Undesired Consequences of Calvo Pricing in a Non-linear World

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Abstract

Applications in modern macroeconomics increasingly require non-linear solution methods. Our paper studies the consequences of the widely used Calvo pricing mechanism in a non-linear world. We introduce the concept of the stability region as a non-linear counterpart to the determinacy region. We show that in non-linear models the Taylor principle is no longer sufficient for inflation stability and stable macroeconomic model moments. The presence of a self-reinforcing price-inflation spiral captured by the non-linear solution presents a new challenge for monetary policy in anchoring inflation expectations.

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NON-TECHNICAL SUMMARY

Inflation in most advanced economies has been rising at a fast pace since the start of 2021. In mid-2022 inflation rates in the United States stood at above 8% with a peak of 9.1% in June 2022. In the Euro Area inflation rates were at 10% by the end of 2022. In some Central and Eastern European countries inflation was even higher, e.g., in Slovakia, annual Consumer Price Index (CPI) inflation was 15% in November 2022. The increase in inflation is accompanied by the rise in both short term and long term inflation expectations[1]. The de-anchoring of inflation expectations from the central bank target adds to the growing evidence on an increasing persistence of inflation (see Reis (2021)) and thus higher trend inflation. In other words, with some simplicity, inflation is heading where people expect inflation to go. Both empirical and theoretical economics predicts, however, that the costs of permanent inflation are small (Cooley and Hansen (1989), McCandless and Weber (1995), Bae and Ratti (2000), Serletis and Koustas (1998)). The argument is that nominal income can adjust for anticipated inflation, leaving people almost as well off as they would have been in the absence of inflation. The idea of medium to long-run money neutrality is a wildly accepted theoretical concept. For instance, McCandless and Weber (1995) find that inflation rates are not correlated with real output growth in most developed countries. On the other hand, a large body of research documents that people hate inflation. Di Tella, MacCulloch, and Oswald (2001) show in a cross-country survey that people's happiness and life satisfaction is negatively correlated to inflation. Shiller (1996), in a detailed survey of public attitude toward inflation, finds that people strongly dislike inflation. The Gallup survey of public opinion repeatedly documents that, in times of fast growing prices, Americans report inflation as the nation's number one problem, dominating even concerns about unemployment. Why, exactly, is higher inflation so bad?

We propose an explanation by showing that the misalignment in relative prices might imply large economic costs of inflation by triggering a self-reinforcing price-dispersion spiral which de-anchors inflation expectations from the central bank's target. The standard prescription for monetary policy to anchor inflation expectations is to follow the Taylor principle, according to which the nominal interest rate should rise more than proportionally with inflation. We show that in times of high inflation when the distribution of prices across products widens and inflation uncertainty rises, the Taylor principle is no longer sufficient for inflation stability[2]. The anchoring of inflation expectations in-

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[1]This can be seen for instance in Survey of Professional Forecasters for 10 year inflation expectations, https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/survey-of-professional-forecasters. 10Y inflation expectations derived from Swaps.

[2]There is a mounting empirical evidence showing that trend inflation, dispersion of prices and inflation

stead calls for an even more aggressive stance on inflation targeting and abstaining from virtually any aim to stabilize the real side of the economy.

In a standard New Keynesian DSGE model with Calvo pricing solved non-linearly we show how the distortion in relative prices might lead to high economic costs of inflation and what, consequently, the implications are for monetary policy. The non-linear solution is an important aspect of our story because it preserves $i)$ the distribution of relative prices in the economy and its influence on the macroeconomy, and $ii)$ inflation uncertainty affecting the consumption saving decision.

A large amount of research has confirmed that prices adjust to a new economic environment in a sluggish way (see Klenow and Kryvtsov (2008), Eichenbaum, Jaimovich, Rebelo, and Smith (2014), Dhyne, Alvarez, Le Bihan, Veronese, Dias, Hoffmann, Jonker, Lunnemann, Rumler, and Vilmunen (2006) and Klenow and Malin (2010) for a survey). It is less clear, however, why prices are rigid. A common way in the literature to model price rigidities is to use the assumption of Calvo pricing. In this framework, a firm is allowed to change its price in a given period only with some probability, and has to keep its price at the previous level otherwise. We may therefore view the Calvo pricing mechanism rather as a useful technical device (which is easy to calibrate), designed to capture the impact of monetary policy on the real economy, rather than capturing or describing the reasons why firms adjust prices in a sluggish way. More concretely, Calvo pricing assumes that every morning when firms open, a lottery is played, determining firms which will be allowed to change their prices. The unlucky firms stay with their old price. Firms which are allowed to change their price do not know, however, for how long this price will remain fixed. Therefore, when setting the price the firm has to take into account its expectation about future prices and future costs of production. This assumption creates a dispersion of prices across product varieties in the non-linear model economy and leads to misalignment in relative prices. The recent spike in inflation provides anecdotal evidence demonstrating how price rigidity creates dispersion of prices in the real world. As an example, consider a particular Czech traditional bricks producer that has recently adjusted its price list to a new set of higher prices; however, there is a number of other producers on the market which sell virtually the identical product still at the old lower prices.

To understand the economic effects of misalignment in relative prices in isolation, let us assume a model where all firms in the economy are the same in every aspect but in

uncertainty are rising since 2021. Albagli (2022) shows firms expectations about future inflation are largely dispersed. This disagreement in inflation forecast is consequently reflected in their price settings. Dispersion in households inflation expectations is growing in US (i.e. Weber et.al. (2022)), EU and Slovakia. Drenik and Perez (2014) show that inflation uncertainty increases price dispersion. Rising inflation risk premia in government bonds points to rising inflation uncertainty.

the price they charge. The assumption of homogeneous firms means that an efficient allocation of scare resources requires that all firms produce the same amount of goods. Nevertheless, because each firm charges a different price each firm faces a different demand for its product. Firms re-pricing their product variety to a new higher price will face, under the (in price) downward sloping demand curve, a lower demand for their product variety and thus produce a smaller level of output. In contrast, firms with low prices will face high demand and produce larger amounts of their product variety. Total output is highest when production inputs are allocated efficiently across producers. In case of homogeneous producers this can be only be the case if each firm produces the same amount of each variety. The distortion in relative prices will therefore lead to misallocation of resources. Let us look at some narrative example demonstrating how misalignments in relative prices lead to economic costs. Let's assume that the car producer, BMW, cannot change its price and is stuck with low price. As a consequence the economy will produce too many BMWs. On the other hand, for instance, Audi has already repriced its cars and, therefore, sells its product at a higher price. As a result, the misalignment in relative prices of these two car producers leads to the production of too many BMWs and too little Audi. This has following consequences. **First**, consumers would be happier if their consumption basket would consist of more Audi and less BMW but the distortion in relative prices motivates them to choose sub-optimal consumption basket. **Second**, the distortion of relative prices means resources do not shift towards the most productive sectors. The firm which has to produce too much to meet demand for its good produces with higher marginal costs as it moves along the concave production function to the right. Going back to the example with car producers, BMW has to stretch its production capacity to an extent which lowers the productivity of its workers. In an economy with decreasing returns-to-scale a firm producing a larger amount of goods operates with higher marginal costs and lower profits. The economy on aggregate would therefore benefit if some of the production is transferred to firms producing a lower amount of goods. **Third**, firms which can change their price can foresee that they might be stuck with their price for some time. In an economy with permanent (trend) inflation they will therefore set, for precautionary reasons, a higher than the currently profit-maximizing price. This will create an additional markup of prices above their marginal costs and reinforce the widening of the distribution of prices across products and thus the distortion in firms' relative prices.

Our non-linear solution allows us to preserve the full distribution of prices across products in the model and thus study the impact of dispersed prices on price formation and inflation. Using our framework, we uncover a previously unknown channel where the misalignment in relative prices de-anchors inflation expectations and leads to a self-

reinforcing inflation spiral. The higher the permanent component of inflation (trend inflation) the higher the chances that the price dispersion inflation spiral is triggered. This is because permanent inflation spreads out the distribution of prices as those firms which cannot change their low prices are left behind further and further from the optimal price as the price level grows. When these firms can finally change their price they create large inflation as they are catching up with the average price level and even setting their prices above the average level so that they can keep their price unchanged for some period again. In fact, instead of decreasing the dispersion of prices, a price re-setting firm further contributes to the widening in the distribution of prices even in the absence of any inflationary shocks in the economy which leads to future higher inflation and de-anchored inflation expectations.

The traditional textbook prescription for the monetary policy authority to anchor inflation expectations based on the linearized macroeconomic models relies on the Taylor principle, according to which the nominal interest rate should rise more than one-to-one with inflation. Blanchard and Kahn (1980) and Bullard and Mitra (2002), among others, generalize the Taylor (1993) principle and define necessary and sufficient conditions for the rational expectations equilibrium to be unique, which is known as the determinacy region. The determinacy region provides a model-consistent definition of what it means to anchor inflation expectations to the central bank's target. However, the presence of the threat that the distortion in relative prices triggers the inflation spiral means that ensuring determinacy is no longer a sufficient condition for the central bank to secure stability of inflation expectations anymore. We instead show that, to anchor inflation expectations and have stable inflation dynamics, the central bank needs to respond to deviations of inflation from its target considerably more aggressively, and give up stabilization of the real side of the economy. The required strength of the response by central bank is defined by the '*stability region*' which we introduce and define as representing the non-linear equivalent to the determinacy region.

One important consequence related to the stability region is that there is no global solution of the model outside of the stability region, as price dispersion and inflation dynamics become explosive for some part of the relevant state space. In other words, in an economy where the central bank allows an inflation spiral to be triggered, the model does not have a stable solution[3]

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[3]The intuitive way to understand this result comes from realizing that the distribution of relative prices defines the solution space but just before we finish the search of the global solution in the space defined by the existing dispersion of prices the dispersion further increases. The analogical way of thinking could be a search for extraterrestrial life in the expanding universe. As the universe expands at faster rate than the spaceship conducting the search we can never find the definite solution.

