac{P_{t+1}}{P_t^2(i)} P_{t+1}(i) Y_{t+1} \right] \right\} \tag{103}$$

by imposing symmetric equilibrium $Y_t = Y_t(i)$, $P_t = P_t(i)$, $MC_t(i) = MC_t$ and $\zeta = \frac{1+\lambda_t}{\lambda_t}$

$$MC_t^{r} = \frac{\lambda_t}{1+\lambda_t} E_t Q_{t,t+1}^{r} \left[\epsilon \left(\frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{Y_{t+1}}{Y_t} \right] + \frac{\lambda_t}{1+\lambda_t} \epsilon \left(\frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} + \frac{1}{1+\lambda_t} \tag{104}$$
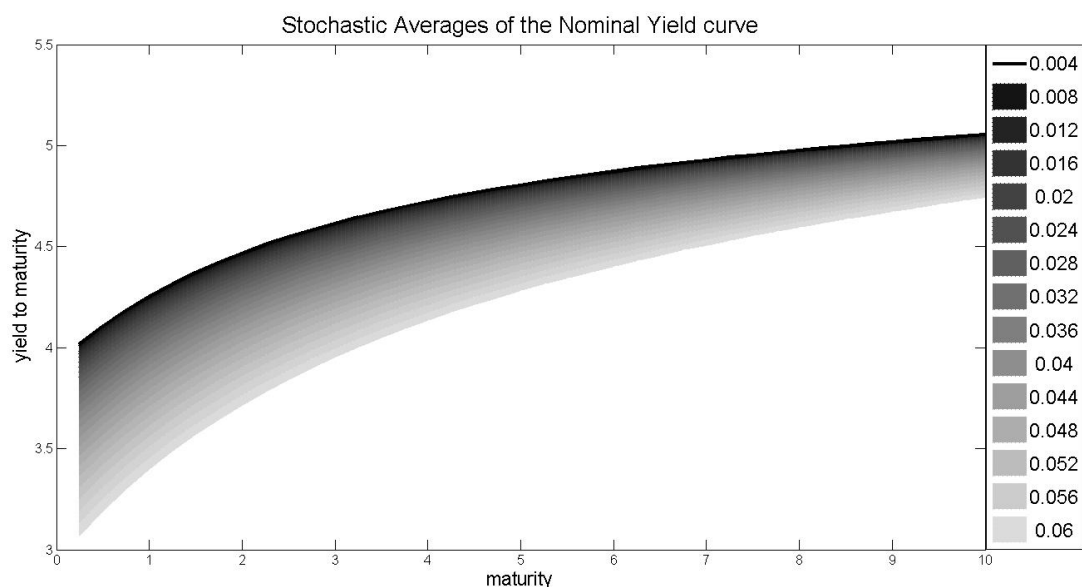
Figure 15: Term structure and varying volatility of G shocks in the benchmark model when central bank puts zero weight on output gap stabilization. In the legend is the volatility of the government spending innovation.
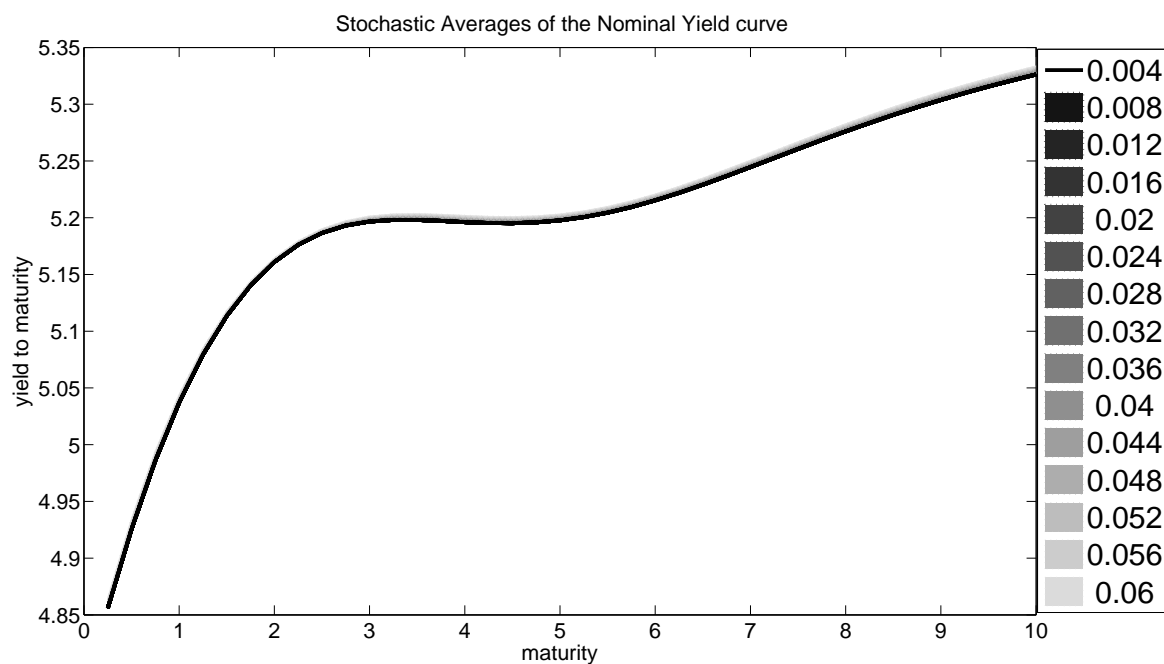

