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PreMISE: DSGE Model of the Slovak Economy Integrated in a Monetary Union

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Executive summary

In this document, we introduce PreMISE – a New Keynesian dynamic stochastic general equilibrium (DSGE) model of the Slovak economy integrated in the euro area that is being developed at the National Bank of Slovakia for the purposes of medium-term projections. The name of the model is a shortcut of the main features of the DSGE model that can be described in short as a ***Pre**diction **M**odel of **I**ntegrated **S**lovak **E**conomy*.

Over the years, impressive amount of expertise in building and practical use of the DSGE models has been accumulated. In our model we draw from the examples of succesfull deployments in forecasting or policy analysis process at other central banks or institutions such as Andrle et al. (2009), Murchison and Rennison (2006), Kilponen and Ripatti (2016), Adolfson et al. (2005), Christoffel et al. (2008) and Coenen et al. (2018).

The Slovak economy is modelled as a small open economy (SOE) integrated in the monetary union of the euro area. Both monetary policy setups are implemented in the model structure – the autonomous monetary policy as well as the monetary union – and the monetary policy regime is switched in the model so as to correspond with the adoption of the euro currency in Slovakia on January 1, 2009.

The domestic economy is modelled with use of following representative economic agents: households, intermediate firms, importers, final goods producers, government and central bank.

Households in the model are Ricardian – they consume, own capital (invest), own firms, have access to financial markets (both domestic and international), supply labour and set the price of their labour services. They also receive transfers from government and pay taxes from consumption (VAT) and labour.

The monopolistically competitive intermediate firms combine capital and labour to produce domestic inetermediate goods. Homogenous bundles of the intermediate goods together with homogenous bundles of imported goods are then used by the final goods producers.

There are four different final production sectors that produce consumption goods, investment goods, government consumption goods and export goods. Firms producing export goods set prices in foreign currency (local currency pricing), which prevents the immediate transmission of real exchange rate movements to prices.

Capital is a homogeneous production factor, which is rented by the firms on a perfectly competitive market. On the other hand, there are infinitely many different types of labour and imported goods. Therefore, importers as well as households have some negotiating power over the prices of imports and wages respectively. All firms also pay taxes levied on their wage costs (social security).

The government collects taxes (both distortionary and lump-sum) paid by households and firms. It uses its revenue for public consumption and to give transfers to households. The difference between revenues and expenditures needs to be financed by the issuance of bonds.

The financial markets are incomplete, which means that households are unable to perfectly insure against unexpected shocks. This structure leads to the uncovered interest parity (UIP) condition which links together the domestic interest rates, foreign interest rates and expected changes in the exchange rate.

The foreign sector is exogenous, which reflects the relatively small size of the domestic economy in comparison to the global economy. A structural vector autoregressive block (SVAR) is used to capture the dynamics of the foreign variables.

In line with the New Keynesian paradigm, a cascade of nominal and real rigidites is assumed throughout the model, which allows us to achieve a gradual pass-through of real exchange rate changes into the domestic price level. For example, firms in all sectors are not allowed to reoptimize prices every period. They follow a pricing mechanism where only a fraction of firms are allowed to reoptimize their prices in a given period à la Calvo (1983). Remaining firms adjust their prices according to past inflation and steady-state inflation. The households adjust their wages in a similar fashion.

The model structure incorporates multiple stochastic trends that are used for stationarisation of the input data within the model. This means that all the input time series are stationarised simultaneously while taking into account their mutual relationships as assumed by the DSGE model.

Model parameters were calibrated in accordance with the literature. However, results of partial estimations of different subsets of model parameters and specific features of the Slovak economic data, such as the *great ratios*, were also taken into account. The parametrization of the model was chosen carefully so as to achieve intuitive behaviour of the model in line with economic theory and was verified using multiple standard methods (impulse response analysis, historical shock decomposition, forecast error decomposition, moments comparison, recursive forecast exercises, etc.).

Currently, the model has been used for evaluating the effects of changes in the underlying assumptions of the official forecast (data revisions, updates in the outlook of exogenous variables, changes in near-term forecast) without necessarily creating an independent model forecast (Mechanical Update). The model is also used to interpret the official or any alternative forecast in terms of the model mechanisms (Shock Decomposition). Finally, after the comprehensive evaluation of the forecasting performance of the model, it can be used for mid-term forecasting.

PreMISE: DSGE Model of the Slovak Economy Integrated in a Monetary Union

Milan Výškrabka *   Martin Železník†   Stanislav Tvrz ‡

November 15, 2019

Abstract

The goal of the paper is to introduce the new structural model (PreMISE) of the National bank of Slovakia and illustrate how it is used for policy analysis. The model derivation and characteristics of its behavior are presented. At the same time procedures that are useful during the prediction process and their contribution to policy analysis are shown.

Keywords: DSGE; general equilibrium; monetary policy; forecasting
JEL-Codes: D58, E32, E58, E47, C53

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1. Introduction

In this document, we introduce PreMISE – a New Keynesian dynamic stochastic general equilibrium (DSGE) model of the Slovak economy integrated in the euro area that is being developed at the National Bank of Slovakia for the purposes of medium-term projections. The name of the model is a shortcut of the main features of the DSGE model that can be described in short as a ***Pre****diction* ***M****odel of* ***I****ntegrated* ***S****lovak* ***E****conomy*.

This model is developed following seminal papers by Christiano et al. (2005) and Smets and Wouters (2007) that provided huge contributions to the development of New Keynesian models that are widely used for policy analysis today. Over the years, impressive amount of expertise in building and practical use of the DSGE models has been accumulated. In our model we draw from the examples of succesfull deployments in forecasting or policy analysis process at other central banks or institutions such as Andrle et al. (2009), Murchison and Rennison (2006), Kilponen and Ripatti (2006), Adolfson et al. (2005) and Christoffel et al. (2008).

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The financial markets are incomplete, which means that households are unable to perfectly insure against unexpected shocks. This structure leads to the uncovered interest parity (UIP) condition which links together the domestic interest rates, foreign interest rates and expected changes in the exchange rate.

The foreign sector is exogenous, which reflects the relatively small size of the domestic economy in comparison to the global economy. A structural vector autoregressive block (SVAR) is used to capture the dynamics of the foreign variables.

In line with the New Keynesian paradigm, a cascade of nominal and real rigidites is assumed throughout the model, which allows us to achieve a gradual pass-through of real exchange rate changes into the domestic price level. For example, firms in all sectors are not allowed to reoptimize prices every period. They follow a pricing mechanism where only a fraction of firms are allowed to reoptimize their prices in a given period à la Calvo (1983). Remaining firms adjust their prices according to past inflation and steady-state inflation. The households adjust their wages in a similar fashion.

The model structure incorporates multiple stochastic trends that are used for stationarisation of the input data within the model. This means that all the input time series are stationarised simultaneously while taking into account their mutual relationships as assumed by the DSGE model.

Model parameters were calibrated in accordance with the literature. However, results of partial estimations of different subsets of model parameters and specific features of the Slovak economic data, such as the *great ratios*, were also taken into account. The parametrization of the model was chosen carefully so as to achieve intuitive behaviour of the model in line with economic theory and was verified using multiple standard methods (impulse response analysis, historical shock decomposition, forecast error decomposition, moments comparison, recursive forecast exercises, etc.).

Currently, the model has been used for evaluating the effects of changes in the underlying assumptions of the official forecast (data revisions, updates in the outlook of exogenous variables, changes in near-term forecast) without necessarily creating an independent model forecast (Mechanical Update). The model is also used to interpret the official or any alternative forecast in terms of the model mechanisms (Shock Decomposition). Finally, after the comprehensive evaluation of the forecasting performance of the model, it can be used for mid-term forecasting.

The rest of the paper is structured as follows: Section 2 describes the main features and stylized-facts of the Slovak economy; Section 3 gives a brief summary of the DSGE model structure; Section 4 contains a detailed exposition of households as representative economic agents in the model and their optimization problems; Section 5 continues the model description with representative economic agents of firms, their different types and pricing behaviour; Section 6 describes the monetary and fiscal policy authorities and in particular the treatment of the monetary policy regime switch; Section 7 presents the foreign sector; Section 8 explains the stationarization of the model variables inside the model with use of trend shocks; Section 9 lists the observable data and presents selected statistics that were used for the calibration of model parameters; Section 10 describes the application of the model for mechanical update of previous forecast based on new assumptions; Section 11 compares the official forecast with data and explain the differences based on the model; final section concludes the paper.

2. A few features of the Slovak economy

The Slovak economy is a small open Central European economy integrated in the Economic and Monetary Union. The Slovak exporters are organically integrated in the supply chains across the Central and Western Europe, especially in the car manufacturing sector. Since Germany is the most important export market for Slovakia,[1] the Slovak economy is tightly interconnected with the core of the euro area and is directly influenced by the economic developments there. The Slovak Republic is a member of the Visegrád group together with its neigbours - Czech Republic, Poland and Hungary. After Germany, these countries are the most important Slovak trading partners, followed by Austria, which is the only neighbouring country of Slovakia that it shares the common european currency with.

Slovakia is still a relatively young market economy. The economic system started to change from socialist planned economy towards the liberal market capitalism only after the Velvet revolution in 1989. Since then, the Slovakia has been converging towards western market economies at a varying pace. In 1995, the real expenditure per capita expressed in purchasing power parity was 48 % of the EU28 average while in 2016 it was 77 %[2]. So far, the Slovak economy converged most rapidly between 1999 and 2008, when the real per capita expenditure increased from 50 % to 71 % of the EU28 average. In this period, the Slovak economy undertook several deep reforms as it prepared to join the European Union in 2004 and to join the monetary union in 2009. Given the values of the real per capita expenditure in neighbouring Austria or in Germany of around 125 %, further convergence and associated structural changes in the Slovak economy can be expected in the future as well.

2.1 Long-term characteristics

The so called *great ratios* are one of the long-term characteristics that capture the specifics of a given economy to a certain extent. They can be defined as the shares of different types of expenditure on the total spending, i.e. expenditure shares, or as fractions of two different price indices, i.e. relative prices. These indicators can be readily calculated using available economic data. Usually, the values of the great ratios are

[1] International trade with Germany accounts for about one fifth of Slovak exports and imports.
[2] Source: Eurostat, http://ec.europa.eu/eurostat/web/products-datasets/-/prc_ppp_ind

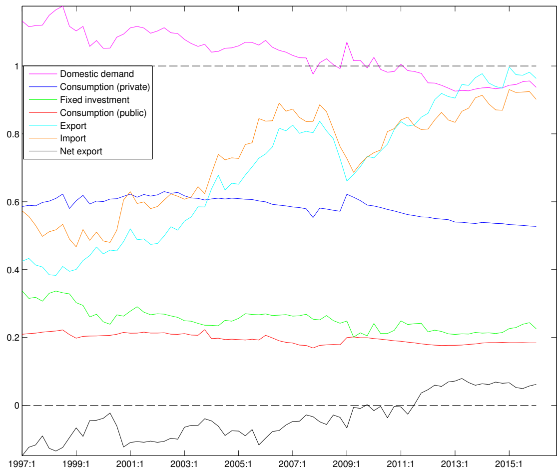
expected to remain roughly constant over time. Therefore, they can be used to calibrate some of the long-run relationships among the model variables on the balanced-growth path (BGP). We define the BGP in a similar way as in King et al. (2001) as the long-run solution of the model where all variables are either constant or grow at a constant pace and nominal expenditure shares are constant.

The well defined BGP is important for the quality of the forecast because it allows us to capture not only the business cycle, but also the medium term behaviour of the economy. Trend-cycle interactions are especially important in emerging economies such as the Slovak Republic, where structural shocks can occur more often, making it hard to disentangle trend and cycle movements in the data, see Aguiar and Gopinath (2004).

Expenditure shares

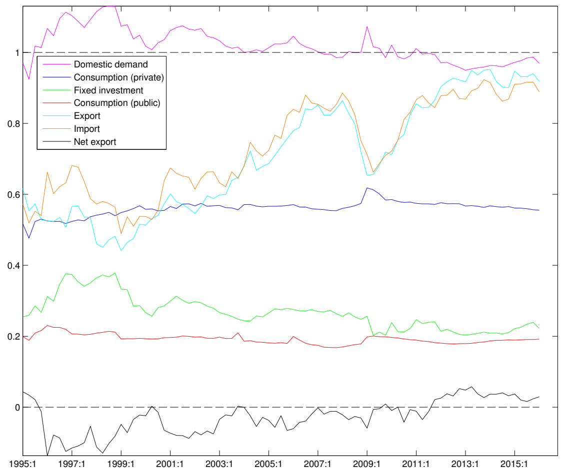
The real and nominal great ratios of the main macroeconomic variables in Slovakia are calculated and shown in Figures 1 and 2, respectively. Numerically, the average values of the great ratios are presented in Table 1. Table 3 presents average annual growth rates of real and nominal aggregates.

Figure 1: Real expenditure shares



Source: Data - Eurostat, Authors' calculations.

Figure 2: Nominal expenditure shares



Source: Data - Eurostat, Authors' calculations.

In Figure 1, the real expenditure shares of private and public consumption and the share of gross fixed capital formation are relatively stable. Nevertheless, there is a noticeable decline in the real private consumption share since the crisis of 2009. A decline in the fixed investment share after the crisis seems to be fading away recently. Remaining shares are developing more dynamically. Most notably, the share of real imports and exports on GDP was growing during the whole depicted period with the exception of the last crisis of 2008–2009. Not only is the Slovak economy becoming ever more open with respect to the international trade, the contribution of the net export to the real GDP has also improved significantly over the past years. Ever since the second half of 2011, the net export of Slovakia has remained distinctively positive. Conversely, the share of total domestic demand on the real GDP declined in 2011 below one as an increasing proportion of the domestic GDP was bought by the foreign residents. Figure 2 gives a similar picture as Figure 1. However, the nominal consumption shares appear to be more stable. That is one of the reasons why we chose to use the nominal expenditure shares in the definition of BGP. Another one being the way of construction of the real aggregates as chain-linked volumes, which makes them non-additive, see Andrle et al. (2009).

Table 1: Expenditure shares

Aggregate	Real expenditure share [%]			Nominal expenditure share [%]		
	1997-2008	2004-2008	2010-2016	1997-2008	2000-2008	2010-2016
Domestic demand	107.49	103.75	95.60	104.00	100.83	97.62
Consumption	80.34	77.99	73.26	75.02	74.39	75.60
– Households and NPISH	60.17	59.19	54.87	55.90	56.46	56.93
— Households	59.25	58.15	53.88	55.03	55.45	55.94
— NPISH	0.93	1.04	0.98	0.87	1.01	0.99
– Public sector	20.14	18.79	18.40	19.11	17.93	18.67
Gross capital formation	28.82	28.38	22.78	29.94	28.67	22.47
– Fixed investment	27.17	25.77	22.33	28.98	26.44	22.02
Export	58.29	74.46	89.74	63.87	77.18	88.96
Import	66.14	80.73	85.73	68.83	80.24	86.98
Net Export	−7.85	−6.28	4.01	−4.96	−3.06	1.98

Source: Data - Eurostat, Authors' calculations.

Due to the fact that Slovakia is a converging economy and also due to the global developments in the international trade (e.g. globalization), export and import have a trend that differs substantially from the common trend of the domestic aggregates. This fact necessitated the incorporation of so called *openness technology* shock into the model. This model variable is used to remove the excessive trend growth of the observed export and foreign demand data inside the model. Explicit definition of the openness shock is given in chapter 9.1.1.