1. Introduction

The staggered contracting introduced by Calvo (1983) is the dominant method to model the macroeconomic effects of nominal rigidities. While traditionally macroeconomic dynamics have been studied primarily in terms of (log-)linear approximations to the true model solution, recent developments in many fields of macro increasingly require non-linear solution techniques[4]. Schmitt-Grohe and Uribe (2006) stress that linear approximations to the model solution ignore the distribution of prices across product varieties implied by Calvo pricing. Using non-linear solutions we show that dispersed prices give rise to a new source of inflation fluctuations not present in linearized models. Inflation widens the dispersion of prices across products which leads to a further increases in inflation, triggering a self-reinforcing price-inflation spiral, which presents a new challenge for monetary policy in anchoring inflation expectations. To avoid self-fulfilling inflation expectations in linearized models as a source of economic instability the central bank must follow the Taylor principle, according to which the nominal interest rate should rise more than proportionally with inflation (Blanchard and Kahn (1980) and Bullard and Mitra (2002)). In times of persistently high inflation, i.e. positive trend inflation, Ascari and Sbordone (2014) and Coibion and Gorodnichenko (2011) show that the Taylor principle is not sufficient and a more aggressive policy response to inflation is required. We find that the presence of the price dispersion driven inflation spiral significantly constrains the central bank further, in particular its ability to stabilize the real side of the economy. In addition, trend inflation prohibits virtually any aim to stabilize the real side of the economy.

Our paper offers two main contributions. First, we introduce the concept of the 'stability region' as a non-linear counterpart to the determinacy region which provides prescriptions on how endogenous monetary policy can preserve stable inflation. Second, building on Fernandez-Villaverde, Ramirez, and Schorfheide (2016) who attest perturbation methods a high level of accuracy, we show that inflation stability is a necessary condition for accuracy of the perturbation solution, and that the global solution does not exist outside of the stability region.

Our analysis proceeds in four distinct steps. In the first step, we show that the price-

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[4]The need to capture the inherent non-linearities of business cycles forced macroeconomists to move beyond traditional linearization methods (see Fernandez-Villaverde, Ramirez, and Schorfheide (2016), Mendoza (2016) and Mendoza and Villalvazo (2020) for a discussion). To understand the macroeconomic effects of, e.g., time-varying volatility, occasionally binding constraints, the effect of non-standard utility functions and off-equilibrium dynamics of big crises, an increasing number of models are solved non-linearly. Non-linear methods are also essential in the asset pricing literature, to capture risk premia and precautionary saving motives (Rudebusch and Swanson (2012), Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018)).

inflation spiral is a global property of the model (no approximation involved) and is independent of the solution method. We document this by studying price dispersion dynamics from its exact (non-approximated) non-linear law of motion. We identify threshold values of inflation across state dependence of past price dispersion. Inflation above the threshold value triggers the price-inflation spiral. We study the analytical approximation of the underlying law of motion governing price dispersion and discuss how it respects the lower and upper bounds on price dispersion and inflation.

In the second step, we use a simple New Keynesian model with Calvo prices similar to Galí (2015), Anderson, Kim, and Yun (2010), Miao and Ngo (2019)) to demonstrate that our findings in step one generalize in a full-scale model. Whenever the price-inflation spiral drives economic fluctuations in the non-linear model solution, the non-linear model produces substantially different (amplified) moments compared to the linearized solution. Following the literature studying the determinacy of linearized solutions (Ascari and Ropele (2009), Coibion and Gorodnichenko (2011), Lubik and Schorfheide (2004)) we simulate the determinacy region and extend it by also displaying the stability region relevant for non-linear models. The stability region substantially narrows down the monetary policy space given to the central bank by the determinacy region. This is because monetary policy needs to contain self-fulfilling inflation expectations induced by the price inflation spiral. The important implication of this result is that there is no global solution outside the stability region even if the corresponding linear solution is determined.

In the third step, we provide a novel analytical decomposition which disentangles the economic channels through which price dispersion (spiral) impacts the real quantities in the non-linear model. Further, we use the decomposition to explain why real rigidity and trend inflation significantly shrink the stability region. The channels we discuss are not new in literature; we, however, generalize them for the non-linear model. We show, as Ascari (2004) and King and Wolman (1996), that marginal-costs and the forward-looking element of price settings are the transmitters of price dispersion.

In the fourth step, we present stochastic simulation results for the full macro-finance model of Rudebusch and Swanson (2012). We do so because the macro-finance literature relies on a higher-order solution and contains highly non-linear modeling features, and thus serves as a particularly relevant field for the findings of this paper. We also replicate the globally solved models by Anderson, Kim, and Yun (2010) and Miao and Ngo (2019) to show that their calibration falls well into the stability region.

1.1. LITERATURE REVIEW

Our paper relates to the literature on recent advances in solution methods for rational expectations dynamic general equilibrium models. The price-inflation spiral was first noticed by Andreasen and Kronborg (2020) in the context of their novel extended perturbation method. They find that: "standard perturbation generates a price-inflation spiral, which eventually may lead to explosive dynamics" ... "because standard perturbation is unable to account for a previously unnoticed upper bound on inflation with Calvo pricing,". Our work extends and further develops Andreasen and Kronborg's findings. We differ in several dimensions; in the depth of the analysis and in showing that the price-inflation spiral is a global property of Calvo pricing independent of perturbation solutions. We show that in the Calvo (1983) pricing model there exists a threshold value on inflation which gives rise to a price-inflation spiral. We also show that the upper bound on inflation is reached because of the price inflation spiral and not the other way around as found by Andreasen and Kronborg (2020). Further, we go beyond Andreasen and Kronborg (2020) by discussing the non-linear model's stability properties. We provide detailed analyses of the price-inflation spiral both in the nonlinearly solved model with Calvo pricing and as a phenomenon outside of the model solution by focusing on the exact non-linear equation for price dispersion.

Our paper also contributes to the empirical and theoretical literature studying Calvo pricing. Nakamura, Steinsson, Sun, and Villar (2018) provide evidence that the costs of inflation in New Keynesian models are elusive in the data. In the Calvo model the cost of inflation corresponds exactly to price dispersion. As a consequence our analysis in a nonlinear setting therefore produces an even larger and counterfactually high cost of inflation.

Our paper also strongly relates to previous research on Calvo pricing and trend inflation. Ascari (2004), Ascari and Ropele (2009) and Ascari and Sbordone (2014) study effects of trend inflation on the deterministic steady state in the models of time dependent contracts (Taylor, Calvo and Rotemberg pricing). Amano, Ambler, and Rebei (2007) study the impact of trend inflation on stochastic means of macroeconomic variables and find that the non-linearity introduced by price dispersion implied by the second order approximate solution increases the welfare costs of inflation. We extend this literature by studying Calvo pricing in a stochastic non-linear setting more generally, by $i)$ considering higher order perturbation solutions and global solutions, $ii)$ considering other forms of non-linearities which, similarly to the introduction of trend inflation, increase the dispersion of prices across product varieties, and by $iii)$ adding a formal exposition of the channels through which the dispersion of prices transmits to the real economy.

Ascari (2004) and Ascari and Sbordone (2014) show that trend inflation significantly affects the cost of inflation. We add to their results our finding that the linear solution of the model significantly underestimates the true cost of inflation in the model. Ascari (2004) concludes that Taylor pricing should be the preferable implementation of price stickiness because of the sensitivity of Calvo pricing to trend inflation. We extend his findings and show that Taylor and Rotemberg pricing are more robust to non-linear solution methods and more in line with the empirical evidence on the estimated Taylor rule.

2. THE BASELINE MODEL

Our baseline model is a standard textbook style New Keynesian DSGE model with Calvo pricing (e.g., Galí (2015), Anderson, Kim, and Yun (2010), Miao and Ngo (2019)) which can be viewed as a simplified version of the Rudebusch and Swanson (2012) (RS) model we adopt later. The model consists of a continuum of firms which operate under monopolistic competition and are subject to nominal rigidities à la Calvo. Firms produce with a technology that uses labor and fixed capital as production inputs. Households have CRRA preferences over consumption and labor, and make optimal choices over consumption, labor supply and bond holdings. The monetary authority follows a Taylor rule. Model dynamics are driven by shocks to total factor productivity.[5] Since the model is quite standard, experienced readers may want to only skim over most of the model description, and jump directly to the section where we discuss price dispersion and its properties, section 3.

2.1. HOUSEHOLDS

The household maximizes lifetime utility of the form

$$U(C_t, N_t) = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} \right], \quad \varphi, \chi > 0, \tag{1}$$

where $\varphi, \chi, \chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$, and the Frisch labor supply elasticity is given by $(1-\overline{N})/\chi\overline{N}$, where $\overline{N}$ is the steady state level

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[5]As we solve the model globally and the global solution is subject to the curse of dimensionality, we consider a parsimonious setup with a low number of state variables.

of hours worked. The households' problem is subject to the flow budget constraint:

$$B_t + P_t C_t = W_t N_t + D_t + R_{t-1} B_{t-1} - \tau_t. \tag{2}$$

In equation (1), β is the discount factor. Utility (U) at period t is derived from consumption (C_t) and leisure ($1 - N_t$). E_t denotes expectations conditional on information available at time t. As the time endowment is normalized to one, leisure time ($1 - N_t$) is what remains after spending some time working (N_t). $W_t N_t$ is labor income, R_t is the return on the one-period nominal bond, B_t. D_t is dividend income, τ_t are lum-sum taxes by the government.[6]

2.2. FIRMS

Final good firms operate under perfect competition with the objective to minimize expenditure subject to the aggregate price level $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)(di) \right]^{\frac{1}{1-\epsilon}}$, where $P_t(i)$ is the price of the intermediate good produced by firm i, using production technology $Y_t = \left[\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(i)di \right]^{\frac{\epsilon}{\epsilon-1}}$. Final good firms aggregate the continuum of intermediate goods i on the interval $i \in [0,1]$ into a single final good. Parameter ϵ determines the elasticity of substitution between varieties. The cost-minimization problem of final good firms delivers demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t. \tag{3}$$

A continuum of intermediate firms operates in the economy. Intermediate firm i produces according to a Cobb-Douglas production function, where θ denotes the capital share, and where $\bar{K}$ refers to the fact that firms have fixed capital:[7]

$$Y_t(i) = A_t \bar{K}^\theta (N_t(i))^{1-\theta}, \tag{4}$$

Technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_t^A, \varepsilon_t^A \sim N(0,1). \tag{5}$$

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[6]The government is assumed to follow a balanced budget each period financing constant government expenditure, G, each period, i.e. $\tau_t = G$.