Figure 16: Term structure and varying volatility of G shocks in the model with spending reversals when central bank puts $\phi_y = 0.075$ on output gap stabilization. In the legend is the volatility of the government spending innovation.
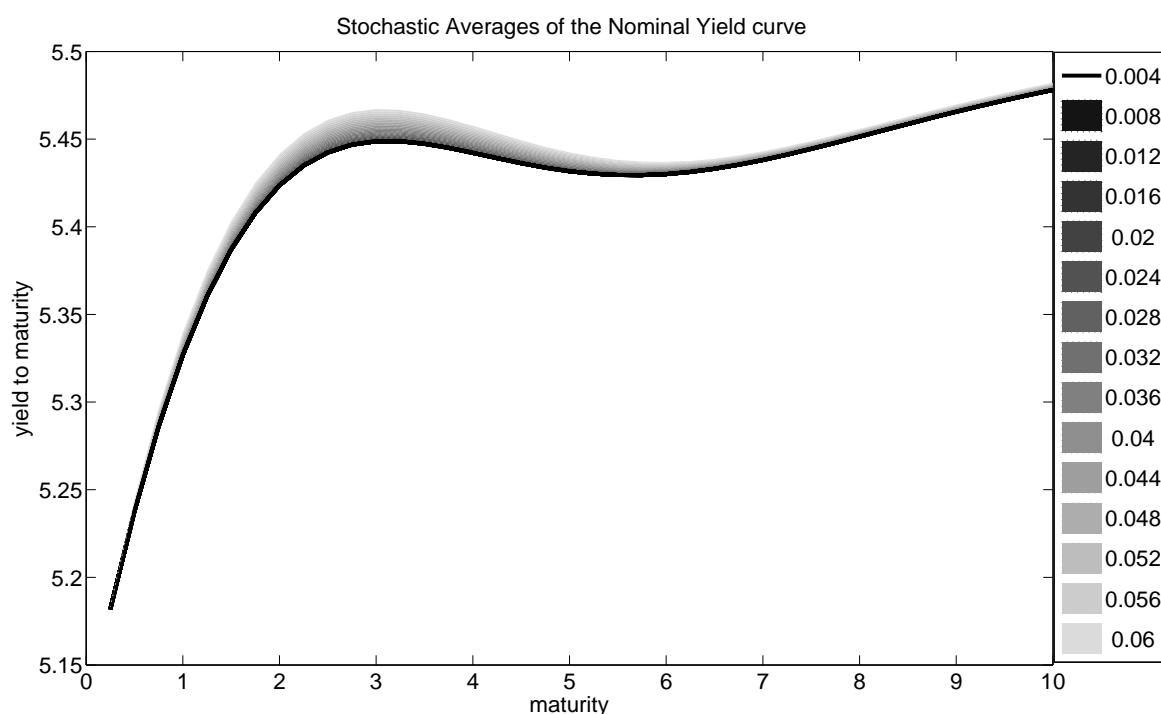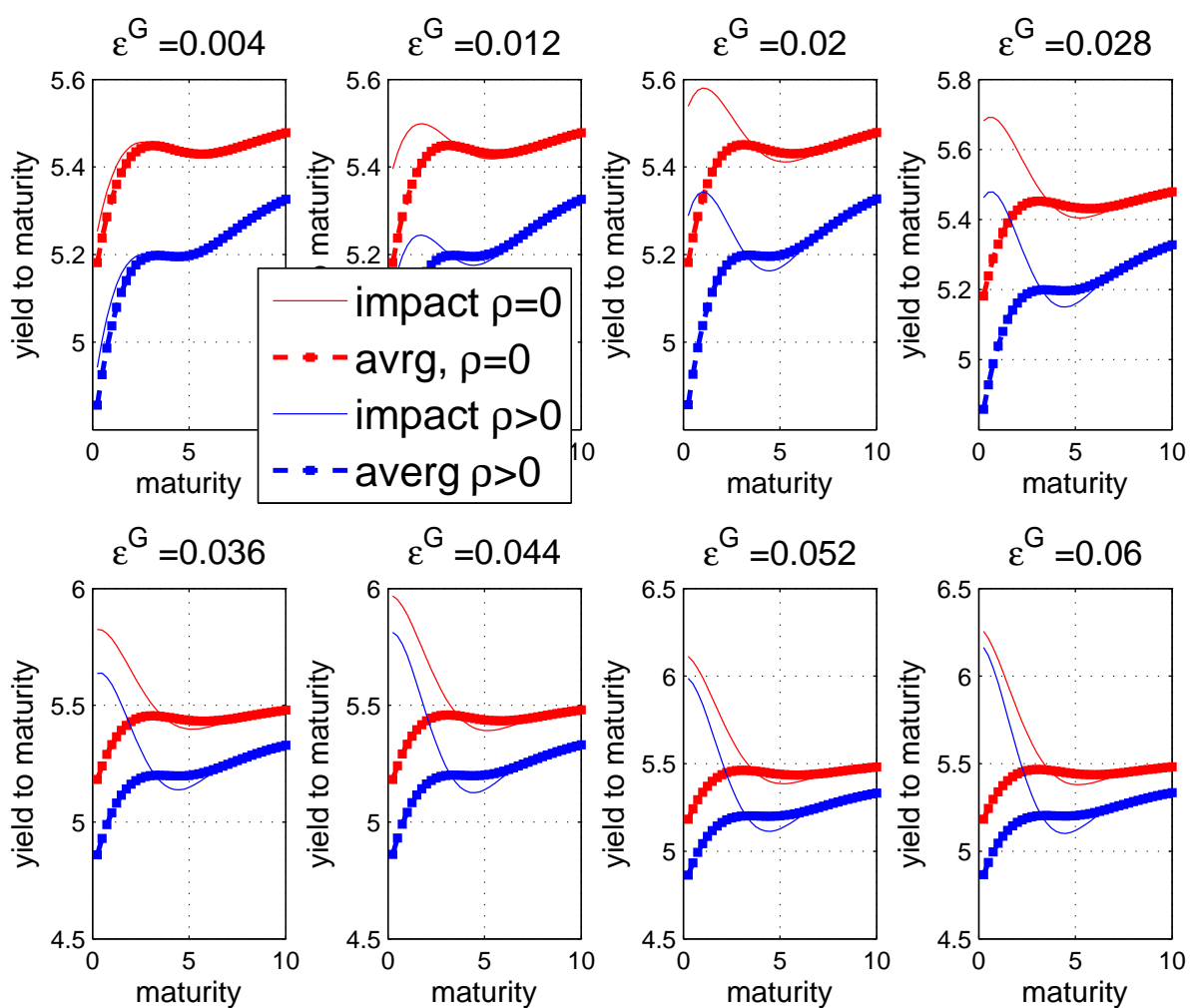
Stochastic Averages of the Nominal Yield curve