Table 2: Annual growth rates

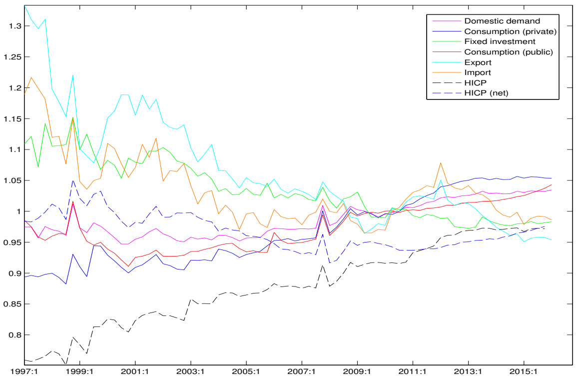
Aggregate	Real growth rate [% p. a.]			Nominal growth rate [% p. a.]		
	1997-2008	2004-2008	2010-2016	1997-2008	2000-2008	2010-2016
GDP	4.90	7.30	2.69	9.52	9.77	3.22
Domestic demand	3.72	5.81	1.71	8.61	9.45	2.95
Consumption	4.37	5.20	0.98	9.81	9.37	2.46
– Households and NPISH	4.68	6.09	0.85	10.39	10.46	2.40
— Households	4.61	6.07	0.86	10.31	10.43	2.42
— NPISH	10.35	7.34	0.31	16.51	12.23	1.30
– Public sector	3.48	2.65	1.40	8.17	6.26	2.67
Gross capital formation	2.40	12.30	4.24	6.53	13.93	4.36
– Fixed investment	1.98	7.83	4.29	5.75	9.70	4.70
Export	9.81	12.08	7.26	11.79	12.12	7.34
Import	7.48	11.69	6.18	10.44	13.16	7.00
Net Export	−7.11	5.08	76.32	−2.13	41.67	24.71

Source: Data - Eurostat, Authors' calculations.

Price developments

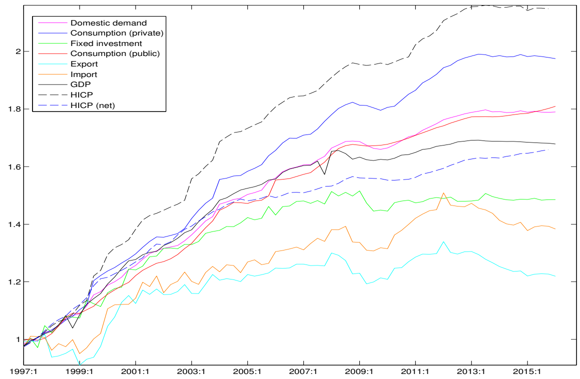
The relative prices of the main macroeconomic aggregates are depicted in Figure 3 and numerical values are presented in Table 3. Development of price indices and deflators is depicted in Figure 4, average annual inflation rates are presented in Table 3.

Figure 3: Relative prices



Source: Data - Eurostat, Authors' calculations.

Figure 4: Deflators and HICP indices



Source: Data - Eurostat, Authors' calculations.

In Figure 3, we can see that while the relative prices of different measures of consumer prices fluctuate in a narrow band around one (except headline HICP[3]), the relative prices of exports, imports and fixed investment have completely different - declining - trajectory. This observation is in line with the (almost ever) increasing expenditure shares of export and import. However, in order to reconcile the price development of fixed investment with its slightly declining expenditure share, we need to add another *investment specific technology* into the model, which acknowledges that capital goods seem to be produced with a different technology than consumption goods, with the growth of technological change in the production of capital goods being higher on average. Together with declining relative price of fixed investment, it can be interpreted as evidence that there have been significant technological advances, which have made the investment less expensive, causing increase in the accumulation of capital in the short and long run. This way, we remove the excessive trend from the investment goods prices. Explicit definition of the investment speciffic shock, as was proposed in Greenwood et al. (1997) or in Whelan (2006), will be given in section 5.4.

[3] This is caused by the fact that it contains prices of gasoline and also regulated prices that grow on average faster than other prices in the economy. As we can see in the Figure 3, HICP (net) is much closer to other aggregates than headline HICP.

Table 3: Relative prices and Annual rates of Inflation

Aggregate	Relative price [%]			Inflation rate [% p. a.]		
	1997-2008	2004-2008	2010-2016	1997-2008	2004-2008	2010-2016
GDP				4.41	2.30	0.52
Domestic demand	96.77	97.22	102.14	4.71	3.44	1.21
Consumption	93.44	95.44	103.25	5.21	3.97	1.47
– Households and NPISH	92.96	95.43	103.83	5.46	4.12	1.54
— Households	92.94	95.41	103.88	5.45	4.11	1.55
— NPISH	93.41	96.51	100.74	5.58	4.55	0.99
– Public sector	94.95	95.49	101.46	4.53	3.52	1.25
Gross capital formation	103.85	101.05	98.56	4.03	1.45	0.11
– Fixed investment	106.40	102.63	98.57	3.70	1.73	0.40
Export	111.33	103.75	99.27	1.81	0.03	0.08
Import	105.04	99.39	101.51	2.75	1.32	0.77
HICP	83.77	87.90	95.50	6.10	3.86	1.54
– HICP (net)	97.73	94.75	95.13	4.12	1.64	1.05
– HICP (energy)	67.91	86.94	99.53	13.42	8.24	0.77
– HICP (food)	85.01	82.94	94.10	4.16	3.04	2.67

Source: Data - Eurostat, Authors' calculations.

3. Overview of the model structure

The model represents a small open Slovak economy using a New Keynesian dynamic stochastic general equilibrium paradigm. Representative agents of households and firms are included in the model together with monetary and fiscal policy rules and other essential macroeconomic linkages. Where possible, the behaviour of economic agents is derived from microeconomic foundations, i.e. as a solution to clearly defined optimization problems under explicit assumptions.

Firms

There are two types of firms, both monopolistically competitive. First, a single tradable differentiated intermediate good is produced using two input factors labour and capital. The type of this production function is Cobb-Douglas. Second, final good is produced combining the domestically produced intermediate goods with imported goods. Moreover there are four different final production good sectors that produce consumption goods, investment goods, government consumption goods and export goods. Final good firms have the CES type of production function. Each firm produces a differentiated good which allows it to charge a price that may differ from the prices charged by its competitors. Nevertheless, firms follow a Calvo pricing mechanism where only a fraction of firms are allowed to reoptimize their prices in a given period. All the remaining firms adjust their prices following the past and steady-state inflation. Moreover, firms producing export goods set prices in foreign currency (local currency pricing), which prevents the immediate transmission of exchange rate movements to prices. Capital is a homogeneous production factor, which is rented by the firms on a perfectly competitive market. On the other hand, there are infinitely many different types of labour and imported goods. All firms also pay taxes levied on their wage cost (social security).

Households

Households are Ricardian – they consume, own capital (invest), own firms, have access to financial markets (both domestic and international), supply labour and set the price of their labour service. They also receive transfers from government and pay taxes from consumption (VAT) and labour.

Monetary authority

There are two regimes of monetary policy implemented in the model. Until the end of 2008, the monetary policy setup reflects the independent central bank with autonomous decision making and full authority over the setting of the interest rates on domestic bonds. In 2009, when Slovakia adopted the common European currency and became a member of the euro area, the monetary policy became exogenous – reflecting the development in the whole euro area economy, of which Slovakia is only a very small part. In both cases, we assume inflation targeting regime.

Fiscal authority

The government collects taxes (both distortionary and lump-sum) paid by households and firms. It consumes part of its income unproductively and pays the rest in a form of transfers to households. The difference between revenues and expenditures is financed by issuing domestically tradable bonds.

Financial markets

The financial markets are incomplete which means that households are unable to perfectly insure against unexpected shocks. This structure leads to the UIP condition which links together the domestic interest rates, foreign interest rates and expected changes in the exchange rate together with risk premium paid on foreign risk-free bonds, which represents, in the sense of Schmitt-Grohé and Uribe (2003), the debt elastic premium and its role to stationarize the level of debt.

Foreign sector

The foreign sector is comprised of interest rates, prices of imported goods and demand for imported[4] goods is exogenous, reflecting the negligible size of the domestic economy.

Balanced-growth path

The real part of the growth at the BGP is driven by a set of nonstationary technology processes:

[4] In both cases we mean *imported* from the point of view of the foreign economy. A part of the foreign demand is satisfied by the domestic export.

- Labour augmenting technology process , A_t - the main driving force of growth
  - evolves according to the following process:

$$\Delta A_t = \rho_A \Delta A_{t-1} + (1-\rho_A) g^A + \epsilon_t^A \tag{3.1}$$

  - $\Delta A_t = A_t / A_{t-1}$ represents the gross rate of labour-augmenting productivity growth with steady state value, $\overline{\Delta A} = \overline{\left(\frac{A_t}{A_{t-1}}\right)} = g^A$, same applies for other four technology processes.
- Willingness to work technology process, ξ_t^N - introduces nonstationarity in hours worked, $\overline{\Delta \xi_t^N} = \overline{\xi_t^N} / \overline{\xi_{t-1}^N} = g^{\xi N}$,
- Investment specific technology process, ξ_t^I - allows relative prices to trend in the steady state, $\overline{\Delta \xi_t^I} = \overline{\xi_t^I} / \overline{\xi_{t-1}^I} = g^{\xi I}$,
- Trade productivity technology process, ξ_t^X - allows imports and exports to grow faster than the output in the steady state, $\overline{\Delta \xi_t^X} = \overline{\xi_t^X} / \overline{\xi_{t-1}^X} = g^{\xi X}$,
- Exports quality technology process, ξ_t^Q - allows higher growth rate of domestic exports relative to the foreign demand proxy in the steady state, $\overline{\Delta \xi_t^Q} = \overline{\xi_t^Q} / \overline{\xi_{t-1}^Q} = g^{\xi Q}$.

Apart from these five real processes, there is also a nominal trend assumed in the model:

- consumer price index, P_t^C - grows on the BGP at a predetermined rate π_C.

4. HOUSEHOLDS

Households are modelled as Ricardian, i.e. they have access to the financial markets, where they can buy domestic and international bonds and they can accumulate physical capital. This allows the members of households to smooth their consumption as a response to shocks. Households also supply differentiated labour services and set their wage in monopolistically competitive markets.

4.1 UTILITY MAXIMIZATION PROBLEM

The representative household is assumed to have a continuum of members, $\mathbb{J}$. Each member j, $(j \in \mathbb{J})$ of representative household maximizes its lifetime utility. The member j chooses the optimal quantities of consumption good purchases, $C_t(j)$, investment good purchases, $I_t(j)$, next periods's physical capital stock, $K_{t+1}(j)$, next period's holdings of domestic and international bonds respectively, $B_{t+1}(j)$ and $B^*_{t+1}(j)$, and utilization of capital which means the intensity with which the existing capital stock is utilized, $u_t(j)$. The household members maximize a given lifetime utility function of the following form:

$$U_{t+k}(j) = E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left(\xi^C_{t+k}(1-\Upsilon) \ln \left(C_{t+k}(j) - \Upsilon * C_{t+k-1} \right) - \xi^N_{t+k} \frac{1}{1+\varphi} \left(N_{t+k}(j) \right)^{1+\varphi} \right) \right\}, \quad (4.1)$$

where β is the discount factor, φ is the inverse of the elasticity of work with respect to the real wage (inverse Frisch elasticity). Following Fuhrer (2000) we introduce external habit in consumption that is expressed by the parameter Υ. Variables ξ^C_t and ξ^N_t denotes preference shock and labour supply shock, respectively. Labour supply shock is included in the model according to Chang et al (2007) in order to deal with nonstationarity of observed hours worked.

This setting shows us that utility of household member depends positively on the difference between the individual consumption in period t, $C_t(j)$ and lagged economy-wide consumption level, C_{t-1} and negatively on individual labour supply.

The household member is confronted with the following budget constraint:

$$\begin{aligned} &(1+\tau^C_t)P^C_t C_t(j) + P^I_t I_t(j) + (\xi^{RP}_t R_t)^{-1} B_{t+1}(j) + \left[(1-\Phi^*_B(A_t))\, R^*_t\right]^{-1} B^*_{t+1}(j) S_t \\ &\quad + T_t(j) = (1-\tau^W_t) N_t(j) W_t(j) + B_t(j) \\ &\quad + B^*_t(j) S_t + R^K_t u_t(j) K_t(j) - \Gamma_u(u_t(j)) P^I_t K_t(j) + Tr_t(j) + Div_t(j) \end{aligned} , \quad (4.2)$$

where P_t^C and P_t^I are prices of the private consumption good and the investment good, respectively. Interest rates used here, R_t and R_t^* are risk-less returns on domestic government bonds and international bonds. ξ_t^{RP} represents domestic risk-premium shock. S_t is nominal exchange rate (expressed in terms of units of the domestic currency per unit of the foreign currency, SKK/EUR) and it is used because internationally traded bonds are denominated in foreign currency. $T_t(j)$ represents lump sum tax. $N_t(j)$ means labour provided to firms for wage $W_t(j)$. τ_t^C represents tax rate on consumption (value added tax, VAT) and τ_t^N denotes tax rate levied on the household's wage income. Function $\Phi_B^*(A_t)$ stands for external financial intermediation premium, which depends on the economy-wide net holdings of internationally traded foreign bonds expressed in domestic currency relative to domestic nominal output $A_t = S_t B_t^*/NGDP_t$ and takes form:

$$\Phi_B^*(A_t) = \gamma_B^* \left(\exp\left(\frac{S_t B_t^*}{NGDP_t} \right) - 1 \right). \tag{4.3}$$

R_t^K is the price that firms pay for renting capital from households. Tr_t represents transfers from government and Div_t are dividends paid by household-owned firms. Varying intensity of utilising the physical capital stock, $u_t(j)$[5] is subject to a proportional cost $\Gamma_u(u_t(j))$ which takes the form:

$$\Gamma_u(u_t(j)) = \gamma_{u,1}(u_t(j) - 1) + \frac{\gamma_{u,2}}{2}(u_t(j) - 1)^2. \tag{4.4}$$

Further they confront capital accumulation law of motion:

$$K_{t+1}(j) = \left[1 - S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j) + (1 - \delta) K_t(j), \tag{4.5}$$

where adjustment cost function $S(\cdot)$ expressed by investment changes follows:

$$S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) = \frac{\gamma_I}{2} \left(\frac{I_t(j)}{I_{t-1}(j)} - g^I \right)^2, \tag{4.6}$$

where γ_I is the investment adjustment cost parameter and $g^I = \overline{\left(\frac{I_t}{I_{t-1}} \right)}$ is the steady-state growth rate of investment.

[5] According to Christiano et al. (2011), the introduction of variable capital utilization is motivated by a desire to explain the slow response of inflation to a monetary policy shock. In any model prices are heavily influenced by costs. Costs in turn are influenced by the elasticity of the factors of production. If factors can be rapidly expanded with a small rise in cost, then inflation will not rise much after a monetary policy shock. Allowing for variable capital utilization is a way to make the services of capital elastic.

4.2 WAGE SETTING

The members of households operate in monopolistically competitive markets which means that they set their wage for their differentiated labour services $N_t(j)$. Wages are detemined by staggered nominal wage contracts à la Calvo, and therefore only a fraction of household members are allowed to reset their nominal wage contract in a given period t. The probability with which the household member is allowed to renegotiate its wage is set to $1 - \omega^W$. Household members that are allowed to renegotiate set their wage to the optimal level, which is the same across the labour market:

$$\widetilde{W}_t(j) = \widetilde{W}_t. \tag{4.7}$$

Remaining fraction of households, who are not allowed to reoptimize their wage in a given period t with probability ω^W, adjust their wages according to the following indexation scheme:

$$W_t(j) = {\pi_C}^{1-\gamma^W} \pi_{C,t-1}^{\gamma^W} W_{t-1}(j). \tag{4.8}$$

Hence, the wage contract is indexed to a consumer-price inflation rate in the period before negotiating and to a steady-state consumer-price inflation rate, π_C, where γ^W is an indexation parameter and $\pi_{C,t} = P_t^C / P_{t-1}^C$.

The representative household members who are allowed to renegotiate their wage contracts to an optimal level in period t maximize their lifetime utility as in equation (4.1), taking into account the indexation scheme(4.8) and the demand for their labour services stated lower.

The maximization problem is:

$$\max_{W_t(j)} E_t \left\{ \sum_{k=0}^{\infty} \left(\omega^W\right)^k U_{t+k}(j) \left(C_{t+k}, N_{t+k}\right) \right\}$$

subject to (4.2), indexation scheme (4.8) and demand for labour variety $N_t(j)$ (4.12).