[7]Fixed capital can be interpreted as a model with endogenous investment that features high adjustment costs in investment. Capital which is treated as fixed implies decreasing return to scale (DRS) and is supported by the arguments in McCallum and Nelson (1999), for instance a very small correlation between capital and output at business cycle frequency, and the fact that capital is in general changing little in the short run.

Intermediate firms facing Calvo contracts maximize the present value of future profits by choosing price, $P_t(i)$,

$$E_t\left\{\sum_{k=0}^{\infty}\zeta^k Q_{t,t+k}\frac{P_t}{P_{t+k}}\left[P_t(i)Y_{t+k}(i)-P_{t+k}W_{t+k}N_{t+k}(i)\right]\right\}, \tag{6}$$

where $Q_{t,t+k}=\beta\left(\frac{C_{t+k}}{C_t}\right)^{-\varphi}$ is the real stochastic discount factor from period t to $t+k$. The Calvo parameter ζ represents the probability that a firm is not able to reset its price in a given period, which controls the average frequency of price changes. W_t denotes the real wage, the term $P_{t+k}W_{t+j}N_{t+j}(i)$ represents the cost of labor in nominal terms. The optimal price chosen by the price resetting firm, P_t^*, relative to aggregate price index, P_t, is $p_t^*=\frac{P_t^*}{P_t}$ and equals a weighted average of current and future expected real marginal costs. The ratio, $p_t^*=\frac{P_t^*}{P_t}$, is often referred to as the price-adjustment gap in the literature (c.f. Ascari and Sbordone (2014)). We follow this terminology throughout the paper, and write:

$$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}}=\frac{\epsilon}{\epsilon-1}\sum_{k=0}^{\infty}\Upsilon_{t+k}MC_{t+k}, \tag{7}$$

where $\Upsilon_{t+k}=\frac{\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}Y_{t+k}}{\sum_{k=0}^{\infty}\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\epsilon-1}Y_{t+k}}$, and where Π_t is gross inflation. Throughout the paper, Π_t denotes the gross inflation rate, defined as $\Pi_t=P_t/P_{t-1}$; lower case variable π_t instead denotes the (annualized) net inflation rate in percent, $\pi_t=100\log(\Pi_t^4)$. Υ_{t+k} is the time varying mark-up implied by price rigidity and $\frac{\epsilon}{\epsilon-1}$ is the mark-up implied by monopolistic competition. MC_t is average real marginal cost, defined as:

$$MC_t=\frac{1}{1-\theta}\left(\frac{W_t}{A_t}\right)\left(\frac{Y_t}{\bar{K}A_t}\right)^{\frac{\theta}{1-\theta}}. \tag{8}$$

2.3. MONETARY POLICY

The model is closed by a monetary policy rule:

$$\log(i_t)=\log(\bar{\imath})+\phi_\pi[\log(\Pi_t)-\log(\overline{\Pi})]+\phi_Y\log\left(\frac{Y_t}{\bar{Y}}\right), \tag{9}$$

where i_t is the quarterly policy rate and $\bar{\imath}=\overline{\Pi}/\beta$. $\overline{\Pi}$ is the inflation target of central bank defining the value of trend inflation. $\bar{Y}$ denotes the steady-state level of output, Y_t.

3. NON-LINEAR IMPLICATIONS OF PRICE DISPERSION

The properties of price dispersion in non-linear settings differ substantially from those of the linearized version. In the following, we discuss how price dispersion, as implied by the exact non-linear law of motion, given by equation (14) below, affects the global properties of the model. Importantly, the analysis of this section is independent of any choice of (approximate) solution method. Section 3.1 derives the recursive formulation of price dispersion and shows that it can be interpreted as a resource cost, reducing output produced per unit of inputs. Section 3.2 discusses the presence and effect of upper and lower bounds on price dispersion. These bounds define a region over which price dispersion is well-defined, but beyond which a (real) solution for price dispersion ceases to exist and where the model economy's production stops. Section 3.3 defines the threshold level of inflation which triggers an inflation spiral and de-anchors inflation expectations in the model. Section 3.4 studies the numerical properties of the true, fully non-linear price dispersion (as described by the actual non-linear law of motion) and its Taylor series expansions. This analysis helps us to build intuition for how the behavior of price dispersion affects solution and stability of the simulation in our non-linearly solved New Keynesian model of section 4.

3.1. RESOURCE COST OF PRICE DISPERSION

Price dispersion is defined by aggregating output across firms (firm i's production functions), where we equalize the supply of intermediate good i, equation (4), with the final good producers's demand curve for intermediate good i, equation (3), to obtain $A_t \bar{K}^\theta N_t^{1-\theta}(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$. Assuming that workers are homogeneous across i, the aggregation of labor input is $N_t = \int_0^1 N_t(i) di$ and together with the definition of price distortion[8], $S_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]^{1-\theta}$, it delivers the aggregate production function,

$$Y_t S_t = A_t \bar{K}^\theta N_t^{1-\theta}. \tag{10}$$

We define S_t so that the cost of prices dispersed across product varieties can be directly expressed in terms of output loss. Variable S_t is, therefore, a model-consistent index

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[8]Note that the literature typically differentiates between price distortion, S_t, and price dispersion, $S_t^{\frac{1}{1-\theta}}$.

of price distortion and summarizes the resource costs induced by the inefficient price dispersion featured in the Calvo model. As a consequence of price dispersion the equilibrium allocation is not optimal. The subset of firms which cannot reset their price, charge either a lower or higher price for their product varieties than price-resetting firms. Since the demand for intermediate output comes from a perfectly competitive final good sector, where intermediate goods enter symmetrically and with equal weight, the firm charging a lower (higher) price than the price-resetting firm is not efficiently using its production capacities and produces too much (little). The efficient resource allocation dictates that each firm produces the same amount of goods. Formally, the loss of GDP due to price dispersion, Δ_t, in percent can be defined as

$$\Delta_t = 100 \left(1 - S^{-1} \right). \tag{11}$$

We can use the Calvo (1983) result and rewrite $S_t^{\frac{1}{1-\theta}}$ recursively as[9]

$$
\begin{aligned}
S_t^{\frac{1}{1-\theta}} &\equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon}{1-\theta}} di, \\
&= (1-\zeta) \left(p_t^* \right)^{\frac{-\varepsilon}{1-\theta}} + \zeta \left(\Pi_t \right)^{\frac{\varepsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}.
\end{aligned}
\tag{12}
$$

3.2. LOWER AND UPPER BOUND ON PRICE DISPERSION

Several points regarding the properties of equation (12) are worth highlighting for future discussion. First, the resource cost of price dispersion, S_t, is a concave function of the ratio of two price indexes, $\left[\int_0^1 P_t(i)^{\frac{-\epsilon}{1-\theta}} \right]$ and aggregate price index $P_t^{\frac{-\epsilon}{1-\theta}} = \left[\int_0^1 P_t^{1-\epsilon}(i) di \right]^{\frac{-\epsilon}{(1-\epsilon)(1-\theta)}}$. The bigger the difference between these indexes the wider is the distribution of prices across product varieties. Yun (1996) and Schmitt-Grohe and Uribe (2006) show that this ratio implies that price distortion S_t is bounded from below by 1, $S_t \geq 1$.[10] When $S_t = 1$ all firms have the same prices in the economy.

Second, the Calvo pricing mechanism also implies an upper bound on inflation, $\Pi_t < \Pi^{upper}$, which consequently implies a maximum admissible level of price dispersion, S_t^{upper}, consistent with this inflation upper bound, for given S_{t-1}. To see this, let us make

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[9] $S_t^{\frac{1}{1-\theta}} = (1-\zeta) \left(\frac{P_t^*}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} + (1-\zeta)\zeta \left(\frac{P_{t-1}}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} + (1-\zeta)\zeta^2 \left(\frac{P_{t-2}}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} \ldots$
where the expansion from time $t-1$ can be summarized by S_{t-1}.