Figure 17: Term structure and varying volatility of G shocks in the model with spending reversals when central bank puts $\phi_y = 0$ on output gap stabilization. In the legend is the volatility of the government spending innovation.

Figure 18: Term structure of interest rates in the economy with spending reversals under various volatility G innovations and monetary policy conduct.
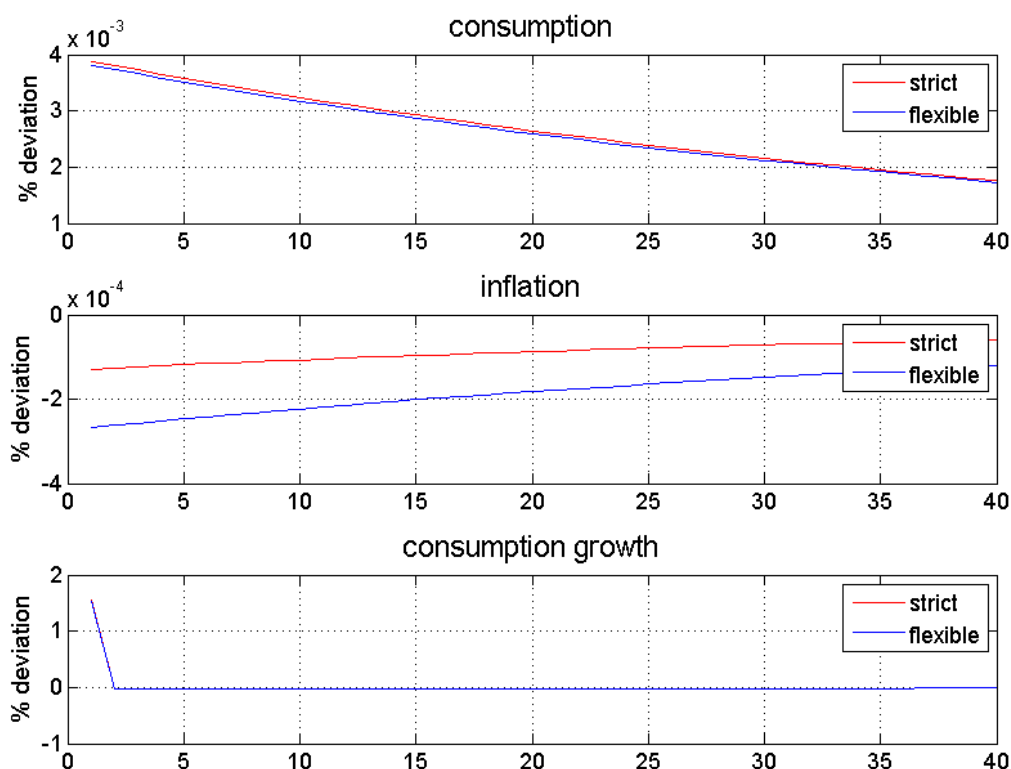
Figure 19: Shows that the TFP shock generates higher inflation risks in the strict inflation targeting regime
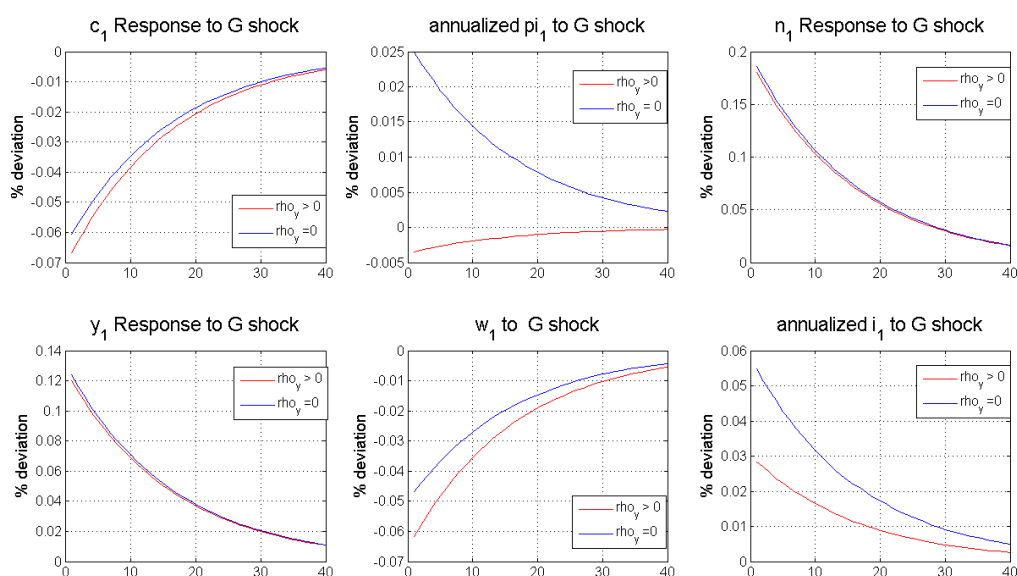
Figure 20: IR functions to 0.8% shock of innovations in government spending. Compares the benchmark model case, $\phi_y = 0.075$ to $\phi_y = 0$