Wage indexation scheme implies for the wages in future period:

$$\begin{aligned} \text{with probability } \omega^W &\rightarrow W_{t+1}(j) = {\pi_C}^{1-\gamma^W} \pi_{C,t}^{\gamma^W} W_t(j) \\ \text{with probability } (1-\omega^W) &\rightarrow \widetilde{W}_{t+1}(j) \end{aligned}$$

We obtain an optimal wage contract:

$$\widetilde{W}_t(j) = \left(\frac{\theta_t^W}{1-\theta_t^W}\right) \frac{\sum_{k=0}^{\infty} \left(\beta\omega^W\right)^k \xi_{t+k}^N N_{t+k}(j)^{1+\varphi}}{\sum_{k=0}^{\infty} \left(\beta\omega^W\right)^k \lambda_{t+k}(1-\tau_t^W) N_{t+k}(j) \pi_C^{1-\gamma^W} \left(\frac{P_{t+k-1}^C}{P_{t-1}^C}\right)^{\gamma^W}}$$

And the total wage level develops according to:

$$W_t = \left[(1-\omega^W)\widetilde{W}_t^{1-\theta_t^W} + \omega^W \left({\pi_C}^{1-\gamma^W} \pi_{C,t-1}^{\gamma^W} W_{t-1}\right)^{1-\theta_t^W}\right]^{\frac{1}{1-\theta_t^W}}$$

4.3 Labour bundling

We assume a continuum of intermediate production firms, $\mathbb{F}$. An intermediate goods producing firm f, $(f \in \mathbb{F})$ uses a bundle $N_t(f)$ of all varieties of heterogenous labour services $N_t(f,j)$, $(j \in \mathbb{J})$, that is created using following CES production function:

$$N_t(f) = \left[\int_0^{\mathbb{J}} N_t(f,j)^{\frac{\theta_t^W - 1}{\theta_t^W}} \, \mathrm{d}j\right]^{\frac{\theta_t^W}{\theta_t^W - 1}}, \tag{4.9}$$

where θ_t^W is the time-varying elasticity of substitution between individual labour varieties, that develops according to

$$\theta_t^W = \rho^{\theta^W} \theta_{t-1}^W + \left(1-\rho^{\theta^W}\right) \overline{\theta^W} + \epsilon_t^{\theta^W}. \tag{4.10}$$

Given the bundling technology, the amounts of labour varieties $N_t(f,j)$ are chosen so as to minimize the cost of the bundle, i.e.:

$$\min_{N_t(f,j)} \int_0^{\mathbb{J}} W_t(j) N_t(f,j) \, \mathrm{d}j, \tag{4.11}$$

such that (4.9) holds. The Lagrangian of this problem is:

$$\min_{N_t(f,j)} \int_0^{\mathbb{J}} W_t(j) N_t(f,j) \, \mathrm{d}j + \lambda_t \left(N_t(f) - \left[\int_0^{\mathbb{J}} N_t(f,j)^{\frac{\theta_t^W - 1}{\theta_t^W}} \, \mathrm{d}j\right]^{\frac{\theta_t^W}{\theta_t^W - 1}} \right),$$

where λ_t is a Lagrange multiplier associated with the technological constraint. The first order condition with respect to $N_t(f,j)$ is:

$$N_t(f,j) = \left(\frac{W_t(j)}{\lambda_t}\right)^{-\theta_t^W} N_t(f).$$

Plug the above expression for $N_t(f,j)$ into the technology equation and find an expression for λ_t, which is

$$\lambda_t = \left(\int_0^{\mathbb{J}} W_t(j)^{1-\theta_t^W}\,\mathrm{d}j\right)^{\frac{1}{1-\theta_t^W}} = W_t\ ,$$

and is equal to total wage. Total demand for j type of labour by firm f thus becomes

$$N_t(f,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta_t^W} N_t(f).$$

Aggregating over all firms $f \in \mathbb{F}$ one obtains total demand for j type of labour

$$N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta_t^W} N_t, \tag{4.12}$$

where N_t is total demand for labour.

5. Firms

There are two types of firms, both monopolistically competitive. First type produces a single tradable differentiated intermediate good, $Ykl_t(f)$. Firms of the second type produce final good by combining the domestically produced intermediate goods with imported goods. There are four kinds of the final goods producing firms that produce either private consumption good, $Y_t^C(f)$, private investment good, $Y_t^I(f)$, public consumption good, $Y_t^G(f)$ or export good, $Y_t^{EX}(f)$.

5.1 Intermediate good producing firms

There is a continuum of intermediate good productng firms, $\mathbb{F}$. Each firm f $(f \in \mathbb{F})$ produces its output using Cobb-Douglas production function that is subject to fixed costs of production[6], fc_t :

$$Ykl_t(f) = \xi_t^{TFP} \left(A_t N_t(f)\right)^{\alpha} \mathcal{K}_t(f)^{1-\alpha} - fc_t, \tag{5.1}$$

where inputs are homogeneous effective capital, $\mathcal{K}_t(f) = u_t K_t(f)$ (u_t being the capital utilization rate) that is rented from households in fully competitive markets and differentiated labour, $N_t(f)$, that is provided by households in monopolistically competitive market. Further, α represents the labour share parameter and ξ_t^{TFP} is transitory technology shock that affects total-factor productivity, while A_t denotes a permanent technology shock that shifts the productivity of labour lastingly. It evolves according to the following process:

$$\Delta A_t = \rho_A \Delta A_{t-1} + (1 - \rho_A) g^A + \epsilon_t^A, \tag{5.2}$$

where $\Delta A_t = \frac{A_t}{A_{t-1}}$ represents the gross rate of labour-augmenting productivity growth with steady-state value $\overline{\Delta A} = \overline{\left(\frac{A_t}{A_{t-1}}\right)} = g^A$.

The intermediate firm optimization problem is the minimization of total cost of production,

$TC_t = (1 + \tau_t^N) W_t N_t(f) + R_t^K \mathcal{K}_t(f)$, taking rental cost of capital R_t^K and the aggregate wage index, W_t as given and subject to the technology constraint (5.1). Let us define $NMC_t^{Ykl}(f)$ as the Lagrange multiplier associated with the technology constraint. $NMC_t^{Ykl}(f)$ then measures the shadow price of varying the use of capital and labour

[6] According to Christoffel et al. (2008) the fixed costs are chosen to ensure zero profits in steady state. This should guarantee that there is no incentive for other firms to enter the market in the long run.

services.

We obtain following combined first order optimality condition:

$$\mathcal{K}_t(f) = \frac{(1+\tau_t^N)W_t}{R_t^K}\frac{1-\alpha}{\alpha}N_t(f). \tag{5.3}$$

Since all firms face the same input prices and same production technology, $NMC_t^{Ykl}(f)$ are identical across firms, $NMC_t^{Ykl}(f) = NMC_t^{Ykl}$.

Nominal marginal costs can be expressed as:

$$NMC_t^{Ykl} = \frac{1}{\xi_t^{TFP} A_t^{\alpha}\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} R_t^{K(1-\alpha)}\left((1+\tau^W)W_t\right)^{\alpha}. \tag{5.4}$$

The intermediate good producers operate at a monopolistically competitive market with time-varying elasticity of substitution between intermediate good varieties of θ_t^{Ykl} (that develops analogously to (4.10)) and they exercise their market power in setting the price of their product $P_t^{Ykl}(f)$. The intermediate good producing firms set their prices in a staggered manner à la Calvo, with probability of being able to reoptimize their price of $1-\omega^{Ykl}$.

Due to similarity and to save space, the pricing decision of intermediate firms is not exposed here and the reader is referred to subsection 5.4, where the pricing decision of a final producer is discussed.

5.2 Importing firms

We assume that there is a continuum $\mathbb{F}^{IM}$ of firms which buy foreign products abroad and sell them in the domestic economy to a national packer who assembles all available goods varieties into a homogenous bundle and prices it. The importing firms use *local currency pricing* strategy to price their products and operate in a monopolistically competitive environment[7]. This allows them to set their desired price given the state of the economy. The fraction of firms are, however, not allowed to adjust their prices every period. They follow a Calvo pricing mechanism similar as shown before.

Those firms that cannot optimally set the price adjust their price from last period according to a weighted average of current inflation and steady state inflation. There are ω^{IM} of those that index their prices. The rest of firms, $1-\omega^{IM}$ fraction of all, solve the

[7] Optimalization of importing firm and changes in their markups can cause the deviations from the law of one price (LOOP).

following optimization problem:

$$\max_{P_t^M(f)} E_t \sum_{k=0}^{\infty} \left(\omega^{IM}\right)^k \Lambda_{t,t+k} IM_t(f) \left[P_{t+k}^M(f) - P_t^{EX*} S_t\right],$$

subject to indexation scheme:

$$P_{t+k}^{IM}(f) = {\pi_{IM}}^{1-\gamma^{IM}} {\pi_{IM,t+k-1}}^{\gamma^{IM}} P_{t+k-1}^{IM}(f),$$

and import demand, that is given by (5.9). P_t^{EX*} is exogenously given foreign price level (index of prices of foreign exporters in foreign currency). For the detailed derivation of the optimal price and aggregate price index see the wage setting problem in section 4.2. Here comes the analogous expressions for import goods.

The new optimal price of imports is set in line with following formula:

$$\widetilde{P}_t^{IM} = \left(\frac{\theta_t^{IM}}{\theta_t^{IM} - 1}\right) \frac{\sum_{k=0}^{\infty} \left(\omega^{IM}\right)^k \Lambda_{t,t+k} P_{t+k}^{EX*} S_t \left[\pi_{IM}^{1-\gamma^{IM}} \left(\frac{P_{t+k-1}^{IM}}{P_{t-1}^{IM}}\right)^{\gamma^{IM}}\right]^{-\theta_t^{IM}} (P_{t+k}^{IM})^{\theta_t^{IM}} IM_{t+k}}{\sum_{k=0}^{\infty} \left(\omega^{IM}\right)^k \Lambda_{t,t+k} \left[\pi_{IM}^{1-\gamma^{IM}} \left(\frac{P_{t+k-1}^{IM}}{P_{t-1}^{IM}}\right)^{\gamma^{IM}}\right]^{1-\theta_t^{IM}} \left(P_{t+k}^{IM}\right)^{\theta_t^{IM}} IM_{t+k}}, \tag{5.5}$$

where $\Lambda_{t,t+k} = \beta^k \dfrac{\lambda_{t+k}}{\lambda_t}$ represents a stochastic discount factor.

The whole import price index evolves according to:

$$P_t^{IM} = \left[(1-\omega^{IM}) {\widetilde{P}_t}^{IM \frac{\theta_t^{IM}}{\theta_t^{IM}-1}} + \omega^{IM} \left[\pi_{IM}^{1-\gamma^{IM}} \pi_{IM,t-1}^{\gamma^{IM}} P_{t-1}^{IM}\right]^{\frac{\theta_t^{IM}}{\theta_t^{IM}-1}}\right]^{\frac{\theta_t^{IM}-1}{\theta_t^{IM}}}. \tag{5.6}$$

In the steady-state it holds that:

$$P^{IM} = \left(\frac{\overline{\theta^{IM}}}{\overline{\theta^{IM}} - 1}\right) \overline{P^{EX*} S}. \tag{5.7}$$

5.3 Bundling of imports

We assume that firms that assemble foreign products into a single homogenous bundle operate outside the borders of the Slovak economy. The technology used to produce bundles is a standard CES function with θ_t^{IM} being the time-varying elasticity of substitution (develops analogously to (4.10)):

$$IM_t = \left(\int_0^{\mathbb{F}^{IM}} IM_t(f)^{\frac{\theta_t^{IM}-1}{\theta_t^{IM}}} \, \mathrm{d}f \right)^{\frac{\theta_t^{IM}}{\theta_t^{IM}-1}} . \tag{5.8}$$

Final composition of the bundle is such that the cost of the bundle is minimal given total demand for imported goods and thus is a solution to the following problem

$$\min_{IM_t(f)} \int_0^{\mathbb{F}^{IM}} P^{IM}(f) IM_t(f) \, \mathrm{d}f,$$

$$s.t. \text{ (5.8)}$$

This leads to the following familiar expression for demand for individual products

$$IM_t(f) = \left(\frac{P_t^{IM}(f)}{P_t^{IM}} \right)^{-\theta_t^{IM}} IM_t, \tag{5.9}$$

and the price index

$$P_t^{IM} = \left(\int_0^{\mathbb{F}^{IM}} P_t^{IM}(f)^{1-\theta_t^{IM}} \, \mathrm{d}f \right)^{\frac{1}{1-\theta_t^{IM}}} . \tag{5.10}$$

5.4 Final good producing firms

There are four different types of final-good firms which combine the purchases of the domestically-produced intermediate goods with purchases of the imported goods into four distinct final goods, namely a private consumption good, $Y_t^C(f)$, a private investment good, $Y_t^I(f)$, public consumption good, $Y_t^G(f)$ and export good, $Y_t^{EX}(f)$.[8]

Final consumption good producing firms

We start by describing the production of final private consumption good, Y_t^C. Each firm f produces its output using a CES production function with intermediate good, $Ykl_t^C(f)$,

[8] It is assumed that there is a continuum of firms in each of these four sectors $\mathbb{F}^S, S \in \{C, I, G, EX\}$. In the following text the particular firm index f is assumed to come from the $\mathbb{F}^S$ of appropriate sector.

and imports, $IM_t^C(f)$, as inputs:

$$Y_t^C(f) = \left[{\alpha_C}^{1/\sigma_C} \left(Ykl_t^C(f) \right)^{\frac{\sigma_C - 1}{\sigma_C}} + (1 - \alpha_C)^{1/\sigma_C} \left(\frac{IM_t^C(f)}{\xi_t^X} \right)^{\frac{\sigma_C - 1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C - 1}} . \quad (5.11)$$

Further, $1 - \alpha_C$ represents the import intensity parameter and σ_C determines the degree of substitutability of inputs. ξ_t^X is export specific technology process that makes the use of imported good less productive and more expensive relative to domestic intermediate production in steady state[9]. It drives a wedge between the real growth rate of exports, imports and value added. At the same time it drives a wedge between steady-state import and export deflators and the common nominal trend of the economy.

The firm solves the optimization problem of minimizing the total cost of its production $TC_t = P_t^{Ykl} Ykl_t^C(f) + P_t^M IM_t^C(f)$ subject to the technology constraint (5.11) and taking the prices as given.

Combined first order optimality condition can be expressed as follows:

$$\left(\frac{Ykl_t^C(f)\xi_t^X}{IM_t^C(f)} \right)^{1/\sigma_C} = \left(\frac{P_t^{IM}\xi_t^X}{P_t^{Ykl}} \right) \left(\frac{\alpha_C}{1 - \alpha_C} \right)^{1/\sigma_C} . \quad (5.12)$$

Nominal marginal cost of final-good production is then following:

$$NMC_t^C = \left[\alpha_C \left(P_t^{IM} \xi_t^X \right)^{(1-\sigma_C)} + (1 - \alpha_C) \left(P_t^{Ykl} \right)^{(1-\sigma_C)} \right]^{\frac{1}{1-\sigma_C}} . \quad (5.13)$$

Pricing behavior

Similarly to the intermediate good producing sector, a monopolistic competition is assumed in the final goods sectors as well. Thus, also the final consumption goods producing firms have certain market power and they can to a certain extent determine the price of their production in order to maximize profit. However, the firms cannot adjust their prices every period. A staggered price setting à la Calvo is assumed, which leads to the introduction of nominal rigidities into the model.

Calvo pricing mechanism implies that only a fraction of firms, $(1 - \omega^C)$ can reoptimize their prices in any given period. The rest of firms cannot update their prices to the new optimal price. The options of these firms are restricted to price indexation only. The probability of price change ω^C is indifferent to time that passed since the last change. Thus the length of the interval between two changes of prices will be a random quantity.

[9] It is true only when $g^{\xi X} > 1$, which is a natural assumption of our model.

The average length of the contracts therefore is $\frac{1}{1-\omega^C}$. Thus, the parameter ω^C determines the degree of price rigidity – the higher it is, the less firms can react to the price impulses and vice versa.