[10] We show in the appendix that the same result holds for price dispersion under decreasing returns to scale, $S_t^{\frac{1}{1-\theta}}$.

use of the definition of the aggregate price index, and express the price-adjustment gap, $p_t^* = P_t^*/P_t$, as a function of inflation only,

$$p_t^* = \left[\frac{1 - \zeta\left(\Pi_t\right)^{\epsilon-1}}{1-\zeta} \right]^{\frac{1}{(1-\epsilon)}}. \tag{13}$$

Substituting this into equation (12) we obtain:

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left[\frac{1 - \zeta\left(\Pi_t\right)^{\epsilon-1}}{1-\zeta} \right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}. \tag{14}$$

The power term in (14), ($\frac{\epsilon}{(\epsilon-1)(1-\theta)}$), implies that price dispersion turns complex unless $\left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta} \right] > 0$, which defines the inflation upper bound as $\Pi_t < \Pi_{max} \equiv \zeta^{\frac{1}{1-\epsilon}}$[11]. The dispersion of prices is therefore bounded both from below and above, creating two bounds on the underlying true policy function of S_t.
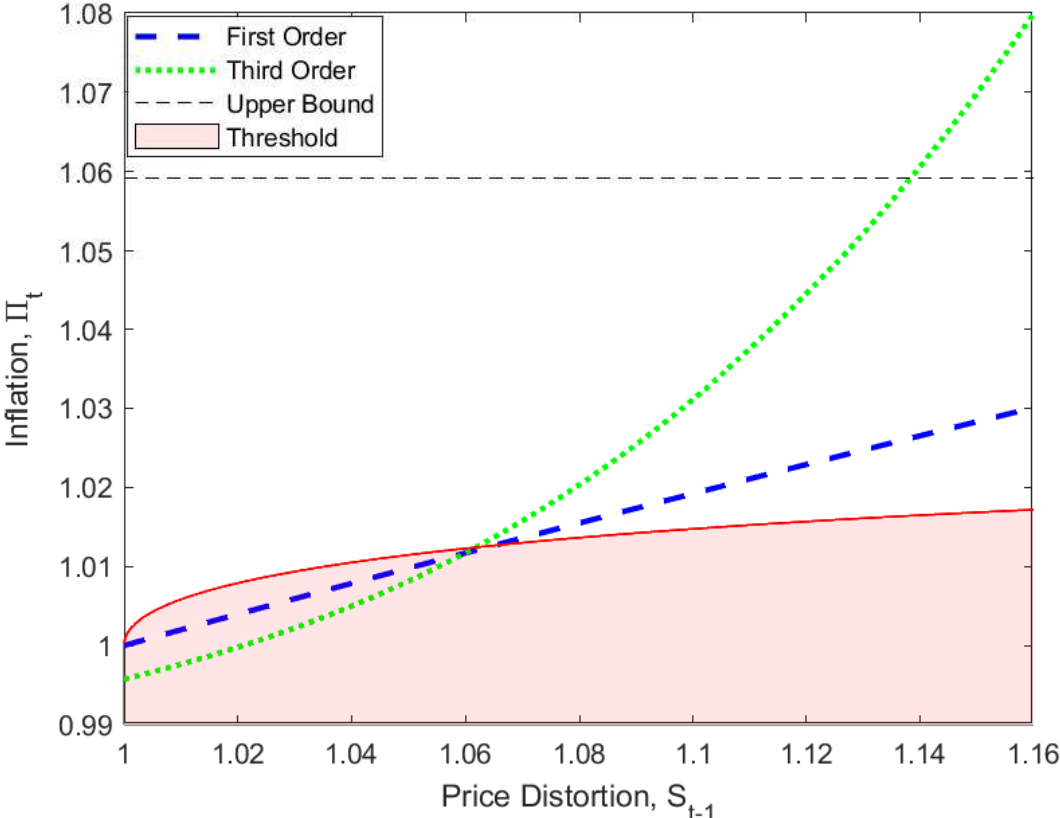
3.3. PRICE DISPERSION INFLATION SPIRAL

Inflation (together with structural parameters ζ, ϵ, θ) determines the dynamic stability of the recursive formulation of price dispersion given by the non-linear first-order difference equation (14). From this equation, we can identify, for each value of S_{t-1}, the specific threshold value of inflation, $\Pi_t^{threshold}$, beyond which $S_t > S_{t-1}$, i.e., inflation for which price dispersion becomes increasing in its own past, so that, the process for S_t is not stable. Figure 1 plots this threshold value of inflation, $\Pi_t^{threshold}$ as a function of S_{t-1}. The red-shaded region shows the stability region of Π_t for which $S_t < S_{t-1}$. Levels of Π_t below inflation threshold $\Pi_t^{threshold}$ imply stable price dispersion dynamics; levels of Π_t above $\Pi_t^{threshold}$ instead trigger explosive price dispersion dynamics. To illustrate the relevance of the inflation threshold for the perturbation model solutions we discuss in section 4, we include policy functions, $\Pi_t = g(S_{t-1}, \overline{A}, \sigma)$ from the first and third perturbation solution as an example[12] Plotted policy functions represent the relationship between Π_t and state variable S_{t-1}, when exogenous shocks are zero. Inflation above the threshold value $\Pi_t^{threshold} < \Pi_t = g(S_{t-1}, \overline{A}, \sigma)$ triggers the price (dispersion) inflation spiral. In this case inflation and price dispersion are self-reinforcing and as a result

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[11]Note that Ascari (2004) derives an inflation upper bound for the deterministic steady state. The bound we present here holds also for the cyclical component of inflation and is often tighter. In our model the inflation upper bound is 11.72% for quarterly steady-state inflation versus 5.92% for for the cyclical deviation of inflation from its steady-state.

[12]Policy functions plotted here depend on a precise model and model parameterization: here they come from the benchmark model (see Table 1).

inflation expectations are no longer anchored. High inflation, Π_t, at time t, leads to high dispersion of prices across product varieties, S_t, as only some firms can adjust their prices. The high dispersion of prices, S_t, in turn calls for a strong adjustment in prices and thus high inflation in the next period, Π_{t+1}. These reinforcing dynamics continue, and hence start the price-inflation spiral. Price dispersion at the same time increases the resource costs implied by dispersed prices (see equation (10)), which effectively leads to a decreased supply of goods, much like a negative TFP realization.

Figure 1: Threshold and Stability Region, Implying Stable Price Dynamics

Note: The (red) line defines the inflation threshold, $\Pi^{threshold}$. Outside the (red-shaded) stability region, where $S_t < S_{t-1}$, price dispersion triggers a self-enforcing inflation spiral. Policy function from first (blue) and third (green) order perturbation solution illustrate that the unstable dynamics may arise already at mild inflation levels far below the inflation upper bound.

We show in the next section that for standard model parameterizations policy functions quickly fall outside of the stability region in a wide variety of (including the textbook) NK models. As in the example in Figure 1, the inflation spiral is triggered already for price dispersion implying mild inflation (above 1.3% quarterly). Quarterly inflation of 1.3% lies far away from the level of inflation implied by the inflation upper bound, of 5.92%. The positive slope of the threshold line is driven by the loading $\zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}}$ on

$S_{t-1}^{\frac{1}{1-\theta}}$ in equation (14). Intuitively, a fraction ζ of firms which cannot adjust their prices towards the optimal price will further increase the dispersion of prices across product varieties, S_t. On the other hand, the price adjustment gap, $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]$, in equation (14) is decreasing with inflation. The fraction $(1-\zeta)$ of price changing firms decreases dispersion by optimally choosing its price by closing the price adjustment gap, $\frac{P_t^*}{P_t}$.

Andreasen and Kronborg (2020) have previously documented the importance of respecting the inflation upper bound for the precision of the approximate model solution. In contrast to Andreasen and Kronborg (2020), we stress here that the unstable dynamics of higher-order solutions of a model with Calvo pricing are not implied by ignoring a upper bound on inflation, but, instead, by the threshold on inflation, $\Pi_t^{threshold}$. The threshold we identify is obtained directly from the actual non-linear equation that characterizes price dispersion dynamics, so that the explosive dynamics beyond that threshold are a true feature of the Calvo model and not an artefact of an approximation or the choice of solution method.

3.4. NON-LINEAR LAW OF MOTION OF PRICE DISPERSION AND APPROXIMATION ACCURACY

Figure 2 provides intuition for our main quantitative results (Table 2 in the section 4). It visually shows how different orders of a polynomial approximation of the underlying price dispersion function respect the lower and upper bound on price dispersion. It also highlights when high and low realization of inflation yields approximations outside of the radius of convergence of Taylor series. Figure 2 plots the true functional form of price dispersion, given by equation (14), together with the analytical approximations given by first, second and third-order Taylor series expansions to that equation. Price dispersion, S_t, is plotted as a function of Π_t at the point $S_{t-1} = \bar{S}$. Dashed-dotted lines are first, second and third-order approximation of S_t at the point $\pi = 0\%$ in panel 2a and $\pi = 4\%$ in panel 2b. Price dispersion, S_t, is a highly non-linear function in inflation which has been historically approximated simply by a line (red dashed-dotted line). Up to a first-order approximation around zero steady state inflation price dispersion is always constant and equal to 1 (panel 2a), and price dispersion does not have an effect on the model solution. Instead, linear approximation around a non-zero inflation steady state or higher-order approximation at any point of steady state inflation introduces the process of price dispersion into the model equilibrium (panels 2a and2b).

Figure 2 illustrates several important properties of the model approximation discussed in the next section. First, a linear approximation is likely to violate the lower bound

on S_t as seen in 2b. Second, at empirically relevant values for steady-state inflation the slope of the line approximating the true S_t implies lower values of approximated $S_t = s(\bar{\pi}) + s'(\bar{\pi})\hat{s}_t$ than the true S_t as Π_t increases and therefore the upper bound on inflation is rarely reached in linearly approximated models. Third, the second-order approximation respects the lower bound on S_t and approximates the underlying function S_t fairly well. Fourth, the third-order approximation matches the underlying function S_t remarkably well for inflation increases but delivers large approximation errors in case of inflation decreases. For large price drops it violates the lower bound on S_t.