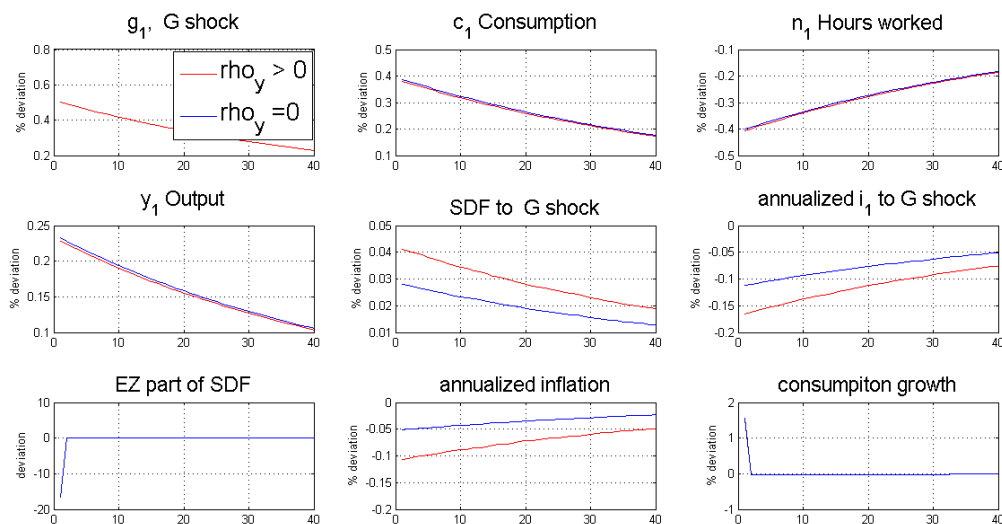
Figure 21: IR functions to 0.5% shock in productivity innovations. Compares the benchmark model case, $\phi_y = 0.075$ to $\phi_y = 0$
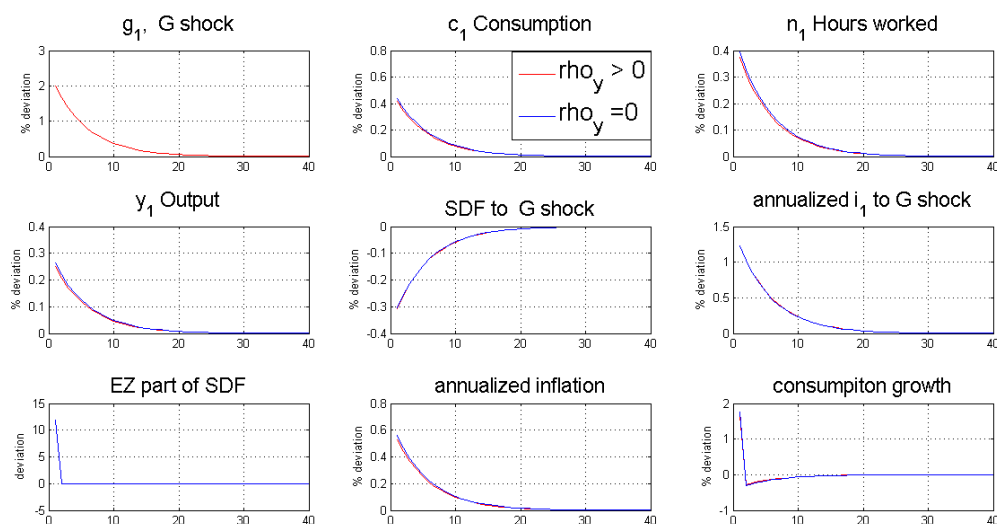
Figure 22: IR functions to benchmark shock calibration of preference shock innovations. Compares the benchmark model case, $\phi_y = 0.075$ to $\phi_y = 0$
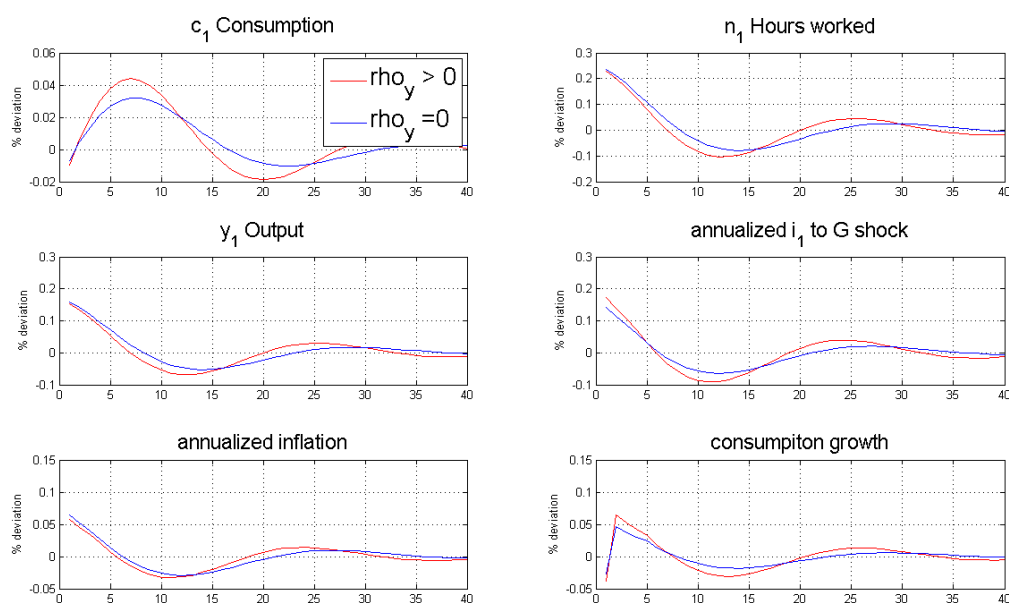
Figure 23: IR functions to benchmark shock calibration of preference shock innovations in case of spending reversals. Compares the benchmark model case, $\phi_y = 0.075$ to $\phi_y = 0$
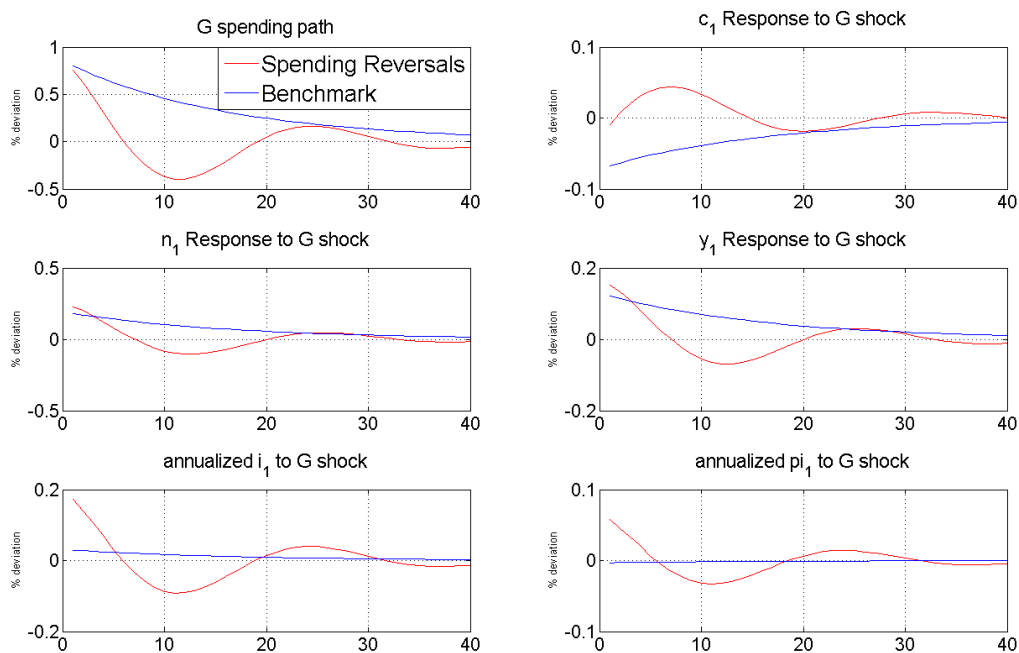