Firms that cannot reoptimize their price in given period index their price according to observed inflation and its steady state value. This can be written in the following formula as:

$$P_{t+1}^C(f) = {\pi_C}^{1-\gamma^C} {\pi_{C,t}}^{\gamma^C} P_t^C(f), \tag{5.14}$$

where γ^C is the rate of price indexation in the final consumption goods sector.

The firm f that can reoptimize its price in given period, chooses the new price in order to maximize expected profit. Hence, the optimization problem becomes:

$$\max_{P_t^C} \left\{ \sum_{k=0}^{\infty} \left(\omega^C\right)^k \Lambda_{t,t+k} Y_{t+k}^C(f) \left[P_{t+k}^C(f) - NMC_{t+k}^C\right] \right\}, \tag{5.15}$$

subject to demand:

$$Y_{t+k}^C(f) = \left[\frac{P_{t+k}^C(f)}{P_{t+k}^C}\right]^{-\theta_t^C} Y_{t+k}^C, \tag{5.16}$$

where θ_t^C is time-varying elasticity of substitution between final consumption goods varieties, that develops analogously to (4.10).

We obtain the following formula for the optimal price:

$$\widetilde{P}_t^C = \left(\frac{\theta_t^C}{\theta^C - 1}\right) \frac{\sum_{k=0}^{\infty} \left(\omega^C\right)^k \Lambda_{t,t+k} NMC_{t+k}^C \left[{\pi_C}^{1-\gamma^C} \left(\frac{P_{t+k-1}^C}{P_{t-1}^C}\right)^{\gamma^C}\right]^{-\theta_t^C} \left(P_{t+k}^C\right)^{\theta_t^C} Y_{t+k}^C}{\sum_{k=0}^{\infty} \left(\omega^C\right)^k \Lambda_{t,t+k} \left[\pi_C^{1-\gamma^C} \left(\frac{P_{t+k-1}^C}{P_{t-1}^C}\right)^{\gamma^C}\right]^{1-\theta_t^C} \left(P_{t+k}^C\right)^{\theta_t^C} Y_{t+k}^C}. \tag{5.17}$$

And the total price index evolves according to:

$$P_t^C = \left[(1-\omega^C) \widetilde{P_t^C}^{\frac{\theta_t^C}{\theta_t^C - 1}} + \omega^C \left[\pi_C^{1-\gamma^C} \pi_{C,t-1}^{\gamma^C} P_{t-1}^C\right]^{\frac{\theta_t^C}{\theta_t^C - 1}}\right]^{\frac{\theta_t^C - 1}{\theta_t^C}}. \tag{5.18}$$

If we assumed that there was no price rigidity and each firm could change the price freely (i.e. $\omega^C = 0$), then the equation (5.17) would simplify to:

$$\widetilde{P}_t^C = \left(\frac{\theta_t^C}{\theta_t^C - 1}\right) NMC_t^C. \tag{5.19}$$

Firms would then set their optimal price according to the desired margin over nominal marginal costs. This margin is in line with the monopolistic competition environment equal to $\frac{\theta_t^C}{\theta_t^C-1}$. Even with price rigidity, however, the following relationship holds in steady state:

$$\overline{P^C} = \left(\frac{\overline{\theta^C}}{\overline{\theta^C} - 1}\right) \overline{NMC^C}. \tag{5.20}$$

Final export good producing firms

A final export good firm uses same inputs as a consumption firm, but with one difference. Now the export specific technology process, ξ_t^X, makes the transformation of domestic intermediate production inputs, $Ykl_t^{EX}(f)$, to final export good more effective.

$$Y_t^{EX}(f) = \left[\alpha_{EX}{}^{1/\sigma_{EX}} \left(Ykl_t^{EX}(f)\xi_t^X\right)^{\frac{\sigma_{EX}-1}{\sigma_{EX}}} + (1-\alpha_{EX})^{1/\sigma_{EX}} \left(IM_t^{EX}(f)\right)^{\frac{\sigma_{EX}-1}{\sigma_{EX}}}\right]^{\frac{\sigma_{EX}}{\sigma_{EX}-1}}, \tag{5.21}$$

Nominal marginal cost of export final-good production now becomes:

$$NMC_t^{EX} = \left[\alpha_{EX}\left(P_t^M\right)^{(1-\sigma_{EX})} + (1-\alpha_{EX})\left(\frac{P_t^{Ykl}}{\xi_t^X}\right)^{(1-\sigma_{EX})}\right]^{\frac{1}{1-\sigma_{EX}}}. \tag{5.22}$$

Final investment good producing firms

Final investment good firms use the same inputs as other final producers in the model (intermediate good and imported good), but they have their own sector-specific investment specific technology process that makes the transformation of both inputs to final investment good more effective. This allows us to have different real growth rate of investment on BGP from common real trend of the economy and it also creates a wedge between steady-state growth of investment deflator and common nominal trend.

$$Y_t^I(f) = \xi_t^I \left[\alpha_I{}^{1/\sigma_I} \left(Ykl_t^I(f)\right)^{\frac{\sigma_I-1}{\sigma_I}} + (1-\alpha_I)^{1/\sigma_I} \left(\frac{IM_t^I(f)}{\xi_t^X}\right)^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}}, \tag{5.23}$$

Nominal marginal cost of investment final-good production now looks like:

$$NMC_t^I = \frac{1}{\xi_t^I}\left[\alpha_I\left(P_t^M\xi_t^X\right)^{(1-\sigma_I)} + (1-\alpha_I)\left(P_t^{Ykl}\right)^{(1-\sigma_I)}\right]^{\frac{1}{1-\sigma_I}}. \tag{5.24}$$

5.5 Demand for exports

The varieties of domestically produced goods that are exported are first bundled into a single country-specific[10] basket using a standard CES aggregator in a perfectly competitive manner:

$$EX_t(c) = \left(\int_0^{\mathbb{F}^{EX}} EX_t(f)^{\frac{\theta_t^{EX}-1}{\theta_t^{EX}}}\,\mathrm{d}f\right)^{\frac{\theta_t^{EX}}{\theta_t^{EX}-1}}. \tag{5.25}$$

The country index c stands for the domestic economy in this case. The national trading agency minimizes the cost of such a basket.

$$\min_{EX_t(f)} \int_0^{\mathbb{F}^{EX}} P_t^{EX}(f)EX_t(f)\,\mathrm{d}f,$$

while respecting the production technology constraint (5.25). Demand for an individual product then evolves according to the formula

$$EX_t(f) = \left(\frac{P_t^{EX}(f)}{P_t^{EX}(c)}\right)^{\theta_t^{EX}} EX_t(c), \tag{5.26}$$

and the price index is

$$P_t^{EX}(c) = \left[\int_0^{\mathrm{F}^{EX}} P_t^{EX}(f)^{1-\theta_t^{EX}}\,\mathrm{d}f\right]^{\frac{1}{1-\theta_t^{EX}}}. \tag{5.27}$$

Total demand for country basket evolves according to

$$EX_t(c) = \left(\frac{P_t^{EX}(c)}{P_t^{IM*}}\right)^{\theta^{IM*}} Y_t^*, \tag{5.28}$$

where P^{IM*} is the price index of competitors of our exporters and Y_t^* is total effective demand (unobserved) of our trading partners for imports. Therefore, the total demand

[10] In general, it is assumed that there is a continuum of countries $\mathbb{C}$. A particular country is denoted by c, $c \in \mathbb{C}$.

for the product of firm f is

$$EX_t(f) = \left(\frac{P_t^{EX}(f)}{P_t^{EX}(c)}\right)^{\theta_t^{EX}} \left(\frac{P_t^{EX}(c)}{P_t^{IM*}}\right)^{\theta_t^{IM*}} Y_t^*. \tag{5.29}$$

Here, we incorporate a mechanism that captures the gains in export market, which Slovak exporters have achieved in recent years. This can be particularly useful in a situation when Slovak exports grow despite a sluggish development of total foreign demand. The explanation of such a counterintuitive development lies in growth of market share of domestic exporters, perhaps due to improved quality. Similar argument can be found in Andrle et al. (2009) in such a situation. This would lead to the following modification:

$$EX_t(f) = \left(\frac{P_t^{EX}(f)}{P_t^{EX}(c)}\right)^{\theta_t^{EX}} \left(\frac{P_t^{EX}(c)}{P_t^{IM*}}\right)^{\theta_t^{IM*}} \xi_t^Q Y_t^{IM*}, \tag{5.30}$$

where ξ_t^Q is the unobserved process for market share gains and Y_t^{IM*} is the observed world demand. Since the quality technology process ξ_t^Q is growth stationary, it can explain the medium-term differences in the trend of Slovak exports and in the trend of foreign demand as a whole.

6. Policy authorities

6.1 Monetary policy

Given the historical development of the Slovak economy, we need to model two regimes of monetary policy.

- In one regime, the central bank sets the interest rate according to a simple Taylor rule, where the policy rate responds to deviations of consumer inflation, consumption[11] and possibly exchange rate growth from their respective targets (steady states):

$$\log\left(\frac{R_t}{\overline{R}}\right) = \rho^R \log\left(\frac{R_{t-1}}{\overline{R}}\right) + (1-\rho^R)\kappa^\pi \log\left(\frac{\pi_{C,t}}{\pi_C}\right) + \kappa^C \log\left(\frac{C_t}{\overline{C}}\right) + \kappa^S \log\left(\frac{\Delta S_{t+1}}{\overline{\Delta S}}\right) + \epsilon_t^R, \tag{6.1}$$

  where ρ^R represents interest rate smoothing parameter, κ^π is the weight of deviation of inflation from its target, κ^C weight of gap in consumption and κ^S weight of expected deviation of exchange rate depreciation. ϵ_t^R stands for monetary policy shock.

- In the other regime we need to model a small country in a monetary union - pegged exchange rate and exogenous interest rate. This is done in two steps:

  1. In the dynamic model we use a simple rule for the interest rate instead of the Taylor rule above,

$$R_t = R_t^*. \tag{6.2}$$

  2. Next we need to disconnect the exchange rate movements from changes in risk premium (UIP condition). To do this we need to exogenize the exchange rate, i.e. we need to get rid of one equation.

In steady state the rate of inflation is given by the central bank target rate of inflation. Furthermore, steady state interest rate is given by the government bond optimality condition. These conditions are

$$\pi_{C,t} = \pi^{target} = \pi_C, \tag{6.3}$$

$$\overline{R} = \frac{\pi_C g^C}{\beta \overline{\xi^{RP}}}. \tag{6.4}$$

[11]We use only disaggregated variables of GDP and GDP itself does not appear in the model.

These two equations pins down the steady state value of both nominal and real interest rates

$$\overline{R} = \frac{\pi^{target} g^C}{\beta \overline{\xi^{RP}}}, \tag{6.5}$$

$$\overline{R^R} = \frac{g^C}{\beta \overline{\xi^{RP}}}, \tag{6.6}$$

where g^C is the steady state growth of consumption.

6.1.1 Regime Change setup

A typical inflation targeting regime model contains (among others) the following three equations:

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta R_t, \tag{6.7}$$

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta \xi_t^{RP*} R_t^* \Delta S_{t+1} (1 - \phi_t^{B*}), \tag{6.8}$$

$$\log\left(\frac{R_t}{\overline{R}}\right) = \rho^R \log\left(\frac{R_{t-1}}{\overline{R}}\right) + (1-\rho^R)\kappa^\pi \log\left(\frac{\pi_{C,t}}{\overline{\pi_C}}\right) + \kappa^Y \log\left(\frac{Y_t}{\overline{Y}}\right) + \epsilon_t^R, \tag{6.9}$$

where the first equation is the first order optimality condition with respect to domestic bonds, while the second equation is the first order optimality condition with respect to foreign bonds. The third equation is a monetary policy rule. In the equations above, λ_t is the Lagrange multiplier associated with the households' budget constraint, R_t and R_t^* are the gross returns on domestic and foreign bonds, ΔS_t is depreciation of the nominal exchange rate, ξ_t^{RP*} is the risk premium paid on foreign risk-free bonds, ϕ_t^{B*} represents the debt elastic premium and its role is to stationarize the level of debt (see Schmitt-Grohé and Uribe (2003)), and ϵ_t^R is the monetary policy shock. The central bank aims to stabilize consumer inflation π_t and output Y_t at their equilibrium values. Finally, β is the subjective discount factor of private agents, ρ^R is the interest rate smoothing factor, κ^π and κ^Y are the weight placed on the two objectives of the central bank. The first two equations together imply an uncovered interest parity condition (UIP):

$$R_t = \xi_t^{RP*} R_t^* \Delta S_{t+1} (1 - \phi_t^{B*}). \tag{6.10}$$

In the regime of the monetary union we assume that the interest rate is set independently of domestic economic developments[12]. In other words we need to get rid of the Taylor rule (6.9) from the model. Moreover, we assume that all cross-border trades are

[12] Given the size of the Slovak economy in the euro area, this assumption is not at odds with the reality.

denominated in euros, and thus, the nominal exchange rate is fixed, which means that $\Delta S_t = 1, \forall t$, and this variable becomes redundant in the model.

However, from the practical point of view it is useful to keep the same number of variables (and equations) in both models. This becomes useful when one wants to run the Kalman filter or calculate the likelihood function over the two regimes - corresponding vectors and matrices in both regimes have the same dimensions. Therefore we keep the change in the nominal exchange rate in the model and the requirement for it being constant replaces the Taylor rule. By dropping the Taylor rule we also drop the monetary policy shock. To keep this shock in the model we place it in the new equation, i.e.

$$\Delta S_t = 1 + \epsilon_t^R. \tag{6.11}$$

Of course, this shock loses its interpretation of being the monetary policy shock. It also makes no sense to run the reaction functions to the shock in the monetary union set-up. When we run the Kalman filter with constant exchange rate, this shock effectivelly cease to exist in the model. The model with autonomuous monetary policy is used to recover the shock until the end of 2008. Since the begining of 2009 the model of monetary union is employed.

A complication then arises in the UIP condition, because the term $(1 - \phi_t^{B*})$ changes over time,[13] while the ratio of interest rates and the change in the exchange rate are constant. To treat this issue we follow the approach of Gomes et. al. (2010) in setting up the monetary union. Namely, they assume that the financial intermediation premium ϕ_t^{B*} applies to both foreign and domestic bond purchases. This term then cancels out in the UIP. The two first order optimality conditions for bonds essentially become identical in the monetary union model. The UIP condition thus becomes an identity that holds at every period and the risk premium shock has zero innovation on the entire sample.

All in all, the model of monetary union differs from the model of autonomuous monetary policy in three equations. The three equations of the latter model are given by equations (6.7)-(6.9) and their monetary union model counterparts are the following:

$$\lambda_t/\lambda_{t+1} = \beta R_t(1 - \phi_t^{B*}), \tag{6.12}$$

$$\lambda_t/\lambda_{t+1} = \beta \xi_t^{RP*} R_t^* \Delta S_{t+1}(1 - \phi_t^{B*}), \tag{6.13}$$

$$\Delta S_t = 1 + \epsilon_t^R. \tag{6.14}$$

[13]This is because the holdings of foreign bonds vary over time and so varies the premium paid on the return.

Steady state interest rate

In both models, the inflation targeting as well as monetary union model, we can define the steady state domestic interest rate in two ways:

1. **The level of domestic interest rate equals the level of foreign interest rate.** In this case the UIP condition becomes:

$$R_t = \xi_t^{RP*} R_t^* \Delta S_{t+1}. \tag{6.15}$$

   and the steady state value of ξ^{RP*} has to be set to 0. However, this shock can be left in the model and it keeps its interpretation of being the risk premium on foreign bonds.[14]

2. **The level of domestic interest rate does not equal the level of foreign interest rate.** This can be achieved through setting the steady-state risk premium ξ^{RP*} to a value different from 0. Then we can easily simulate, for example, effects of long-lasting relaxed monetary policy (domestic interest rate is then forced to be equal to the foreign interest rate through series of risk premium shocks).

Kalman filter/smoother

Since we defined the two versions of the model in the previous section and we would like to incorporate all the data available before the regime switch occured in order to identify the exogenous shocks throughout the history, we need to adjust the Kalman filter/smoother.