Figure 2: Analytical approximation of S_t
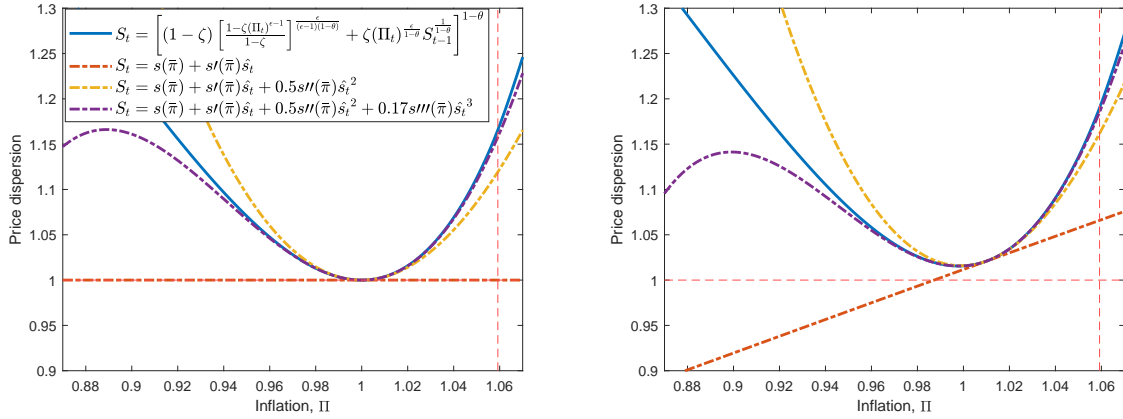
(a) Approximation at $\pi = 0\%$.   (b) Approximation at $\pi = 4\%$.

Note: Blue solid line, S_t, represents price dispersion as a function of Π_t, as given by equation (14), at point $S_{t-1} = \bar{S}$. Dash-dotted lines are first (red), second (yellow) and third (purple) order approximations of S_t at $\pi = 0\%$ and at $\pi = 4\%$. Light red Horizontal and vertical dashed lines represent the lower bound on S_t and upper bound on Π_t.

4. RESULTS FROM BASELINE NK MODEL

Section 3 established the non-linear properties of price dispersion by studying the equation that governs its true non-linear law of motion in isolation. This section instead shows that the consequences of Calvo pricing we outlined dominate the dynamic behavior of the non-linearly solved dynamic general equilibrium New Keynesian model with important implications for monetary policy. For this purpose, we solve and simulate the example model from section 2 with both perturbation (local) and with global approximation methods.[13] First, we calibrate the model by following a standard choice of parameters in the literature. Second, we introduce the concept of the stability region

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[13]The perturbation solutions are obtained as implemented in Dynare routines. Simulated time series from higher-order solutions are pruned. As a global approximation method we adopt a policy function/

as a non-linear analogue to the determinacy region. Third, we show that solutions outside of the stability region deliver explosive dynamics. Forth, we build on the findings of Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011) that the Taylor principle breaks under trend inflation and show that the stability region quickly shrinks with positive trend inflation. Fifth, we follow the literature on real rigidities as in Levin, David Lopez-Salido, Nelson, and Yun (2008) and discuss the effect of decreasing returns to scale (as the most wide spread form of real rigidity) on the stability region.

4.1. PARAMETERIZATION

Table 1: Parameterization and Scenarios

β	Discount factor	0.99	ζ	Calvo parameter	0.76
φ	Coeff. of RRA	2	ρ_A	Autocorrelation, TFP shock	0.95
ϵ	Elasticity of subst.	6	σ_A	Volatility, TFP shock	0.007
$\frac{1-\bar{N}}{\bar{N}}\frac{1}{\chi}$	Frisch elast.	0.28	G/Y	government spending to GDP	0.17

Benchmark (A): zero trend inflation, decreasing returns-to-scale: $\bar{\pi}=0$, $\theta=1/3$:

case A1:	$\phi_\pi=2.5$	$\phi_Y=0$	*Simple TR, Woodford(2003)*
case A2:	$\phi_\pi=2.2$	$\phi_Y=0.43$	*Coibion and Gorodnichenko (2011)*
case A3:	$\phi_\pi=1.53$	$\phi_Y=0.77$	*Taylor (1999)*

Trend inflation (B): 2% trend inflation, decreasing returns-to-scale: $\bar{\pi}=2$, $\theta=1/3$:

case B1:	$\phi_\pi=2.5$	$\phi_Y=0$	*Simple TR, Woodford(2003)*
case B3:	$\phi_\pi=2.5$	$\phi_Y=0.5$	*Guerrieri and Iacoviello (2015)*

Real rigidity (C): zero trend inflation, linear-in-labor production: $\bar{\pi}=0$, $\theta=0$

case C1:	$\phi_\pi=2.5$	$\phi_Y=0$	*Simple TR, Woodford(2003)*
case C2:	$\phi_\pi=1.53$	$\phi_Y=0.77$	*Taylor (1999)*

Note: The first part presents baseline parameters common across all model parameterization scenarios. The second part specifies the parameters for cases A1-C2, where parameters ϕ_π, ϕ_Y, $\bar{\pi}$, θ differ across scenarios.
Scenarios A consider zero trend inflation and DRS in production.
Scenarios B consider positive trend inflation and DRS in production.
Scenarios C consider zero trend inflation and no DRS in production.

We summarize the choice of parameters in Table 1. Parameter values fall well into the region typically chosen in the literature (e.g. Galí (2015), Rudebusch and Swanson (2012)). The bottom part of Table 1 presents three model scenarios we consider to demonstrate the role of monetary policy in anchoring inflation expectations. The benchmark scenarios (cases A) rely on a standard assumption of zero trend inflation and decreasing returns to scale. Scenarios B relax the assumption of zero trend inflation and scenarios C consider the economy in the absence of real rigidity (adopting

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time iteration method developed by Coleman (1990, 1991) and adopted, e.g. in Rabitsch (2012, 2016). Appendix A provides a description of the algorithm for our example model. Alternatively, we also implement a projection method (Chebyshev collocation) as in Anderson, Kim, and Yun (2010), which yields virtually identical results.

instead a linear-in-labor production function). We analyze several cases for the Taylor rule calibrations. We choose popular estimates of the Taylor rule so that they lie $i)$ in, $ii)$ on the border and $iii)$ outside of the stability region. Consequently, we demonstrate the resulting model dynamics.

4.2.   EQUILIBRIUM STABILITY AND THE ANCHORING OF EX-PECTATIONS

The standard way of closing a macroeconomic NK model is the assumption that monetary policymakers set interest rates based on a simple Taylor rule. It is well known that to anchor inflation expectations in the linearized version of the model the weight on inflation must follow the Taylor (1993) principle, according to which the nominal interest rate should rise more than proportionally with inflation. Blanchard and Kahn (1980) and Bullard and Mitra (2002), among others, generalize the Taylor (1993) principle and define necessary and sufficient conditions for the rational expectations equilibrium to be unique, which is known as the determinacy region. The determinacy region provides a model-consistent definition of what it means to anchor inflation expectations by the central bank. In the empirical work Weber, D'Acunto, Gorodnichenko, and Coibion (2022) defines anchored expectations as: "changes in short-run inflation expectations that are largely uncorrelated with changes in long-run expectations". In other words, the central bank achieves its targeted inflation in the medium run. In NK macro model shocks which might, e.g., temporally lead to increased inflation are consequently offset by a monetary tightening, so that inflation returns to its steady state. The determinacy region ensures that the monetary policy response is strong enough.

We add to these well-known results by showing that model determinacy is not a sufficient condition for model stability in non-linearly solved models with Calvo prices. We show numerically that ensuring that $S_t \leq S_{t-1}$ over the largest part of the state-space is the necessary condition for model stability. This result holds both for globally solved models and higher order (local) perturbation methods.

Using the insights and results from section 3 we define the stability region as a non-linear analogue to the determinacy region. The stability region is the space spanned by ϕ_π and ϕ_y for which $S_t \leq S_{t-1}$, i.e., for which inflation falls below the inflation threshold. Alternatively, the stability region can be thought of as the region for which the global solution exists and for which perturbation methods generate stable model

dynamics. Formally, the region defined by

$$H\left(\Pi|(S_{t-1} = S_t)\right) \geq \max\left(\hat{h}(P_1(S, A), \gamma, \sigma), \ldots, \hat{h}(P_{99}(S, A), \gamma, \sigma)\right) \qquad (15)$$

provides a sufficient condition for the stability of simulated model dynamics. The function $\hat{h}$ is the third order approximation to the function mapping the state vector to inflation. γ is the vector of model parameters. σ scales the amount of uncertainty in the model. The inflation policy function is evaluated over the relevant state space. As the relevant state space we take values that lie between the 1st and 99th percentile of the simulated bivariate distribution of S_{t-1} and A_t, denoted by $P_1(S, A)$ and $P_{99}(S, A)$.[14] Function H calculates the inflation threshold based on the exact non-linear form of price dispersion in equation (14) conditional on $S_t = S_{t-1}$.

We use the following algorithm to calculate the stability region. First, we compute the threshold inflation from the non-approximated recursive form of price dispersion using numerical solver. Threshold inflation represents inflation for which $S_t = S_{t-1}$ in equation (14). Second, we use the third order perturbation solution[15] to simulate the model and calculate the state space as implied by the 1st to 99th percentile of the simulated bivariate distribution of S and A. Third, we use the inflation policy function from the third order solution to calculate the implied levels of inflation over this state space, and find the maximum implied inflation. Fourth, we evaluate whether the implied inflation lies below or above the inflation threshold (i.e., within or outside the stability region (see Figure 1). Inflation outside of the stability region implies unstable dynamics.

4.3. STABILITY UNDER BENCHMARK SCENARIO

To assess stability of the non-linear model we build on the example of Ascari and Sbordone (2014) and Coibion and Gorodnichenko (2011). We simulate the determinacy region for the calibration of our benchmark example model and extend it by displaying the stability region relevant for non-linear models. The gray area in Figure 3 depicts the

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[14]This choice is driven by the experience that model moments are well contained at this percentile. In contrast, we experienced first signs of instability (amplification of moments) when using the $P_5(S, A)$ and $P_{95}(S, A)$.