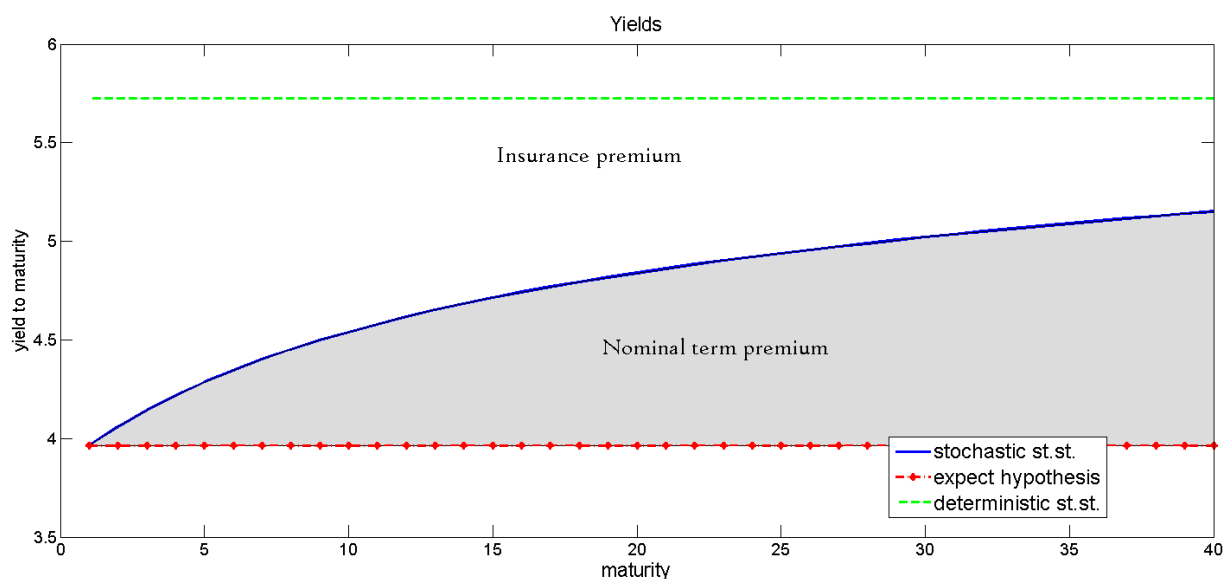Figure 24: Compares IRFs of macro variables for the benchmark model and benchmark with spending reversals