A general form of the linear state space models takes the form:

$$\begin{aligned} s_t &= \Phi s_{t-1} + C + Q\epsilon_t \\ y_t &= A + B s_t + H u_t \end{aligned} \tag{6.16}$$

where s_t is an $n \times 1$ vector of state variables, ϵ_t is an $ne \times 1$ vector of structural shocks with $E(ee') = \Sigma$, y_t is an $ny \times 1$ vector of observed variables and u_t is and $ny \times 1$ vector of measurement errors with $E(uu') = \Theta$.

[14] If both variables R and R^* are among observables ξ_t^{RP*} must be turned on. If $R = R^*$ at all times, one can omit the domestic interest rate from the set of observables. It is then necessary to turn ξ_t^{RP*} off.

The Kalman filter/smoother algorithm works also in a framework when the model changes over time. This encompasses the expected/unexpected monetary policy regime changes. For the time-dependent model it is necessary to add appropriate time indices to the model matrices Φ, C, Q, A, B, H. Thus, the valid model takes the form

$$\begin{aligned} s_t &= \Phi_t s_{t-1} + C_t + Q_t \epsilon_t \\ y_t &= A_t + B_t s_t + H_t u_t \end{aligned} \tag{6.17}$$

Announced vs. Unannounced regime switch

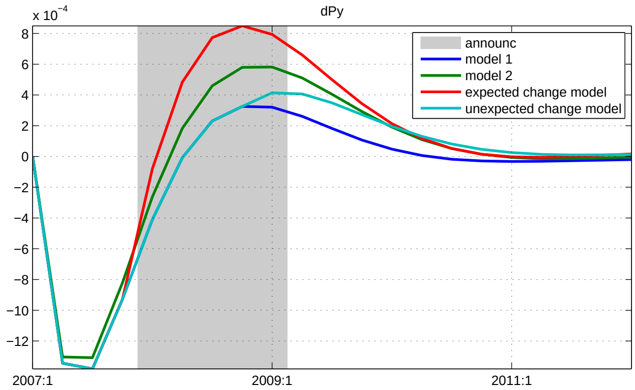
To deal with an announced change in the regime of monetary policy we employ the solution method proposed by Kulish and Pagan (2014), which is briefly introduced in the next paragraph (see Appendix A for details).

For the sake of simplicity let's assume that there is a single structural change implemented at time T^s which is announced at time T^a where $T^a < T^s$. Let's call the model that is in place up until period $T^s - 1$ $M1$. From T^s onwards the relevant model is called $M2$. At any period $t < T^a$ agents in the model believe that the model $M1$ will hold forever and form their expectations accordingly. When agents are informed about the intended change, in period T^a, they need to account for it in expectations formation process. Finally, from T^s on agents form expectations in line with the model $M2$.

Putting it backwards, from T^s the model $M2$ holds forever. Then in period $T^s - 1$, when model $M1$ is in place, agents form their expectations in period E_{T^s-1} about the next period T_s using the solution matrices of model $M2$. When we plug this into the linearized model $M1$ we find the solution of period $T^s - 1$. Repeating this process for periods $t = T^s - 2, ..., T^a$ we find the solution for all transition models. Finally, for the pre-announcement period we find the model using the previous section method and model $M1$ with its coefficient matrices. All this resolved models can be then plugged in the Kalman filter/smoother.

An illustration - Impulse response functions: We start off with a simple impulse-response function that illustrates how the change in the model structure changes reactions of the model economy to a stationary technology shock ξ_t^{TFP} (5.1). Specifically, we plot response of the inflation rate in the price of value added (price of intermediate sector). The Figure consists of four responses coming from four different simulations. The blue line is the response when model $M1$ holds on the whole simulation period. Similarly the green line is the response when model $M2$ holds on the whole simulation period. The red line is the response when model $M1$ is in 2009Q1 replaced by model $M2$ and agents are informed about this change 4 quarters before that. Finally, the turquoise line is the response when model $M1$ is in the beginning of 2009Q1 replaced by model $M2$ and agents learn about this change immediatelly after it takes place. The shock hits the economy in 2007Q2.

Figure 5: IRF stationary TFP shock (ξ_t^{TFP}) - Inflation in value added price



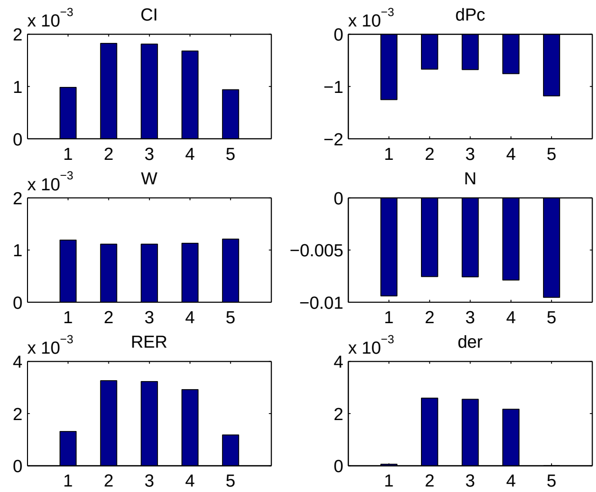
Note: Deviations from the steady state are depicted.

Figure 5 shows that the reaction of the inflation in the monetary union model (MU, $M2$ model, green line) is higher than the model with autonomous monetary policy (ATR, $M1$ model, blue line). When the change of regimes is unexpected (turquoise line), the reaction follows the reaction of the model 1 until the structural change occurs in 2009Q1 when it starts to deviate and gets closer to the reaction of model 2 from the lower side. When the change of regimes is expected (red line), the reaction starts to deviate from the model 1 in the period of the announcement of the regime change, in this case in 2008Q1, and gets closer to the reaction of the model 2 from the upper side.

More insights into the problem: An obvious disadvantage of this method is that while we have two structural models (for the initial regime and the final regime) we do not have a structural model for the transition period. All we have for the transition period is a series of reduced form models, which means we are unable to explain the working of the economy during the transition period using usuall model language.

To illustrate the problem we report an impulse-response function using all reduced form models - i.e. model for the pre-announcement period, model of the monetary union regime as well as all models of the transition period. More specifically, we report responses of the economy in the first period becasue each transition period model is used in one period only. Hence, in our particular application we report responses of a few selected variables in the period when a shock hits the economy. We do this using all five models in one figure.

Figure 6: IRF stationary TFP shock (ξ_t^{TFP}) - response on impact



Note: The figure reports responses of six variables (consumption, consumption price inflation, real wage, employment, real exchange rate and nominal exchange rate depreciation) in the period when the shock hits the economy. Each bar corresponds to a single model. Model 1 is the model with autonomuos monetary policy, model 5 is the model of monetary union, models 2-4 are models of autonomuos monetary when agents are informed about the coming regime change. Model 4 corresponds to the situation in which the change will take place in the next period. In model 3 the change will take place in two periods, while in model 2 the new regime will be adopted in three period.

It is apparent that the model economy behaves quite differently during the transition period. Even if the responses under model 1 and 5 are similar to each other, in most cases responses under transition period models 2-4 are different. In particular, the nominal exchange rate (der, and subsequently the real exchange rate, RER) behaves very differently. It is hard to trace the mechanism driving the model responses because we do not have the structural model for the transition period available. Intuition, however, can be that when agents are informed about the forthcoming regime change, they try to adjust their decisions accordingly. In a situation when agents do not have plenty of time for adjustment they respond more aggressively. This shows up especially in the strong reaction of the nominal exchange rate. Possible consequences - rather large effects in historical decomposition.

We also compared the identified exogenous shocks obtained from the Kalman smoother between the two solutions of the regime change (expected vs. unexpected). We found out that the filtered shocks are very similar with negligible differences. Thus, the drawbacks of the solution of expected regime change outweighs over its advantages. Disadvantages are realized as the unexplained shock decomposition during the announcement period because of the reduced form models used in this period. Thus, we decided to use the unexpected regime switch.

6.2 Fiscal policy

The government levies taxes on households' labour income, τ_N, private consumption, τ_C (i.e. VAT), and firms' labour costs, τ_w. Furthermore it earns seignorage on money and collects lump-sum taxes, which ensure a constant debt-to-output ration in steady state. Income is used to finance its consumption, G_t, and to make transfer payments to households, Tr_t. The government also issues bonds, B_t, in order to refinance its debt. The following budget constraint must therefore hold every period:

$$P_t^G G_t + B_{t-1} NGDP_{t-1} = B_t NGDP_t \frac{1}{R_t} + \tau_t^C P_t^C C_t + (\tau_t^N + \tau_t^W) W_t N_t + T_t NGDP_t. \quad (6.18)$$

The nominal government consumption share in GDP is given exogenously via AR(1) process ξ_t^{GY}:

$$P_t^G G_t = \xi_t^{GY} NGDP_t, \quad (6.19)$$

$$\xi_t^{GY} = \rho^{GY} \xi_{t-1}^{GY} + (1 - \rho^{GY}) \overline{\left(\frac{P^G G}{NGDP}\right)} + \epsilon_t^{GY}, \quad (6.20)$$

where ρ^{GY} is persistence and $\epsilon_t^{GY} \sim N(0, \sigma^{GY})$.

Lump-sum taxes are used to stabilize the debt-to-output ratio in line with the following fiscal rule

$$\frac{T_t}{NGDP_t} - \overline{\left(\frac{T}{NGDP}\right)} = \phi_B \left(\frac{B_t}{NGDP_t} - \overline{\left(\frac{B}{NGDP}\right)}\right), \quad (6.21)$$

where ϕ_B is the elasticity of lump-sum taxes with respect to deviation of debt-to-output ratio from target.

7. Foreign economy

Foreign economy is represented by exogenous processes and is identified separately, thus the domestic economy can not influence foreign economy, which we assume because the Slovak Republic is a small country that has no impact on foreign variables that in our model represents the rest of the Euro Area. We use four variables:

- foreign demand (Y_t^{IM*})
- foreign short term interest rate (3M-EURIBOR, R_t^*)
- competitors prices on the export side (CXD, π_t^{IM*})[15]
- competitors prices on the import side (CMD, π_t^{EX*}) [16]

We chose competitors prices on the export side as the main foreign price instead of HICP inflation in Euro Area. The reason is that CXD is the main driver of our export deflator and thus have the impact on our domestic export sector.

$$log\left(\frac{\pi_t^{IM*}}{\overline{\overline{\pi^{IM*}}}}\right) = \rho^{\pi^{IM*}} log\left(\frac{\pi_{t-1}^{IM*}}{\overline{\overline{\pi^{IM*}}}}\right) + (1-\rho^{\pi^{IM*}})\left[\kappa^{\pi^{IM*}R^*} log\left(\frac{R_t^*}{\overline{\overline{R^*}}}\right)\right] + \epsilon_t^{\pi IM*} \tag{7.1}$$

$$log\left(\frac{Y_t^{IM*}}{\overline{\overline{Y^{IM*}}}}\right) = \rho^{Y^*} log\left(\frac{Y_{t-1}^{IM*}}{\overline{\overline{Y^{IM*}}}}\right) + (1-\rho^{Y^*})\left[\kappa^{y^*R^*} log\left(\frac{R_t^*}{\overline{\overline{R^*}}}\right)\right] + \epsilon_t^{Y*} \tag{7.2}$$

$$log\left(\frac{R_t^*}{\overline{\overline{R^*}}}\right) = \rho^{R^*} log\left(\frac{R_{t-1}^*}{\overline{\overline{R^*}}}\right) + (1-\rho^{R^*})\left[\kappa^{R^*\pi^{IM*}} log\left(\frac{\pi_t^{IM*}}{\overline{\overline{\pi^{IM*}}}}\right) + \kappa^{R^*Y^*} log\left(\frac{Y_t^{IM*}}{\overline{\overline{Y^{IM*}}}}\right)\right] + \epsilon_t^{R*} \tag{7.3}$$

[15]CXD is composed as an weighted average of prices of countries that are competing with domestic export companies in countries where they export domestic goods. See Hubrich & Karlsson (2010).
[16]CMD is obtained as a simple weighted average of competitors' export prices. See Hubrich & Karlsson (2010).

8. Stationarization

There are five real (aggregate labour augmenting progress, willingness to work, trade openness shock, exports quality shock and investment specific shock) and one nominal (inflation) sources of non-stationarity in the model. Let us establish the notation for the steady-state growth rates of model variables that stand for quantities: $\overline{\left(\frac{X_t}{X_{t-1}}\right)} = \overline{\Delta X} \equiv g^X$, and for corresponding price indices: $\overline{\left(\frac{P_t^X}{P_{t-1}^X}\right)} = \overline{\Delta P^X} = \overline{\pi^X} \equiv \pi^X$.

The consumption deflator is the target price index for the central bank, which pins down the equilibrium inflation in consumption price index:

$$\pi_C = \pi^{\text{target}}. \tag{8.1}$$

Optimality condition for domestic government bonds implies that

$$\xi_t^{RP} \beta R_t = \left(\frac{C_t(j) - bC_{t-1}}{C_{t+1}(j) - bC_t}\right) \frac{P_{t+1}^C}{P_t^C}.$$

In equilibrium the nominal interest rate is thus defined as

$$R = \frac{\pi_C g^C}{\beta \overline{\xi^{RP}}}. \tag{8.2}$$

Having the growth rate of final consumption good price, the growth rate of nominal marginal cost of production in the consumption good sector can be calculated. The steady state equation for consumption good price (5.20) implies that

$$\overline{\Delta NMC^C} = \pi_C,$$

From the equation for nominal marginal cost of production in the final consumption good sector (5.13), also the components of NMC_t^C have the same steady state growth π_C:

$$\overline{\Delta P^{Ykl}} = \pi_C, \tag{8.3}$$

$$\overline{\left(\frac{P_t^{IM} \xi_t^X}{P_{t-1}^{IM} \xi_{t-1}^X}\right)} = \frac{\overline{\Delta P^{IM}}}{\overline{\Delta \xi^X}} = \pi_C \Rightarrow \overline{\Delta P^{IM}} = \pi^{IM} = \frac{\pi_C}{g^{\xi X}}. \tag{8.4}$$

Given that private consumption and government consumption production sectors employ the same production technologies, it holds that:

$$\overline{\Delta P^G} = \pi^G = \pi_C. \tag{8.5}$$

In the investment production sector, which includes the investment specific technology process in the production function (5.23), the steady state growth of final investment good price is:

$$\overline{\Delta NMC^I} = \overline{\Delta P^I} = \pi^I = \frac{\pi_C}{g^{\xi I}}. \tag{8.6}$$

The equilibrium price of the export good is equal to a constant mark-up over marginal cost of production expressed in foreign currency (because domestic exporters operate on foreign markets). The steady state inflation rate in this sector is thus proportional to steady state rate of change of marginal cost and exchange rate. From the nominal marginal cost equation (5.22) it follows that the rate of growth of marginal cost is not the same as in other sectors, but it is $\pi_C/g^{\xi X}$ (in domestic currency). The steady state growth of nominal exchange rate follows from its definition.[17] Then the price of export good, expressed in the foreign currency, grows at the rate:

$$\overline{\Delta P^{EX}} = \overline{\left(\frac{P_t^{EX}}{P_{t-1}^{EX}}\right)} = \overline{\left(\frac{NMC_t^X}{NMC_{t-1}^X} \Big/ \frac{S_t}{S_{t-1}}\right)} = \frac{\pi_C}{g^{\xi X}} \Big/ \overline{\Delta S} = \frac{\pi_C}{g^{\xi X}} \frac{g^{\xi X}\pi^{IM*}}{\pi_C} = \pi^{IM*}.$$

Domestic importers buy goods on foreign markets priced in foreign currency and sell it at home in domestic currency. Since both foreign price indices have the same growth rate in steady state ($\pi^{IM*} = \pi^{EX*}$), we obtain from (5.7) the following steady state growth of import prices expressed in the domestic currency :

$$\overline{\Delta P^{IM}} = \overline{\left(\frac{P_t^{IM}}{P_{t-1}^{IM}}\right)} = \overline{\left(\frac{P_t^{EX*}}{P_{t-1}^{EX*}}\frac{S_t}{S_{t-1}}\right)} = \pi^{EX*}\overline{\Delta S} = \pi^{EX*}\frac{\pi_C}{g^{\xi X}\pi^{IM*}} = \frac{\pi_C}{g^{\xi X}}$$