[15]Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) show that higher order perturbation and projection methods deliver very similar results. We use the third order perturbation solution, because, as we show, it delivers virtually the same policy functions as a global solution within the stability region, yet global solutions cannot be obtained when model dynamics imply instability. Appendix A explains this is the case because the relevant state space in dimension S_{t-1} over which the solution would need to be computed becomes unbounded for regions where $S_t > S_{t-1}$. Appendix B shows policy function solutions for the various scenarios of this section to develop these points further.

determinacy region which demonstrates the well-known generalized Taylor principle that the central bank should respond more than one-to-one to inflation. The green area in Figure 3 instead is specific to the non-linearly solved model and shows the stability region for which $S_t \leq S_{t-1}$. In other words, the determinacy region represents the first order condition for inflation stability whereas the condition $S_t \leq S_{t-1}$ defines the stability region in non-linear models. The gray area of the ϕ_y and ϕ_π space defines the region where the model satisfies Blanchard and Kahn (1980) conditions and can be solved by perturbation methods. Once out of the green region, the perturbation solution, nonetheless, features large local instabilities in the simulation and consequent wild dynamics of second moments as we present in Table 2. In practice, even a small part of the state space in a model simulation plagued by unstable price dispersion dynamics leads to unreasonable amplification in simulated model moments. For these reasons, obtaining a global solution outside of the stability region is not feasible. The red triangles in Figure 3 represent the calibration of the Taylor rule coefficients employed in benchmark scenarios A1-A3 (c.f., Woodford (2003), Coibion and Gorodnichenko (2011), Taylor (1999)). The gray marks depicts other seminal estimates from a literature survey (summarized in Table 8). While these examples of Taylor rule coefficients fall well into the determinacy region, once the non-linear aspects of the solution (e.g. dispersion of prices and inflation uncertainty) are taken into account many of the empirically relevant estimates quickly leave the stability area. Namely, a higher weight on the output gap[16], plays a potentially destabilizing role and calls for a more aggressive policy response to inflation even if an economy stays in the determinacy region.

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[16]$\phi_y > 0.2$ is an common estimate, see Taylor (1999), Judd and Rudebusch (1998) and Clarida, Gali, and Gertler (2000) and others.

Figure 3: Determinacy vs. Stability Region for the Simple Model, Scenario A
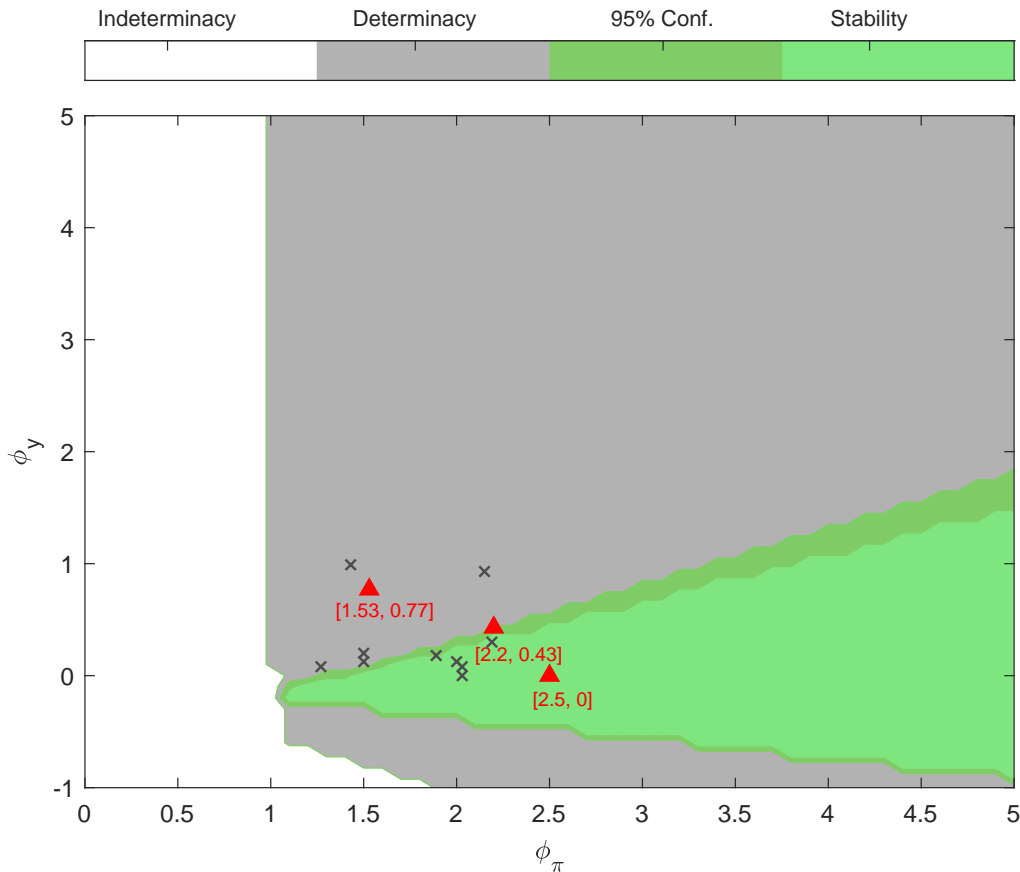
Note: Simulated determinacy (gray) and stability (green) region. Red triangles represent scenarios A1-A3. The stability region denotes the region where $S_{t-1} < S_t$ over the 1st to 99th percentile of the simulated bivariate distribution state variables, denoted $P_{1,99}(S)$ and $P_{1,99}(A)$. The region 95 % Conf. does the same for $P_{5,95}(S)$ and $P_{5,95}(A)$. Gray 'x' marks correspond to other empirically found Taylor rule coefficients in the literature (Table 8).

Table 2 accompanies Figure 3 and shows simulated moments for the benchmark scenarios represented by the red triangles. The simple feedback to inflation-only Taylor rule in case A1 falls well into the stability region. The model solutions of third order and global solution method yield almost identical and stable moments (and policy functions). This attests higher order perturbations a high solution accuracy. This is in line with previous findings by Fernandez-Villaverde, Ramirez, and Schorfheide (2016), who show that higher order perturbation methods feature small approximation errors. Our results confirm these findings but only within the stability region. The lower part of Table 2 shows how much output in percent is lost due to dispersion of prices across products. Well within the stability region price dispersion is contained and in the range of empirical estimates (i.e. Nakamura, Steinsson, Sun, and Villar (2018)) and the lower and upper bounds on π and S are respected.

Table 2: Model-Based Unconditional Moments: Benchmark scenarios (A1-A3)

	case A1: $\phi_\pi = 2.5$, $\phi_Y = 0$			case A2: $\phi_\pi = 2.2$, $\phi_Y = 0.43$		case A3: $\phi_\pi = 1.53$, $\phi_Y = 0.77$	
	1st order	3rd order	global	1st order	3rd order	1st order	3rd order
Moments of macroeconomic variables							
$std(C)$	2.73	2.74	2.74	2.43	4.47	2.16	16.37
$std(N)$	0.93	0.94	0.94	1.21	2.20	1.47	11.89
$std(i)$	1.76	1.76	1.76	3.72	6.23	5.55	39.54
$std(\pi)$	0.70	0.70	0.70	2.89	5.01	4.94	36.12
Behavior of price dispersion and implied output loss, $\Delta_t = 100\left(1 - S^{-1}\right)$							
$mean(\Delta)$	-0.00	0.05	0.05	-0.00	0.87	-0.00	1.85
$std(\Delta)$	0.00	0.06	0.06	0.00	2.11	0.00	14.02
$max(\Delta)$	0.00	0.42	0.43	0.00	16.85	0.00	67.47
$min(\Delta)$	-0.00	0.00	0.00	-0.00	-7.76	-0.00	-143.34
$\pi_t > \pi^{upper}$	0.00	0.00	0.00	0.00	0.67	0.00	20.35
$S < 1$	0.00	0.00	0.00	0.00	12.63	0.00	44.98

Note: Implied model moments for calibration from the stable (A1), borderline (un)stable (A2) and unstable (A3) but determined region. All variables are quarterly values expressed in percent, apart from inflation and interest rates, which are expressed at an annual rate. The mean, standard deviation, minimum and maximum value of price dispersion are expressed in terms of implied output loss. We report the percentage of simulation periods, in which price dispersion travels to regions of the state space that are in violation of the upper and lower bound of π and S. Macro moments are for consumption, hours worked, the nominal interest rate and inflation.

The Taylor rules of scenarios A2 (borderline (un)stable) and A3 (unstable) document that the simulated moments of macro variables show a largely elevated volatility under the third order solution compared to the linear solution; as (a substantial part of) the relevant state space is plagued by unstable price dispersion dynamics no global solution can be obtained. The third order solution in scenarios A2 and A3 implies significant output losses each quarter (0.87% and 1.85% respectively). Even though mean output losses are far from what is suggested by empirical evidence (i.e. Nakamura, Steinsson, Sun, and Villar (2018)), the volatility of Δ_t rises to clearly counterfactual levels. Higher-order solutions produce states of the world when the quarterly lost output due to price dispersion reaches 16.85% (A2) or respectively 67.47% (A3). The 'negative losses' of output -7.76% or -143.34% in the third-order solution reflect the perturbation solution's inability to respect the bound of $S_t \geq 1$, which is violated in 12.63% or 44.98% of cases. The percentage of simulation periods with violations of bounds on $\pi_t \leq \pi^{upper}$, lies at

0.67% or 20.35%.[17]

To better visualize the part of the model's state space that suffers from a local non-stability, we also inspect policy functions under the various settings. To save space this analysis is relegated to appendix B for interested readers.