Figure 25: The blue line is the stochastic steady state of the nominal term structure of interest rates. The space between the deterministic steady state (green line) and yield curve represents the insurance premium investors are willing to pay. The difference between the yield curve and and yields consistent with expectations hypothesis constitute the nominal term premium. The intersection of the blue and red line is the one period nominal interest rate
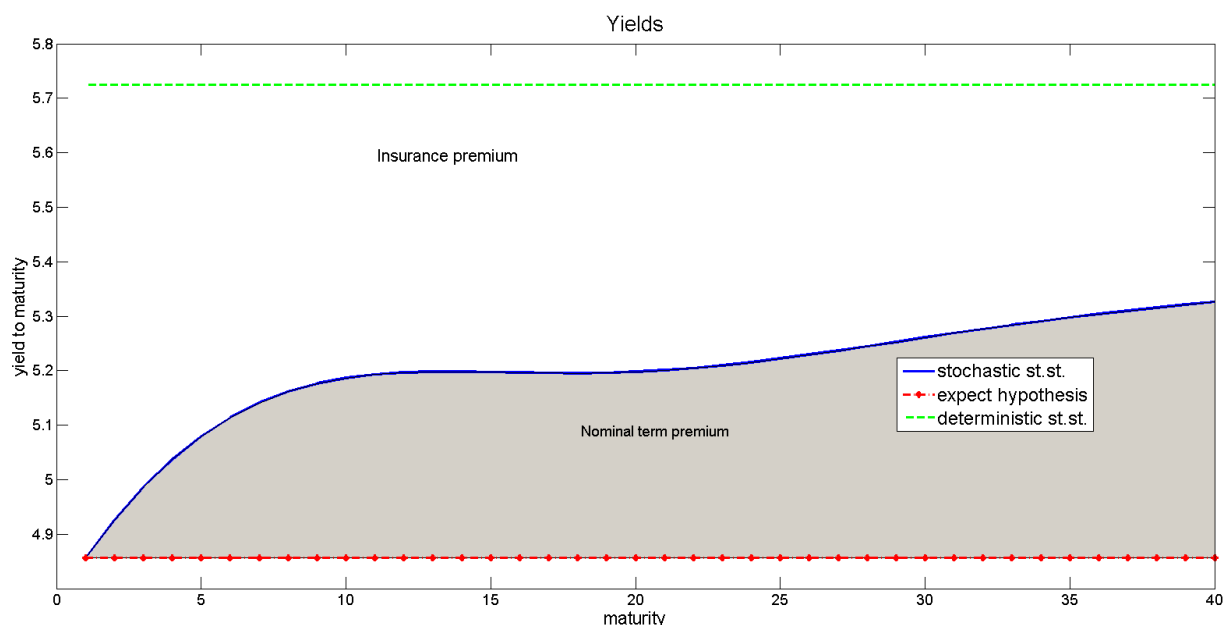