From the investment optimality condition it follows that $Tobin's\ Q$ has the same trend as the price of the investment good and using the optimal condition for capital utilization rate one finds that the price of capital also shares the trend with price of investment, i.e.

$$\overline{\Delta R^K} = \overline{\Delta Q} = \frac{\pi_C}{g^{\xi I}}. \tag{8.7}$$

Capital law of motion (4.5) implies that the growth of capital is the same as the growth of investment

$$\overline{\Delta K} = \overline{\Delta I} = \overline{\Delta Y^I}. \tag{8.8}$$

[17] $S_t = \frac{RER_t P_t^C}{P_t^{IM*}}$. We assume that exchange rate does not grow in the steady state (because since 2009 Slovak economy joined the EA), thus home and foreign prices have to have same growth rate which is secured through the export specific productivity process, then $\frac{S_t}{S_{t-1}} = \frac{\pi_C}{\pi^{IM*}g^{\xi X}} = 1$

In order to have a constant growth rate of Y_t^I $\left(\overline{\Delta Y^I} = g^{YI}\right)$, both factors $Ykl_t^I(f)$ and $\frac{IM_t^I(f)}{\xi_t^X}$ need to have the same trend growth. From market clearing conditions, the trend of $Ykl_t^I(f)$ must be the same as g^{Ykl}, and from the production function of final investment good (5.23), we obtain $g^{YI} = g^{\xi I} g^{Ykl}$. Using intermediate production function (5.1), where A_t is labour augmenting technology that grows at rate g^A, N_t grows according to the willingness to work technology[18] at rate $g^{\xi N}$, and capital grows at the same rate as investment, this implies that:

$$g^{Ykl} = \left(g^A g^{\xi N}\right)^{\alpha} \left(g^{\xi I} g^{Ykl}\right)^{1-\alpha}, \tag{8.9}$$

which simplifies to

$$g^{Ykl} = g^A g^{\xi N} g^{\xi I\left(\frac{1-\alpha}{\alpha}\right)}. \tag{8.10}$$

Since $g^I = g^{YI}$ due to market clearing conditions, the steady-state growth of final investment good is then:

$$g^I = g^{YI} = g^{\xi I} g^{Ykl}. \tag{8.11}$$

From the production function of final investment good (5.23), we know that the growth rate of second input has to be the same as g^{Ykl} so we see that:

$$\frac{g^{IM}}{g^{\xi X}} = g^{Ykl}, \tag{8.12}$$

$$g^{IM} = g^{Ykl} g^{\xi X}. \tag{8.13}$$

From the production function of final consumption good (5.11) and from market clearing conditions:

$$g^C = g^{YC} = g^{Ykl}. \tag{8.14}$$

We assume that the government production sector employs the same technology and consequently has the same growth rate as the consumption sector,

$$\overline{\left(\frac{G_t}{G_{t-1}}\right)} = g^G = g^C = g^{Ykl}. \tag{8.15}$$

From the production function of the final export good (5.21), we find that the final

[18]In our case $g^{\xi N} = 1$ which means that there is no stochastic trend growth in hours worked in steady state. It only captures the non-stationarity of the data.

export good production has the same trend as import good:

$$\overline{\left(\frac{EX_t}{EX_{t-1}}\right)} = g^{EX} = g^{Ykl} g^{\xi X} = g^{IM}. \tag{8.16}$$

Nominal GDP is defined as follows:

$$NGDP_t = P_t^C C_t + P_t^I I_t + P_t^G G_t + P_t^{EX} EX_t - P_t^{IM} IM_t. \tag{8.17}$$

Time-differencing this equation in steady-state yields:

$$\pi^Y g^Y \overline{Y} = \pi_C g^{Ykl} \left(\overline{C}_t + \overline{I}_t + \overline{G}_t\right) + \frac{\pi_C}{g^{\xi X}} g^{Ykl} g^{\xi X} \left(\overline{EX}_t - \overline{IM}_t\right),$$

and this yields

$$\pi^Y g^Y \overline{Y}_t = \pi_C g^{Ykl} \left(\overline{C}_t + \overline{I}_t + \overline{G}_t + \overline{Ex}_t - \overline{Im}_t\right),$$

which implies that

$$\pi^Y g^Y = \pi_C g^{Ykl} = \pi_C g^C.$$

Similar to Andrle et al. (2009), we do not have real GDP defined in the model as a single variable. This is due to the fact, that we decided to model individual sectors of GDP with their specific price levels. Therefore, it is not possible to retrieve the growth rates of real GDP directly, but we can calculate the GDP outside of the model using the individual components.

Nevertheless, the main source of value added is in the domestic intermediate sector produced by labour and capital services. In the final good sectors, the value added comes from monopolistic profits only, because these sectors operate mainly as aggregators and distributors.

9. DATA

The available data on Slovak economy has its limitations. Data before the year 2000 is excluded from our analysis because it covers a period with substantial structural changes that we do not treat in the model explicitly. Therefore we use time-series at quarterly frequency from $2000q1 - 2016q4$. During this period the Slovak economy underwent further fundamental structural changes, some were instantaneous, such as the adoption of the common european currency and joining the monetary union, and other continuous, such as the post-transformation processes common in the emerging markets. Thus, not all the data can be easily used for model calibration.

The model was calibrated using several complementary approaches. Since empirical estimates for Slovak economy are rare in the literature our initial calibration was based on standard values common in DSGE models. Where possible, parameters were set according to their emprical counterparts. In the next step we tailored the calibration to the specifics of the Slovak economy.

We used several tools to evaluate the behaviour of model variables and to achieve results consistent with our prior beliefs, stylized facts and observed data dynamics. We analyzed the model from many points of view, we used: *(a) impulse-response analysis*, *(b) shock decomposition of historical data*, *(c) comparison of model's moments with data* and *(d) recursive filtering and forecasts*.

We focused on the storytelling potential of the model. That is why we tried different parameter setting to identify the structural shocks of the model.

Great care was devoted to analytical identification of the stationarized steady state, which allows us to set the long-run growth of the main variables and *great ratios*.

9.1 TRANSFORMATION OF DATA

First, we would like to concentrate on data transformation. First, we take logarithms of all variables except interest rates, both home and foreign and trade balance that is calculated as a ratio of net exports to nominal GDP.

Then we use Hodrick-Prescott filter on these levels of data with lambda set to 1 in order to get rid of the noise[19]. The reason is that we do not want the model to replicate every bit of noise in the data. Another reason is that even after seasonal adjustment the data

[19]The HP filter with lambda set to 1 is approximately equal to the Band Pass filter which retains the frequencies between [5,Inf].

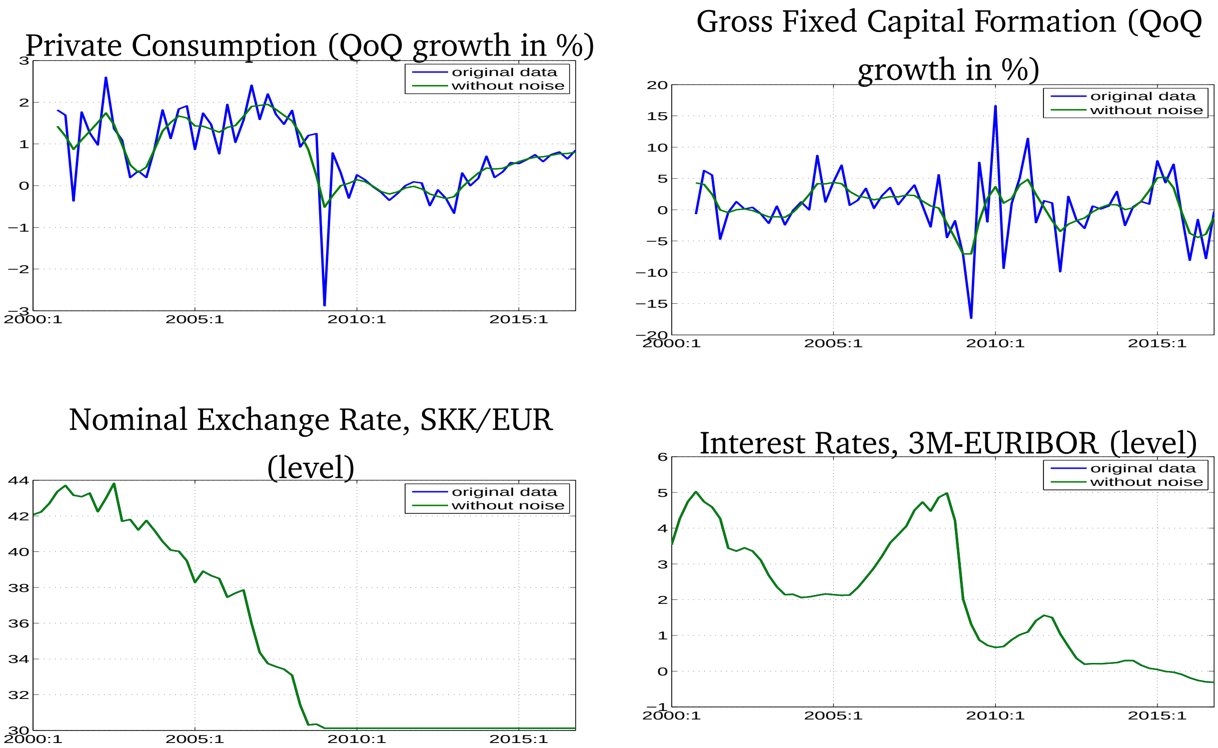
are still very volatile containing high share of noise, which does not play an important role in forecasting trends.

Last step is that we transform all data into quarter-on-quarter growth rates, approximated by the first difference of their logarithm again except interest rates, home and foreign, and trade balance.

Figure 7 demonstrates the idea of using less noisy data on example of several variables. We do not apply this filtering to exchange rate or interest rates which we consider perfectly measured.

Our goal is the forecast of the business cycle tendencies, which means that we focus on medium term. Benefits of this approach are demonstrated in the table 4, where we compare prediction errors of the same model using two different data sets. First the original data series and second with the removed noise with HP filter.

Figure 7: Original and HP-filtered data



Source: Data - Eurostat, Authors' calculations.

In the PreMISE model, following 18 seasonally adjusted time-series obtained from Eurostat[20] and ECB[21] enter the model as observed variables:

[20]We get C, I, G, EX, TB, Pc, Pi, Pg, Px, Pm, N, W, R, S, R^* from Eurostat.

[21]We get Y^{IM*}, P^{EX*} and P^{IM*} from ECB

- private consumption (C)
- gross fixed capital formation (I)
- government consumption (G)
- exports (EX)
- trade balance (TB)
- private consumption deflator (Pc)
- gross fixed capital formation deflator (Pi)
- government consumption deflator (Pg)
- export deflator (Px)
- import deflator (Pm)
- total hours worked (N)
- compensation per hour (W)
- nominal interest rate (R)
- nominal exchange rate (S)
- foreign demand (Y^{IM*})
- foreign interest rate (R^{*})
- competitors prices on the import side (P^{EX*})
- competitors prices on the export side (P^{IM*})

Domestic model variables are linked to their observed counterparts in the data. Foreign demand Y^{IM*} is linked to the indicator of world demand for Slovak exports (WDR), π^{IM*} is linked to a weighted average of prices of competitors to Slovak export producers (CXD), and π^{EX*} is linked to a weighted average of prices of foreign exporters to Slovakia (CMD).

9.1.1 Trade openness

Further transformation of the data is applied to the trade variables, specifically, exports, imports and world demand. The Balanced growth path would indicate that the nominal expenditure share of exports and imports in value added are constant, but the Figure 2 shows that it is not true.

For treating the model trend behaviour consistently, we introduce an openness technology process ξ_t^O to take into account the growing trend in the nominal expenditure share of the trade in value added that we observe in the data. In the model we introduce this technology shock to identify the common component of growth of export, import and foreign demand.

It is defined in measurement equations of these variables which means, that agents perceive trade variables already deflated by the openness shock. This ensures the consistency of BGP definition.

Thus, the observed time series have the following trend:

$$EX_t^{obs} = IM_t^{obs} = Y_t^{*obs} = g^{Ykl} g^{\xi X} g^{\xi O}. \quad (9.1)$$

9.1.2 Prediction errors

In the next table we present the results of the exercise where we compute prediction errors of the model for the first 4 quarters for two different data sets. One with the original data and one with filtered data. The table shows the ratio between the two results. If the ratio is higher than one it means that the first model with original data has higher prediction errors than the second model.

Table 4: Prediction Errors: Original/Filtered

	Quarter of forecast			
Variable Name	1st	2nd	3rd	4th
Consumption Growth	1.18**	1.11***	1.05***	1.05**
Investment Growth	1.33***	1.06	1.01	1.02*
Government Expenditure Growth	1.03	1.06**	1.04	1.03
Export Growth	1.00	1.03	1.01	1.02***
Import Growth	1.06**	1.05	0.99	1.01**
Consumption price Growth	1.17	1.21**	1.13***	1.06
Investment price Growth	1.02	1.08	1.03	1.03
Government price Growth	1.03	1.11**	1.07**	1.03**
Export price Growth	1.04	1.02	1.02	1.01
Import price Growth	1.05	1.04*	1.01	0.98**
Nominal Wage Growth	1.11	1.09*	1.06	1.05
Employment Growth	1.18**	1.14***	1.06***	1.08*

Source: Authors' calculations, Diebold and Mariano (1995).

*Note: at significance level: *10%, **5%, ***1%*

From the table above, it is clear that the prediction of the model with filtered data is almost always better than model with original data for all four quarters. The Diebold Mariano test tells us that at the significance level $\alpha = 5\%$, the null hypothesis about no difference between the two forecasts is rejected when the value of DM test is outside the interval $(-1.96, 1.96)$. The test shows that in many cases the two forecasts are significantly different.

9.2 Moments of the model

Another way to assess the quality of the model's fit is to compare standard deviations of key macroeconomic variables in model and in the data, as in Cuche-Curti et al. (2009). In Table 5, we evaluate the results for the two periods: period of independent monetary policy with active Taylor Rule (ATR) from $2000q1$ until $2008q4$; and the period, where the Slovak economy is already a part of the monetary union (MU), i.e. from $2009q1$ onwards. The numbers in the table represent the standard deviations of the quarter on quarter growths of the observed data with their model counterparts. We can see that both models replicate the moments in observed data reasonably well. Nevertheless, there is a room for improvement in labour market variables, investment or export prices.

Table 5: Standard Deviations: Model vs. Data

	Data			Model	
Name	Total	ATR	MU	ATR	MU
Consumption Growth	0.71	0.44	0.39	0.73	0.71
Investment Growth	2.87	2.16	3.35	2.26	2.01
Export Growth	2.58	2.83	2.16	2.53	2.43
Consumption price Growth	0.64	0.51	0.45	0.73	0.60
Investment price Growth	0.63	0.53	0.44	0.92	0.72
Export price Growth	0.91	0.93	0.81	0.87	0.67
Import price Growth	0.83	0.68	0.92	1.52	0.72
Nominal Wages Growth	1.14	1.21	0.79	0.46	0.43
Hours Worked Growth	0.58	0.66	0.45	1.73	1.48

Source: Data - Eurostat, Authors' calculations.

Note: Standard deviations in percentage points.

10. Mechanical Update

In this section we present an application of the model which is used regularly in every forecasting round.

The end of the forecasting horizon is set to the fourth quarter of year $T+2$, where T is the current year. The whole forecasting horizon can be divided into two periods:

- Near Term Forecast (NTF) horizon from the quarter $t+1$ to quarter $t+3$,
- Medium Term Forecast (MTF) horizon from quarter $t+4$ until the end of forecasting horizon,

where t is the current quarter. The DSGE model is designed to be used at the medium-term forecast horizon (MTF).