In the rest of this section, we investigate how two – for monetary policy major and empirically important – modifications of the model affect the stability region.

4.4. STABILITY UNDER POSITIVE TREND INFLATION AND ABSENCE OF REAL RIGIDITY

Recent theoretical and empirical work of Ascari and Sbordone (2014), Coibion and Gorodnichenko (2011), Ascari and Ropele (2009), Bauer and Rudebusch (2017) among others shows that the conduct of monetary policy should be analyzed by appropriately accounting for positive trend inflation targeted by policymakers. Using different theoretical and empirical models the literature on trend inflation finds that positive trend inflation tends to destabilize inflation expectations and requires a much stronger response of monetary policy to inflation.

In a similar fashion, the monetary policy literature (see e.g. chapter 3 of Woodford (2003), Levin, David Lopez-Salido, Nelson, and Yun (2008) and Marencak (2022)) has emphasized that the degree of real rigidity in the economy is another key element forming the determinacy space and anchoring inflation expectations. Woodford (2003) argues that nominal rigidities in quantitative models for monetary policy analysis need to be accompanied by some form of real rigidity to be able to amplify the real effects of monetary disturbances. Real rigidities are the mechanism which lowers firms' incentive to increase prices in the face of a rise in nominal demand or productivity. To analyze the way monetary policy anchors inflation expectations in the non-linear model we choose, for illustration, the most wide spread way of introducing real rigidity in the form of decreasing return to scale.

In Figure 4 we show that $i)$ positive trend inflation significantly shrinks the stability region and requires monetary policy to renounce responding to the real side of the economy; $ii)$ the absence of real rigidity widens the stability region and significantly improves the ability of monetary policy to anchor inflation expectations.

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[17]We should note at that the amplification in moments in the unstable cases does not arise only because of violation of lower and upper bounds on price dispersion and inflation. The counterfactual amplification remains when excluding simulation periods where bound were violated from the sample.

Figure 4a shows, in line with Ascari and Sbordone (2014), that already with mild positive annualized trend inflation of 2% the determinacy region shrinks significantly. After taking the non-linear aspects of the solution into account the model prescribes monetary policy to abstain from virtually any aim to stabilize the real side of the economy and respond solely to inflation. In the next section we analyze in detail the channels through which trend inflation contributes to the instability of the non-linear solution. Ascari and Sbordone (2014) show that the determinacy of the linear solution shrinks with trend inflation because price-setting firms are more forward-looking and the inflation rate becomes less sensitive to current economic conditions. In non-linear settings, with an active price-inflation spiral, the current output levels do not reflect future inflation induced by price dispersion and the stability region shrinks even more substantially.

Figure 4b shows that the presence of real rigidity has no impact on the determinacy region and as soon as the Taylor principle is satisfied inflation expectations are anchored. However, the absence of real rigidity has a significant impact on the stability region. Without the real rigidity, the ability of monetary policy to respond to the real economy increases. A lower degree of real rigidity makes stability sustainable at higher levels of ϕ_y. In the next section we show in detail that the reason is the wedge between optimal marginal costs at the firm level and aggregate marginal costs (see equation (19)). Firms with higher market share produce with higher marginal cost as they move along on the concave (as opposed to linear) production function to the right which makes the inability to adjust prices more costly in terms of aggregate output. Decreasing returns to scale magnify the dispersion of prices and the power on S_t). Signals from the real side of the economy do not fully reflect the price inflation spiral. Monetary policy responding to deviations of output does not anchor inflation expectations in the model with real rigidities.

Figure 4: Stability region under Positive Trend Inflation and Absence of Real Rigidity
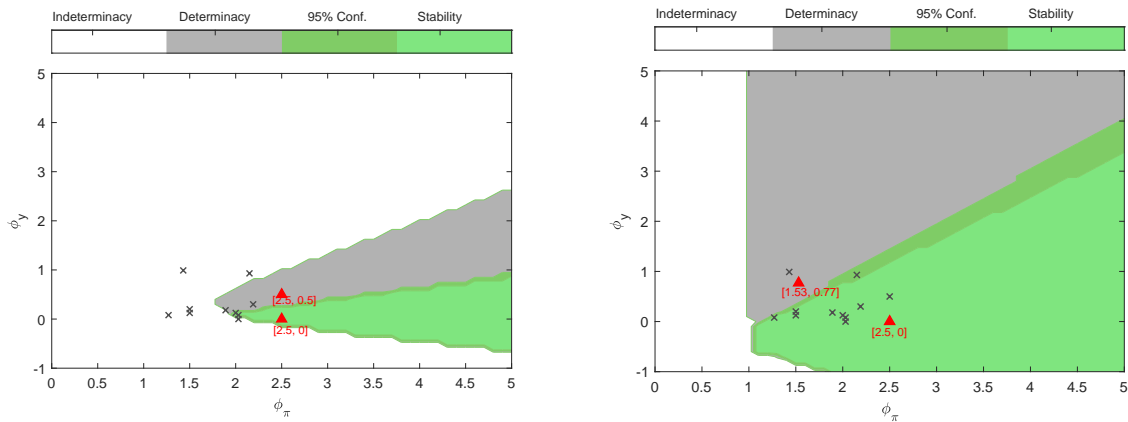
(a) Trend Inflation $\pi = 2\%$.

(b) DRS $\theta = 0$.

Note: Simulated determinacy (gray) and stability (green) region for scenarios B (left) and C (right). Red triangles represent model parameterization scenarios B1-B2 and C1-C2 respectively. The stability region denotes the region where $S_{t-1} < S_t$ over the 1st to 99th percentile of the simulated bivariate distribution state variables, denoted $P_{1,99}(S)$ and $P_{1,99}(A)$. The region 95 % Conf. does the same for $P_{5,95}(S)$ and $P_{5,95}(A)$. Gray 'x' marks correspond to other empirically found Taylor rule coefficients in the literature (Table 8)

Table 3 shows the implied (cases B1 and B2) model moments from the stable and unstable calibration for positive trend inflation and moments for the benchmark model without decreasing returns to scale (case C2)[18]. The unstable scenario B2 demonstrates the strong impact of trend inflation on model dynamics. For instance, the standard deviation of consumption grows from 3.13% quarterly standard deviation to 104.67 % when moving from the stability to the instability region by increasing the response to the real side of the economy in the Taylor rule. Moments for case C2 demonstrate that even when the NK model is characterized only by mild non-linearities (no trend inflation, no decreasing returns to scale), standard Taylor rule specifications can easily fall outside the stability region.

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[18]Case C1 is stable and moments of third order and global solution coincide.

Table 3: Model-Based Unconditional Moments: Scenarios B and C (B1-B2, C2)

|	case B1: $\phi_\pi = 2.5, \phi_Y = 0.$			case B2: $\phi_\pi = 2.5, \phi_Y = 0.5$		case C2: $\phi_\pi = 1.53, \phi_Y = 0.77$	
	1st order	3rd order	global	1st order	3rd order	1st order	3rd order
Moments of macroeconomic variables							
$\text{std}(C)$	3.11	3.13	3.13	5.12	104.67	2.13	10.52
$\text{std}(N)$	1.05	1.06	1.06	2.21	47.91	1.00	5.54
$\text{std}(i)$	1.80	1.79	1.79	6.11	108.81	9.02	42.60
$\text{std}(\pi)$	0.72	0.72	0.72	5.01	93.33	8.60	41.12
Behavior of price dispersion and implied output loss, $\Delta_t = 100\left(1 - S^{-1}\right)$							
$\text{mean}(\Delta)$	0.94	1.01	0.54	0.97	-27.33	-0.00	1.72
$\text{std}(\Delta)$	0.35	0.37	0.37	2.44	223.17	0.00	9.01
$\text{max}(\Delta)$	1.94	2.72	2.23	7.86	99.95	0.00	52.27
$\text{min}(\Delta)$	0.01	0.48	0.01	-5.61	-3999.1	-0.00	-71.58
$\pi_t > \pi^{upper}$	0.00	0.00	0.00	0.00	26.25	0.50	23.38
$S < 1$	0.00	0.00	0.00	35.57	38.42	0.00	42.56

Note: Implied model moments for calibration from the stable and unstable but determined region. All variables are quarterly values expressed in percent, apart from inflation and interest rates, which are expressed at an annual rate. The mean, standard deviation, minimum and maximum value of price dispersion are expressed in terms of implied output loss. We report the percentage of simulation periods, in which price dispersion travels to regions of the state space that are in violation of the feasible regions. Macro moments are for consumption, hours worked, the nominal interest rate and inflation.

5. ECONOMIC INTERPRETATION - TRANSMISSION CHANNELS

In section 3 we explained that efficient resource allocation dictates that each firm produces the same amount of goods. This section discusses in detail the economic mechanism through which the non-linear model solution, together with trend inflation and decreasing returns-to-scale, amplify economic shocks and lead to excessive model dynamics. Let, in the following, variables carrying an asterisk denote prices and quantities of a firm that, in period t, is allowed to re-set its price optimally. Let variables without asterisk denote aggregate, economy-wide variables, that include firms that are not allowed to re-set their price in the current period and are stuck with prices from the past.