Figure 26: The blue line is the stochastic steady state of the nominal term structure of interest rates. The space between the deterministic steady state (green line) and yield curve represents the insurance premium investors are willing to pay. The difference between the yield curve and and yields consistent with expectations hypothesis constitute the nominal term premium. The intersection of the blue and red line is the one period nominal interest rate
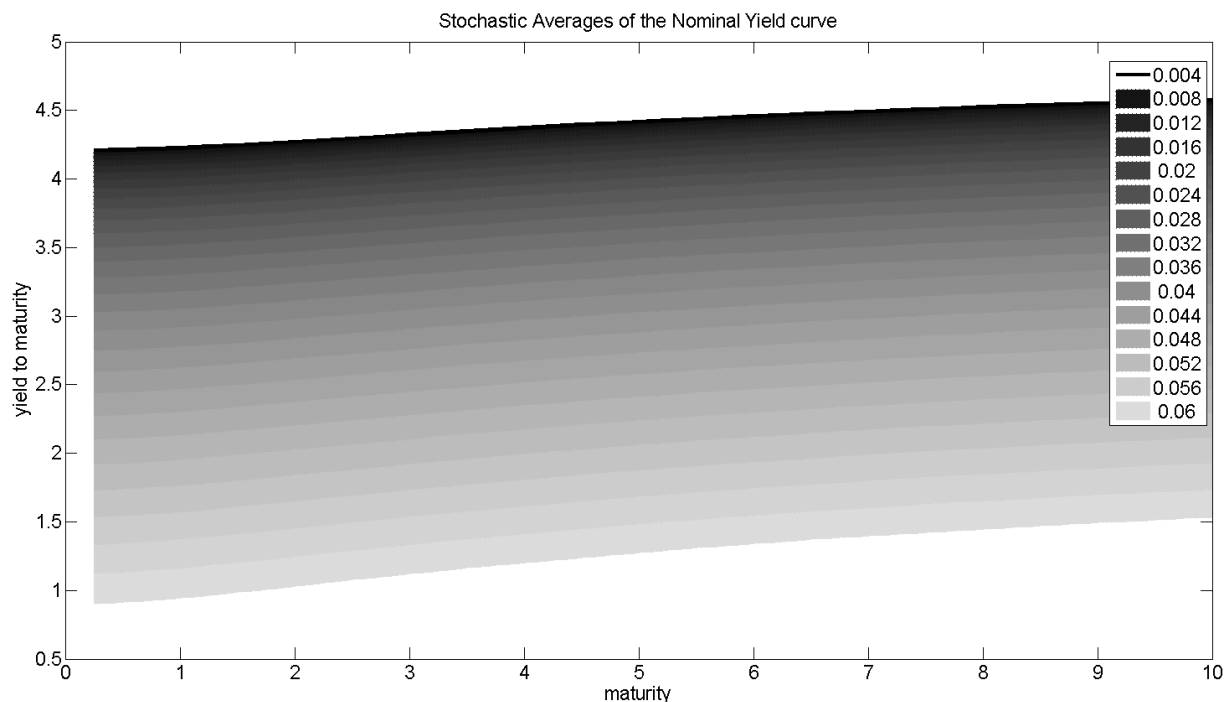
Figure 27: Term structure and varying volatility of G shocks in the Rudebusch and Swanson (2012) for the benchmark calibration. In the legend is the volatility of the government spending innovation.
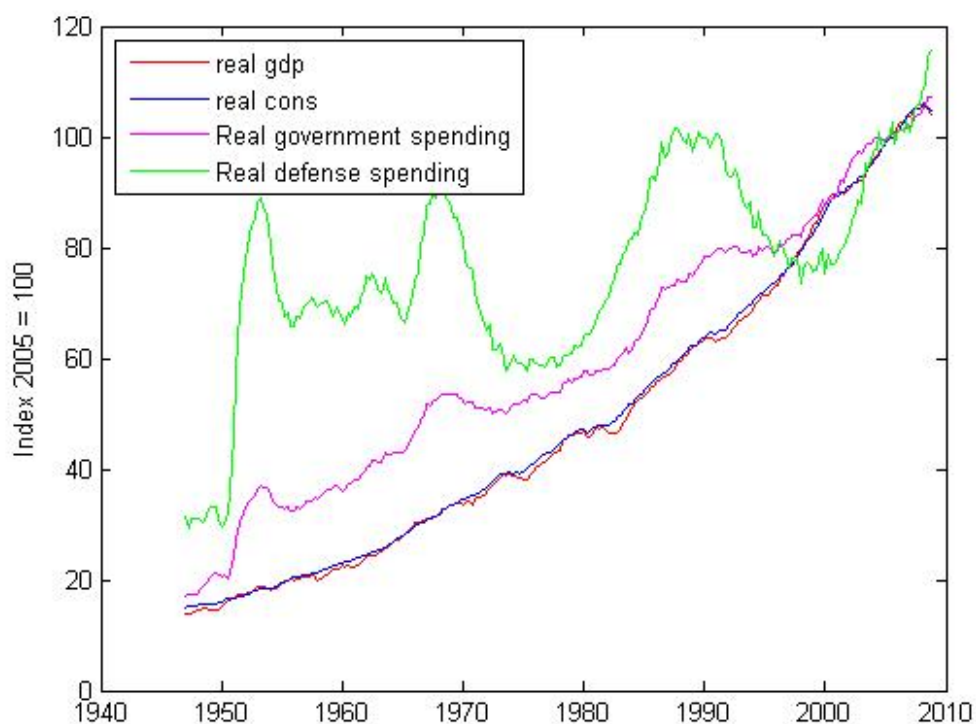

Figure 28: Breakdown of government spending based on the Ramey (2011).

Stand alone factors	Benchmark								Government spending							
	$\phi_y = .075$ $\sigma_g = .004$		$\phi_y = 0$ $\sigma_g = .004$		$\phi_y = .075$ $\sigma_g = .6$		$\phi_y = 0$ $\sigma_g = .6$		$\phi_y = .075$ $\sigma_g = .004$		$\phi_y = 0$ $\sigma_g = .004$		$\phi_y = .075$ $\phi_y = .6$		$\phi_y = 0$ $\phi_y = .6$	
	Levels	%	L	%	L	%	Level	%	Level	%	Level	%	Level	%	Level	%
$-\frac{\gamma^2}{2n}Var_t(\Delta C_{t+n})$	-0.012	(1.3)	-0.012	(1.3)	-0.021	(1.1)	-0.019	(1.3)	0	(0.9)	-0	(1)	-0.009	(0.9)	-0.007	(1)
$-\frac{1}{2n}Var_t(\sum_{j=1}^n \pi_{t+n})$	-0.814	(85.8)	-0.789	(79.3)	-1.396	(72.5)	-1.22	(79)	-0.003	(59.7)	-0.002	(78)	-0.58	(59.7)	-0.437	(78)
$-\frac{\alpha^2}{2n}Var_t(S_{t+n})$	0.077	(-8.1)	0.076	(-7.7)	0.126	(-6.6)	0.12	(-8.1)	0	(-5.1)	0	(-9)	0.05	(-5.1)	0.049	(-9)
$-\frac{1}{2n}\mathrm{Var}_t \sum_{j=1}^n (\zeta_{t,t+j})$	-0.02	(2.1)	-0.02	(2)	-0.02	(1)	-0.02	(1.3)	0	(0)	0	(0)	0	(0)	0	(0)
Factor interactions																
$-\frac{\gamma}{n}Cov_t(\Delta C_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.001	(-0.2)	-0.002	(0.2)	0.001	(-0)	0.003	(-0.2)	-0	(0.1)	0	(-1)	-0.001	(0.1)	0.005	-1
$-\frac{\gamma\alpha}{n}Cov_t(\Delta C_{t+n}, S_{t+n})$	-0.277	(29.2)	-0.277	(27.9)	-0.68	(35.6)	-0.635	(41)	-0.002	(41.8)	-0.0016	(65)	-0.41	(41.8)	-0.36	(65)
$-\frac{\alpha}{n}Cov_t(S_{t+n}, \sum_{j=1}^n \pi_{t+n})$	0.419	(-44.1)	0.35	(-35.2)	0.39	(-20.4)	0.54	(-35)	-0	(2.6)	0.001	(-35)	-0.025	(2.6)	0.193	(-35)
$+\frac{\alpha}{n}Cov_t(S_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	-0.35	(36.9)	-0.34	(35.1)	-0.35	(18.2)	-0.35	(22.6)	0	(0)	0	(0)	0	(0)	0	(0)
$+\frac{\gamma}{n}Cov_t(\Delta C_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	0.017	(-1.7)	0.02	(1.8)	0.017	(-0.9)	0.017	(-1.1)	0	(0)	0	(0)	0	(0)	0	(0)
$+\frac{1}{n}Cov_t(\sum_{j=1}^n (\zeta_{t,t+j}), \sum_{j=1}^n \pi_{t+n})$	0.01	(-1.1)	0.01	(-1.1)	0.01	(-0.5)	0.01	(-0.7)	0	(0)	0	(0)	0	(0)	0	(0)
Total																
$E[ytm_t^n] - \overline{ytm_t}$	-0.948	(100)	-0.994	(100)	-1.925	(100)	-1.55	(100)	-0.004	(100)	-0.002	(100)	-0.98	(100)	-0.56	(100)