At the beginning of the forecasting process we have the final forecast from the previous round at our disposal. We also know what the story behind the last forecast according to the shock decomposition is. The macroeconomic outlook and the story behind the forecast can, and usually will, change between the subsequent forecasting rounds for several reasons. The final forecast is conditioned on the external environment variables on the whole forecasting horizon. Therefore, one source of changed forecast of the domestic variables are the changes in the external assumptions. Another source of changes come from the revisions[22] of the domestic variables made by the Statistical Office. According to the rules of the forecasting process we use seasonally adjusted data as published by the Statistical Office. Thus, the change of historical data between the subsequent forecasting rounds can also change the underlying story. In general there will also be some forecast error produced by the model itself. After publication of new data about the domestic economy, the beginning of the forecasting horizon is shifted forward and the forecast error in domestic variables is realized.

This being said, we can formulate the following question that we would like to address by the mechanical update procedure: **What is the implication of the new data according to the model mechanisms without introducing any new judgements?**[23] Or less formally we can ask: How should we update/revise the forecast according to the model based on the new data, without changing the underlying story? This means that we accept the changes in the assumptions of the forecast arising from the publishing and revising the data, while at the same time we keep some assumptions unchanged.

[22]New releases of the domestic variables.

[23]Judgement is an additional assumption made by the forecasting team in order to incorporate information that is not explicitly modelled.

In terms of the DSGE model, these would namely be the assumptions about the development of the underlying shock processes on the forecasting horizon.

Mechanical Update - Data

In detail, the sources of new data are listed below:

- ESA 2010 data[24] - domestic variables, historical data revisions and publication of the new data.
- Near Term Forecast - domestic variables for three quarters produced outside of the model.
- Fiscal report - exogenous (outside of the model) information about government consumption in nominal terms, the time-series cover the history as well as the whole forecasting horizon.
- External assumptions - foreign demand, foreign prices, foreign interest rates, oil prices, food prices, the time-series cover the history as well as the whole forecasting horizon.

In order to differentiate the effects that stem from the individual sources of the data we nee to handle the new data accordingly. The revisions of the historical data and their impact on the current position of the economy is an apposite example.Thus we input the new information in stages that are listed below.

Mechanical Update - Main Algorithm

We update the previous forecast by changing the exogenous assumptions in *stages*, in order to see the effects of changes in particular assumptions:

- stage 1 - new exogenous variables on the history only, [25]
- stage 2 - new exogenous variables on NTF & MTF,
- stage 3 - new government consumption on history only,
- stage 4 - new government consumption on NTF & MTF,
- stage 5 - new endogenous domestic variables on history only,
- stage 6 - new endogenous domestic variables on NTF (NTF forecast itself).

[24] The European System of National and Regional Accounts
[25] That means up to the period before the start of NTF.

It is important to emphasize that the results are conditional on the order of stages.

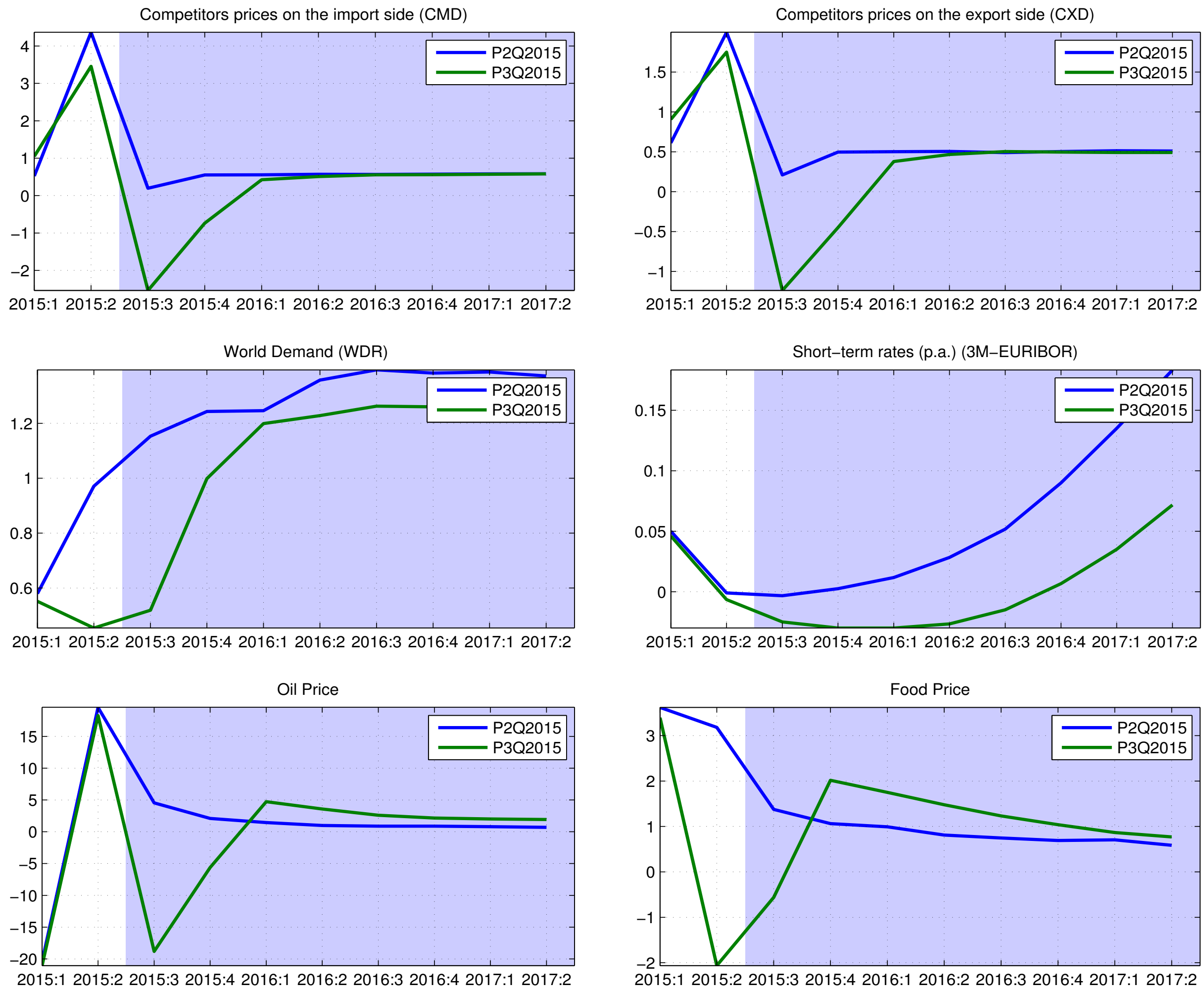
Mechanical Update - Scenario P3Q2015

Now we will describe an example of how this whole procedure works. **We present the results as a difference of the given variable's quarter on quarter growth with respect to the previous forecasting round, which was P2Q2015**.

We chose the forecasting round called P3Q2015 that refers to the third quarter of 2015. At that time, we had the data available until the second quarter of 2015. The near term forecast spanned the third and fourth quarters of 2015 and the first quarter of 2016. We run the exercise with the *new* data that was available at that time (in the third quarter of 2015) and we look into what the model tells us about the effect of the new external assumptions as well as the new ESA Data (revisions of home variables until 2015Q2) on the domestic variables. To keep things simple, we do not take into account the effects of the new Near Term Forecast. That means that we evaluate only cases $1 - 5$. The forecasting horizon in this exercise was until 2017Q4, but we compare the observed data from ESA Data release 2017Q2 with revisions.

The outlook of the external variables was revised down, Figure 8. The competitors' prices on the import side (CMD) were expected to drop by 2 percent in the third quarter of 2015. Similar pattern was expected to happen to competitors prices on the export side (CXD), only with a smaller dip. The world demand (WDR) was lowered too. The expected interest rates (3M-EURIBOR) at that time were lower compared to the previous forecasting round. The oil price growth was expected to drop masivelly by almost 25 percentage points from what was expected in the previous forecasting round. The food prices dropped in 2015Q2 and were expected to be higher after 2016Q1 until the end of the forecasting horizon.

Figure 8: External assumptions (QoQ growth in %)



Source: Data - Eurostat, Authors' calculations.

Note: Except for the short-term interest rates, all the variables are depicted as quarter on quarter growths in percent.

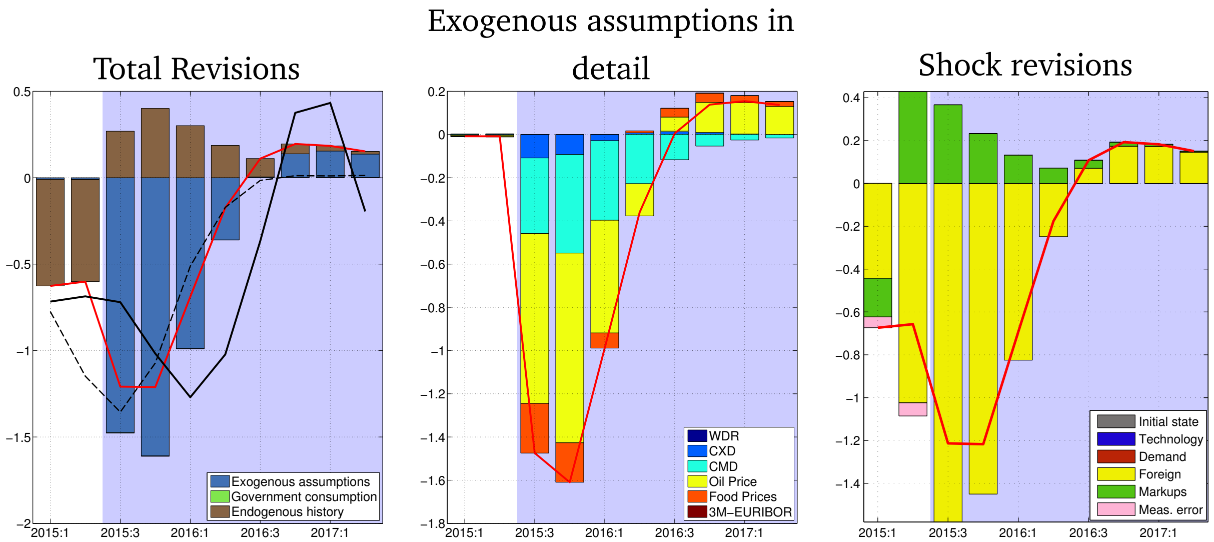
We present the results of this excercise for the import and private consumption deflators and for real private consumption in Figures 9, 10 and 11 respectively. The figures are structured as follows. Panel 'Total Revisions' compares the model revisions (red line), the official forecast P3Q2015 (black dashed line) and the ESA Data 2017Q2 (black solid line) all depicted as differences with respect to P2Q2015. At the same time the model revisions are decomposed into the contributions of three main groups of observed variables - exogenous assumptions, government consumption and endogenous history revisions. Panel 'Exogenous assumptions in detail' shows a detailed decomposition of the 'Exogenous assumptions' group from the previous subgifure into the contributions of particular exogenous variables and panel 'Shock revisions' shows the total revisions decomposed into the contributions of revised exogenous shocks as identified by the model. We categorize shocks into four groups: 'Technology' group contains shocks that are linked with the labour productivity, long-term and temporary; 'Demand' group consists of shocks, which represents consumer preferences in general as well as in particular individual sectors; 'Foreign' group represents external assumptions; 'Markup' group contains shocks that drive the wedge between firms' unit costs and prices in all production sectors.

In 'Total revisions' panel of Figure 9, all exogenous assumptions result in deflationary pressures for the first year of the forecasting horizon and then positively influence the import deflator.

The middle panel is more specific and shows the exogenous assumptions in detail. The expected lower competitors prices on both sides (export and import) brought deflationary pressures on imported prices which then created deflationary pressures on domestic prices. Additional deflationary effects at that time came from the oil prices that dropped rapidly and a smaller effect came also from the food prices. The model forecast (red line) is not very far away from the official forecast at that time (black dashed line). From today's point of view, the import deflator dropped by the same magnitude, but two quartares later than the model suggested.

Unlike previous two panels, which show the effect of observed variables revision, the 'Shock revisions' panel reflects how the new information from all sources changed the identification of unobserved shocks on the history and their further development on the forecast. The first thing to realize is that the import deflator is influenced only by the foreign environment (yellow bars) and by the markup (green bars) of domestic import firms in the model. Thus, with the new data stemming from the exogenous assumptions and the history of import deflator, the current position of markups in the last quarter of the history (2015Q2) changed. The time varying profit margin of import firms rose in this quarter because the importers did not lower their prices according to the foreign prices fully. The import firms might have been unwilling to decrease their prices in the prospect of expected future price growth. On the forecast horizon, the model introduces small declining trajectory of this effect subsequently.

Figure 9: Revisions: Import Deflator (QoQ growth difference in p.p.)



Source: Data - Eurostat, Authors' calculations.

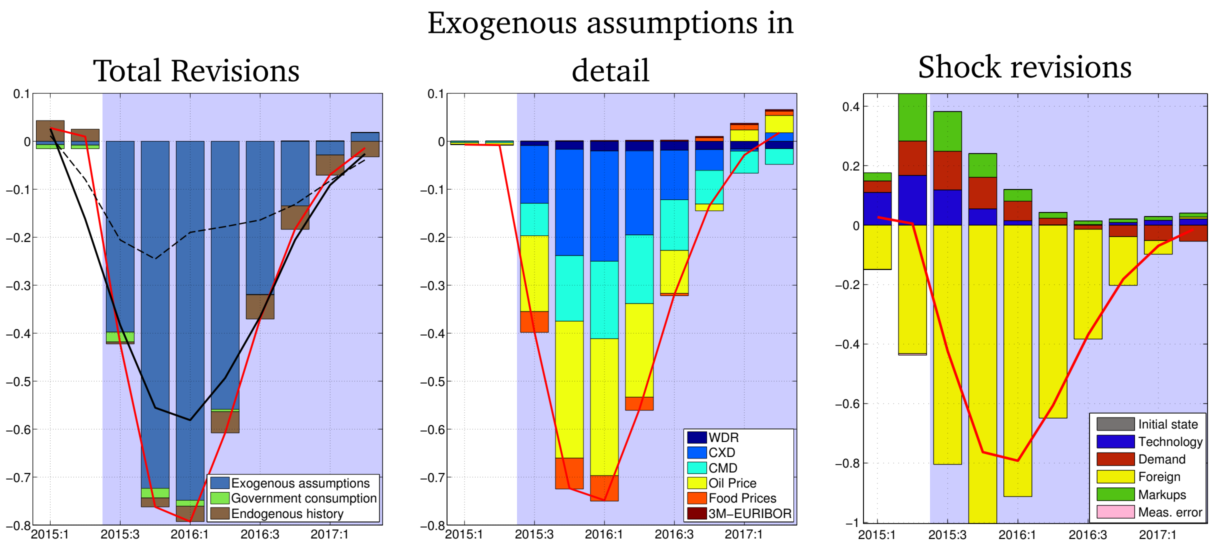
Note: RED line - Model Revisions $P3Q2015$*; DASHED BLACK line - Official Forecast* $P3Q2015$*; SOLID BLACK line - ESA Data* $2017Q2$*.*

Looking at the result of the model revisions for the consumption deflator, depicted in Figure 10, oil prices, food prices and competitors' prices take effect in the same direction as one would expect. The difference is, however, in the magnitude. The official forecast at that time (black dashed line) suggested lower consumer prices inflation than it was in the previous forecasting round, nevertheless, the model implied even greater deflationary pressures (red line). According to today's up-to-date ESA data (black solid line), the model revisions turned out to be slightly more accurate in updating the previous forecast than the official forecast.

In the third panel ('Shock revisions') we can see that, in contrast to import deflator, domestic inflation is influenced by all categories of shocks. The negative effect of the 'foreign' group of shocks (yellow bars) reflects the change in the exogenous assumptions that was described above. This effect is partially offset by the revisions of the remaining shock categories. The revision of the markup shocks (green bars) across the sectors pushed the consumption deflator up in the last quarter of the history. This reflects the increased overall nominal rigidity of prices, where importers and also firms in other sectors did not strictly follow the decline in foreign prices. Similarly to import deflator the model implies a diminishing contribution of these shocks on the forecast horizon. As the new ESA data showed, the potential growth of the economy slowed down at the end of the history. This has adverse effect on the labour productivity and implies pro-inflationary pressures. In the model, the labour productivity is represented by the technology shocks (blue bars), which fell down in the last two quarters of history and had a positive effect on the forecast horizon, where we assume their consecutive return towards the steady state. The domestic demand shocks (red bars) reflect the positive revision of the domestic demand in the new ESA data, mainly the investment that rose almost 3 p.p. more than in the previous forecast round, which causes pro-inflationary pressures.