Table 8: Shows the factors attribution to the stochastic component of the yield curve for benchmark model and government shock only. The reported numbers are averages over the maturity profile $n = 40$, each column shows the contribution of the attributes to the mean of ergodic distribution of the yield curve in levels and percents ($\frac{f_i}{\sum_i f_i}$). The higher order interactions are zero up to the 2nd order approximation. The benchmark model in the second column is decomposed to contribution of single shocks to overall risk. $L * 100$ means that numbers in the column are multiplied by 100 for better readability. $\alpha = -108$, $\gamma = 2$

Stand alone factors (in %)	Benchmark $\phi_y = 0.075$	Benchmark $\phi_y = 0$	TFP
$-\frac{\gamma^2}{2n}\Delta Var_t(\Delta C_{t+n})$	0.9	1.3	1.4
$-\frac{1}{2n}\Delta Var_t(\sum_{j=1}^{n}\pi_{t+n})$	59.7	78.5	63.3
$-\frac{\alpha^2}{2n}\Delta Var_t(S_{t+n})$	-5.1	-8.8	-8.8
Factor interactions (in %)			
$-\frac{\gamma}{n}\Delta Cov_t(\Delta C_{t+n}, \sum_{j=1}^{n}\pi_{t+n})$	0.1	-0.9	-1
$-\frac{\gamma\alpha}{n}\Delta Cov_t(\Delta C_{t+n}, S_{t+n})$	41.8	64.6	73.2
$-\frac{\alpha}{n}\Delta Cov_t(S_{t+n}, \sum_{j=1}^{n}\pi_{t+n})$	2.6	-34.7	-28.2
Total			
$\Delta E_t[ytm_t^n]$	-0.976	-0.55	-0.978

Table 9: Quantifies the factors attribution to the total drop in the level of the term structure of interest rates when the volatility of the innovations to government spending increases from 0.4% to 6% (volatility of TFP is design to deliver similar size of the drop in the yield curve, from 0.214% to 0.687%). The reported numbers are averages over the maturity profile n, each column shows the percentage contribution of the attributes to the total change in the level of the yield curve.

Stand alone factors	B	G	G $\phi_y = 0$
$-\frac{\gamma^2}{2n}Var_t(\Delta C_{t+n})$	1.3	0.9	1.3
$-\frac{1}{2n}Var_t(\sum_{j=1}^n \pi_{t+n})$	85.4	59.7	78.5
$-\frac{\alpha^2}{2n}Var_t(S_{t+n})$	-8.1	-5.1	-8.8
$-\frac{1}{2n}\mathrm{Var}_t \sum_{j=1}^n (\zeta_{t,t+j})$	2.1	0	0
Factor interactions			
$-\frac{\gamma}{n}Cov_t(\Delta C_{t+n}, \sum_{j=1}^n \pi_{t+n})$	-0.2	0.1	-0.1
$-\frac{\gamma\alpha}{n}Cov_t(\Delta C_{t+n}, S_{t+n})$	29.4	41.8	64.6
$-\frac{\alpha}{n}Cov_t(S_{t+n}, \sum_{j=1}^n \pi_{t+n})$	-43.5	2.6	-34.7
$+\frac{\alpha}{n}Cov_t(S_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	36.4	0	0
$+\frac{\gamma}{n}Cov_t(\Delta C_{t+n}, \sum_{j=1}^n (\zeta_{t,t+j}))$	-1.7	0	0
$+\frac{1}{n}\Delta Cov_t(\sum_{j=1}^n (\zeta_{t,t+j}), \sum_{j=1}^n \pi_{t+n})$	-1	0	0
Total			
$E_t[ytm_t^n] - \overline{ytm_t}$	-0.53	-0.052	0.0066

Table 10: The table shows the factors attribution to the stochastic component of the yield curve and to the transmission of fiscal uncertainty. The reported numbers are averages over the maturity profile n, each column shows the percentage contribution of the attributes to the mean of ergodic distribution of the yield curve. These proportions are constant in the size of uncertainty thus we interpret them as prices of risk. The higher order interactions are zero up to the 2nd order approximation. B represents the benchmark model, G is B with only government spending shock