In general, the contribution of shock groups on the forecast horizon depends on which particular shocks were revised. This can lead to more or less rapid reversal of the original direction of the shock that occured at the end of history in line with the model impulse responses.

Figure 10: Revisions: Private Consumption Deflator (QoQ growth difference in p.p.)



Source: Data - Eurostat, Authors' calculations.

Note: RED line - Model Revisions $P3Q2015$*; DASHED BLACK line - Official Forecast* $P3Q2015$*; SOLID BLACK line - ESA Data* $2017Q2$*.*

Regarding the private consumption, the model forecasted lower consumption growth compared to the previous forecast round and to the official *P3Q2015* forecast (see Figure 11). The main drivers were the lower competitors' prices on the export side (CXD) and the new ESA data $2015Q2$. Lower CXD prices pressure our exporters to lower their prices in order to keep the same level of competitiveness. Due to the nominal price rigidities, our exporters lower their prices only gradually, which leads to a lower demand for our export production, lower hours worked and eventually to lower consumption of households.

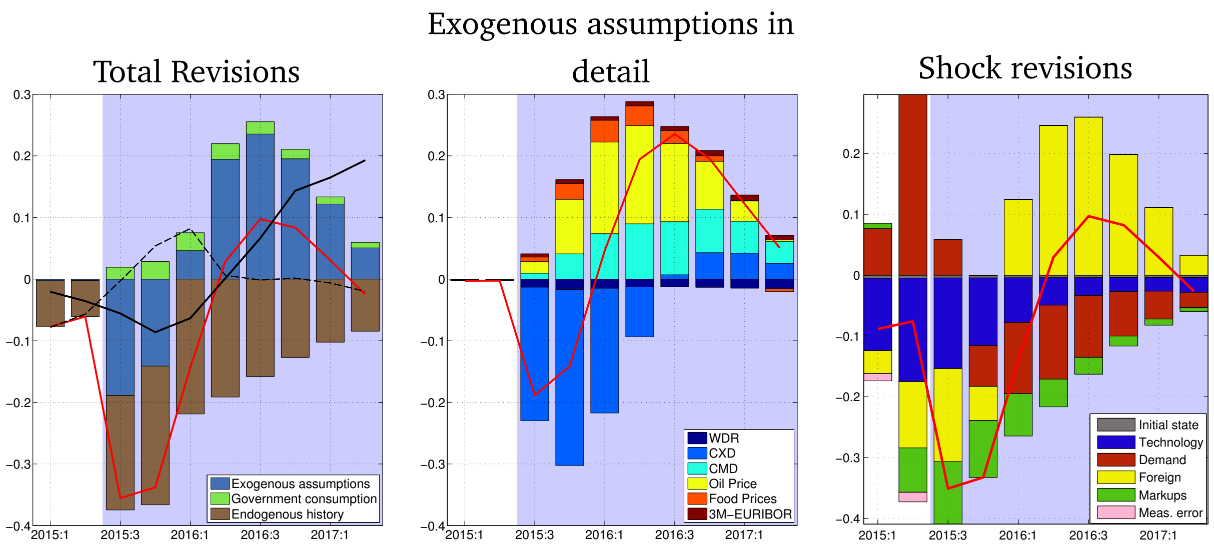
The 'Shock revisions' panel clearly shows how the new data changed the current position of the economy in terms of unobserved shocks and their future evolution on the forecast horizon. The 'Foreign' group of shocks (yellow bars) reflect the evolution of the exogenous assumptions in previous panel.

As we described earlier, the new ESA data result in lower potential growth of the economy, which has negative impact on the labour productivity. The lower labour productivity in the model as identified by the technology shocks (blue bars) in the last two quarters of the history has a negative effect on the forecast horizon, where we assume their consecutive return closer to the steady state.

Above described revision of the domestic demand in the new ESA data is reflected in the development of the domestic 'Demand' shocks (red bars). On the forecast horizon, this effect reverses and affects the consumption negatively, because the demand shocks only shift the demand in time in general. As these preference shocks do not increase the disposable income of households, the immediate positive impact is smoothed out in the medium term by a negative contribution of this shock group.

'Markups' (green bars) were revised up, increasing the profit margins in consumption sector in the last quarter of the history, which has a negative effect on real consumption growth. Then again, this effect fades away on the forecasting horizon.

Figure 11: Revisions: Real Private Consumption (QoQ growth difference in p.p.)



Source: Data - Eurostat, Authors' calculations.

Note: RED line - Model Revisions $P3Q2015$; DASHED BLACK line - Official Forecast $P3Q2015$; SOLID BLACK line - ESA Data $2017Q2$.

11. Ex-post evaluation of the official forecast P3Q2015

Another possible application of the model is a ex-post comparison of two different official forecasts, or an official forecast with the realized data in terms of unobserved shocks. In Figure 12, we present the difference between the shock decomposition of the data observed until the second quarter of 2017, and the official forecast P3Q2015 that was used in the previous exercise. We show the difference in development of private consumption deflator which is one of the key macroeconomic variables. In the figure, we indicate the forecast horizon that corresponds to the P3Q2015.

The growth of consumption deflator was slower than was expected in P3Q2015, mainly during 2016. The main deflationary effects stem from the foreign environment. However, these deflationary pressures were not fully reflected and the domestic prices became more disconnected from the imported inflation because the domestic prices turned out to be more rigid than expected. Another factor acting against the deflationary pressures from the foreign environment is the more sluggish potential growth than was expected in P3Q2015.

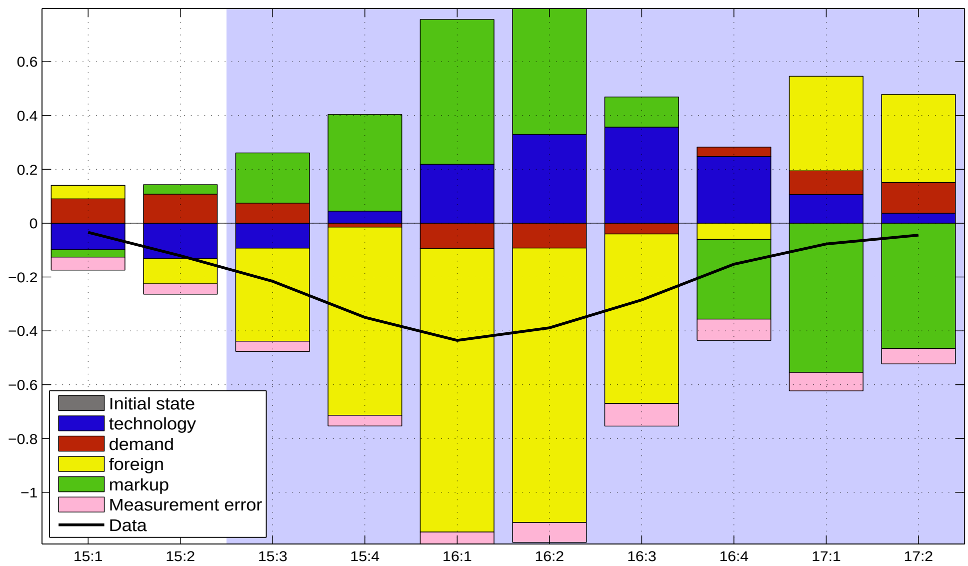
In the rest of this section we describe these effects in detail. The main deflationary pressures stemming from the foreign environment (yellow bars) are caused by the change of the exogenous assumptions about competitors prices on the export side (CXD) that were expected to grow in 2016 in P3Q2015, but instead fell rapidly. In 2017, the situation changed and foreign prices grew faster in reality. Behind the inflationary effect in 2017, there is also higher than expected growth of competitors prices on the import side (CMD) that were expected to grow, but in the data the expectations were revised up, which pushed up the outlook of domestic prices.

The inflationary effects in the first year of the forecast horizon reflect the fact that the official forecast P3Q2015 expected markups (green bars) in import and consumption sector to slowly return to its steady state, but we observed the opposite. Rather quick rise of profit margin in import sector reflects that despite the declining costs (represented by foreign prices) the import prices did not decrease accordingly. Similarly, in the consumption sector the slight decline in import prices, which are an input in this sector, was also partly compensated by higher profit margin. In other words, it means that in 2016 the lower input costs were not fully reflected in prices (in both sectors), which led to higher profit margins. Later on in 2017, the consumption deflator was influenced negatively by markup in the consumption sector, which was lower than expected in the P3Q2015. Effect of the markups in the first year and half can represent the fact that the downward price rigidity was higher than the official forecast expected at that time.

Blue bars represent the technology shocks that are linked with labour productivity, long term or temporary. In 2016 we observed in the data that labour productivity fell below the expected growth, which caused inflationary pressures and this effect started to fade away in 2017. This means that the official forecast P3Q2015 expected the higher potential growth, which did not materialize and this lower potential growth had inflationary effect on the private consumption deflator.

Small effects arising from the demand side can be seen as well. They mainly reflect the revisions of the demand in the investment sector. The observed growth of investment fell down in 2016 - after the end of the second programming period of the EU funds, while in 2017, the investment growth started to return closer to the expectations of P3Q2015.

Figure 12: Shock Decomposition diff P3Q2017 vs P3Q2015: Private Consumption deflator (QoQ growth difference in p.p.)



Source: Data - Eurostat, Authors' calculations.

Conclusion

In this paper, we have presented a detailed overview of the structure of the new medium-term forecasting NK-DSGE model of the National Bank of Slovakia - the PreMISE model.

The model has several desirable properties. Majority of the model's relations are derived from theoretical microeconomic foundations as solutions to optimization problems of representative economic agents. The model assumes forward-looking agents with rational expectations. Having been constructed as a system, the model structure is internally consistent. The model can be used for story-telling purposes because the development of the endogenous model variables can be decomposed into the individual contributions of well-defined structural shocks with straightforward economic interpretation. The model can be used to extract information about unobserved endogenous model variables of interest.

The model was calibrated to fit the specifics of the small open Slovak economy and it is being used for interpretation of the official forecast (shock decomposition). The model's forecasting performance and story-telling potential is subject to ongoing testing and evaluation. The empirical results obtained so far suggest that the model can be used as an alternative forecasting tool capable to provide additional insight into the inner workings of the Slovak economy. The results of the *mechanical update* exercise showed that the model forecast based on the PreMISE can potentially add useful information to the official forecast.

As the research in the area of macroeconomic modelling constantly advances in pursuit of ever changing economic reality, every macroeconomic model that aims to be seriously used for the purposes of forecasting or policy analyses is necessarily a work in progress. The same holds for the PreMISE model and the DSGE models in general. Further extensions of the model should take advantage of the substantial research in the area of labour market or financial frictions modelling within the DSGE framework.

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A. Modelling expected monetary policy regime switch: solution method

To deal with an announced change in the regime of monetary policy we employ the solution method proposed by Kulish and Pagan (2014). Here we describe only the essential details of the method. In the next subsection we present a general solution method of a linear rational expectations model with no structural changes. After that we show how to employ this method to find a solution when agents expect a change in the structure of the model. In the final section we demonstrate the method on a few examples.

The general method: The general solution method employed is that of Binder and Pesaran (1997). The linearized rational expectations model is written in the following form

$$A_0 y_t = C_0 + A_1 y_{t-1} + A_2 E_t y_{t+1} + B_0 \epsilon_t, \tag{A.1}$$

where y_t is a vector of all model variables, ϵ_t is a vector of exogenous variables, A_0, A_1, A_2, C_0, B_0 are matrices of parameters. If the solution to this model exists and is unique, it takes the form

$$y_t = C + Q y_{t-1} + G \epsilon_t, \tag{A.2}$$

Assuming that the expected values of future shocks are all zeros, $E_t \epsilon_{t+1} = 0$, and that equation (A.2) indeed represents the solution of the model it must hold that $E_t y_{t+1} = C + Q y_t$. Plugging this equation into equation (A.1) one obtains that

$$A_0 y_t = C_0 + A_1 y_{t-1} + A_2 (C + Q y_t) + B_0 \epsilon_t.$$

It follows that

$$y_t = (A_0 - A_2 Q)^{-1} (C_0 + A_2 C + A_1 y_{t-1} + B_0 \epsilon_t). \tag{A.3}$$

Comparing the solution form (A.2) with equation (A.3) and using the idea of undetermined coefficients one can guess what the solution matrices C, Q, G are:

$$C = (A_0 - A_2 Q)^{-1} (C_0 + A_2 C) \tag{A.4}$$

$$Q = (A_0 - A_2 Q)^{-1} A_1 \tag{A.5}$$

$$G = (A_0 - A_2 Q)^{-1} B_0 \tag{A.6}$$

Equation (A.5) is a quadratic equation in Q. Once the solution for Q is found, equations (A.4) and (A.6) can be used to calculate C and G.

The method with announced structural changes: For the sake of simplicity let's assume that there is a single structural change implemented at time T^s which is announced at time T^a where $T^a < T^s$. Let's call the model that is in place up until period $T^s - 1$ $M1$ and its coefficient matrices are $A_0^1, A_1^1, A_2^1, C_0^1, B_0^1$. From T^s onwards the relevant model with coefficient matrices $A_0^2, A_1^2, A_2^2, C_0^2, B_0^2$ is called $M2$. At any period $t < T^a$ agents in the model believe that the model $M1$ will hold forever and form their expectations accordingly. When agents are informed about the intended change, in period T^a, they need to account for it in expectations formation process. Finally, from T^s on agents form expectations in line with the model $M2$.

Putting it backwards, from T^s the model $M2$ holds forever hence the solution can be found using the above section approach, i.e.

$$y_t = C^{T^s} + Q^{T^s} y_{t-1} + G^{T^s} \epsilon_t, \tag{A.7}$$

where the solution matrices $C^{T^s}, Q^{T^s}, G^{T^s}$ satisfy equations (A.4)-(A.6) using model $M2$ coefficient matrices. Then in period $T^s - 1$, when model $M1$ is in place, agents know that $E_{T^s-1} y_{T^s} = C^{T^s} + Q^{T^s} y_{T^s-1}$. Plugging this expression into the linearized model $M1$ and following the reasoning laid out in the previous section one finds the solution of period $T^s - 1$ model:

$$y_{T^s-1} = C^{T^s-1} + Q^{T^s-1} y_{T^s-2} + G\epsilon_{T^s-1}, \tag{A.8}$$

where

$$C^{T^s-1} = (A_0^1 - A_2^1 Q^{T^s})^{-1}(C_0^1 + A_2^1 C^{T^s-1}) \tag{A.9}$$

$$Q^{T^s-1} = (A_0^1 - A_2^1 Q^{T^s})^{-1} A_1^1 \tag{A.10}$$

$$G^{T^s-1} = (A_0^1 - A_2^1 Q^{T^s})^{-1} B_0^1 \tag{A.11}$$

Notice that equation (A.10) is now not a quadratic equation in Q^{T^s-1} because the right hand side counterpart of this matrix is dated T^s and is already known at this stage. Repeating this process for periods $t = T^s - 2, ..., T^a$ we find the solution for all transition models C^t, Q^t, G^t. Finally, for the pre-announcement period we find the model using the previous section method and model $M1$ with its coefficient matrices.

The whole solution thus consists of time varying matrices C^t, Q^t, G^t where

$$C^t = C^1, Q^t = Q^1, G^t = G^1 \qquad for\ t = 1, ...T^a - 1 \tag{A.12}$$

$$C^t = C^t, Q^t = Q^t, G^t = G^t \qquad for\ t = T^a, ..., T^s - 1 \tag{A.13}$$

$$C^t = C^2, Q^t = Q^2, G^t = G^2 \qquad for\ t = T^s, ..., T \tag{A.14}$$