Figure 5 plots the distribution of prices of optimizing firms (P_t^*) relative to aggregate

Figure 5: Simulated Distribution of Price-adjustment Gap
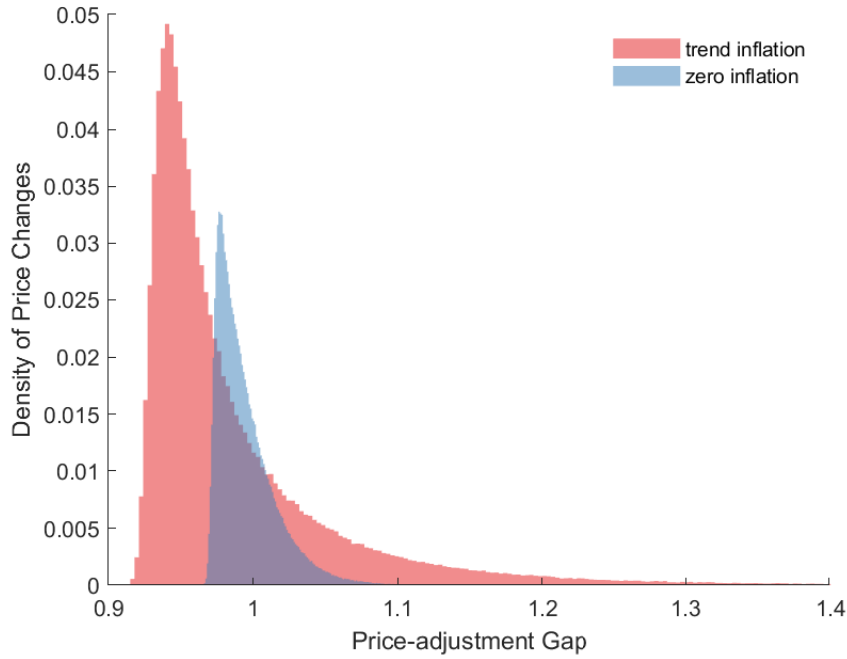
Note: The shaded areas plot the simulated distribution of optimal relative prices, $p_t^* = \frac{P_t^*}{P_t}$, which captures the changes in the prices of the optimizing firms relative to aggregate price in the economy at every period with (red) and without (blue) trend inflation.

price (P_t) and visualizes that in a non-linear Calvo pricing model firms with many different prices co-exist.[19] Trend inflation adds a drift into the evolution of prices and, thus, drives the distribution of optimal prices, P_t^*, further apart from the average price index P_t. Aggregate output in the New Keynesian model is driven by the downward sloping demand for product variety, $Y_t(i)$. Firms always adjust their supply to meet demand for good i given their price, $P_t(i)$. As only a fraction of firms are allowed to re-set their prices optimally, each firm produces a different amount of goods. In particular, a fraction $1 - \zeta$ is allowed to set its price; for the fraction ζ of non-optimizing firms for which $P_t(i) < P_t^*$ (respectively $P_t(i) > P_t^*$) the produced output is $Y_t(i) > Y_t^*$ (respectively $Y_t(i) < Y_t^*$).

Firms with low prices produce more than firms with high prices. In other words, the dispersion of prices across products leads to $Y_t(i) \lessgtr Y_t^*$ which implies that resources are not allocated optimally across firms. As shown in section 3, on the aggregate level this introduces a wedge, S_t^{-1}, between aggregate output and the amount of inputs, $Y_t = S_t^{-1} A_t K^\theta N_t^{1-\theta}$.[20]

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[19]The cross-sectional distribution of prices is captured by the measure of price dispersion, S_t, equation (12). The distribution of prices across time is linked to cross sectional distribution of prices because prices optimally set in time $t - 1$, P_{t-1}^* are fixed at time t with probability $1 - \zeta$.

[20]This wedge was emphasized in the literature by Ascari (2004), Ascari and Sbordone (2014)). Ascari and Sbordone (2014) discuss the steady-state implications of trend inflation, whereas our focus is on the

We disentangle the effects through which price dispersion amplifies the impact of macroeconomic shocks on prices and quantities into three channels: i) a marginal-cost channel, ii) a trend-inflation markup channel. The marginal cost channel shows how the real rigidity (DRS) amplifies the impact of the fact that firms cannot reoptimize and are stuck with prices from the past. The trend-inflation markup on the other hand highlights the effects of trend inflation on firms that can currently reoptimize and how they act in setting their new price.

We should note that these two channels are already to some extent present in the literature. Our contribution is to outline the channels' mechanism in the newly structured exposition tailored to non-linear settings. The marginal-cost channel was already mentioned in Ascari (2004). The fact that price dispersion creates a inefficiency wedge between production inputs and aggregate output is stressed in King and Wolman (1996), Schmitt-Grohe and Uribe (2006) and Ascari and Sbordone (2014). The upper bound on inflation is discussed also in Ascari and Sbordone (2014) and from numerical perspective in a non-linear setting by Andreasen and Kronborg (2020). The trend inflation mark-up channel is related to the discussion of the price adjustment gap in Ascari and Sbordone (2014).

5.1. THE MARGINAL-COST CHANNEL

We show here that, in a world with trend inflation, the subset of firms which cannot change their price and are stuck with a price from the past faces higher marginal costs, which aggravates the impact of exogenous shocks on the economy. The wedge between optimal marginal costs of a current price-resetter, MC_t^*, and aggregate marginal costs, MC_t increases with trend inflation and the curvature of the production function (i.e., the degree of DRS). This wedge is present only in a non-linear model +solution.

A firm which can change its price in period t, will set its price equal to $P_t(i) = P_t^*$, which, from equation (3) and (4), implies that $Y_t^* = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t = A_t \bar{K}^\theta N_t^{*(1-\theta)}$. This expression defines labor demand of a price changing firm as

$$N_t^* = \left(\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} \frac{Y_t}{A_t \bar{K}^\theta}\right)^{\frac{1}{1-\theta}}. \tag{16}$$

Relating the optimal labor demand N_t^* to aggregate labor demand N_t – which we ex-

dynamics, which is crucial for asset pricing.

press from the aggregate production function – delivers

$$N_t^* = \phi_{n,t} N_t \qquad \text{where} \qquad \phi_{n,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di}, \tag{17}$$

where $\phi_{n,t}$ can be interpreted as a measure of labor market inefficiency and is determined by the ratio of two price indexes.[21]

Using the wedge between labor demands, we can similarly describe the wedge between marginal costs for the price re-setting, $MC_t^* = \frac{W_t}{(1-\theta)A_t K^\theta N_t^{*-\theta}}$, and the average firm, $MC_t = S_t \frac{W_t}{(1-\theta)\left(A_t K^\theta N_t^{-\theta}\right)}$[22], as

$$MC_t^* = \phi_{mc,t} MC_t \qquad \text{where} \qquad \phi_{mc,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di}. \tag{18}$$

Average marginal costs are proportional to marginal costs of the price re-setting firm and this proportion is determined by a similar ratio of price indices as in the case of the labor market wedge.

In Appendix 2 we show that

$$\phi_n, \phi_{mc} \begin{cases} < 1 & \text{for } \pi_t > 0, \\ = 1 & \bar{P} = P_t^* = P_t(i) = P_t, \\ > 1 & \text{for } \pi_t < 0, \end{cases} \tag{19}$$

where $\bar{P}$ is the deterministic steady state of the price level. In a setting *without trend inflation* ($\bar{\pi} = 0$), the ratios involving the price adjustment gap to price dispersion in equations 17 and 18 will be smaller than one, $\phi_{n,t} < 1$, $\phi_{mc,t} < 1$, in states of the economy with positive inflation realizations, $\pi_t > 0$; equivalently it is larger than one, $\phi_{n,t} > 1$, $\phi_{mc,t} > 1$ in states in which $\pi_t < 0$. In a non-linear model solution the average value of the wedges is smaller than 1, $\phi_{n,t} < 1$, $\phi_{mc,t} < 1$, even when inflation is centered around zero at steady state. This comes from the fact that, in a non-linear world, when resources are used unequally, this results in a welfare loss. Therefore, as on average

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[21]We develop these results in the baseline setting with fixed capital, which is a case of decreasing returns to scale (DRS). The appendix develops a similar set of results for the case of variable capital or constant returns to scale (CRS).

[22]Using aggregate labor demand $N_t = \left[\frac{S_t Y_t}{A_t K^\theta}\right]^{\frac{1}{1-\theta}}$, we can calculate total costs, $W_t N_t$, from which, using the definition of marginal costs, $MC_t = \frac{\partial W_t N_t}{\partial Y_t}$, the expression in the text follows.

$S_t > 1$ and an average price adjustment gap of one, $\frac{P_t^*}{P_t} = 1$, it follows $\phi_{n,t}$, $\phi_{mc,t} < 1$ on average as well.

In the case of *positive trend inflation*, $(\bar{\pi} > 0)$, the probability of realizations of deflationary states of the world, $\pi_t < 0$, decreases, because to reach an overall negative realization of inflation the decrease in prices would need to exceed the positive steady-state inflation value. Equivalently, the likelihood of observing states of nature where $\pi_t > 0$ increases. For this reason, the values of $\phi_{n,t}$ and $\phi_{mc,t}$ will be less than one, $\phi_{n,t} < 1$ and $\phi_{mc,t} < 1$, for most of the states of the world and accordingly also move the average value of $\phi_{n,t}$ and $\phi_{mc,t}$ (more strongly) below one. The intuition is straightforward: in periods with growing prices, firms that can react and adjust their prices will be able to reflect the price growth in their profit-maximizing-price and consequently employ a lower optimal amount of labor inputs and produce with lower marginal costs. The fact that the average firm will produce at a higher marginal cost than the price optimizing firm at time t adds an additional inefficiency in the production and amplifies the real costs of price dispersion in the economy. Positive trend inflation thus amplifies the inefficiency coming from price stickiness. As firms produce under different cost conditions some of them are better equipped to accommodate exogenous (productivity) shocks. The wedge between aggregate and optimal quantities, $1/\phi_{n,t}$ and $1/\phi_{mc,t}$ non-linearly increases with decreasing-returns-to-scale, pinned down by parameter θ, and the elasticity of substitution between good varieties, ϵ. The intuition is again straightforward: to satisfy the higher demand for $Y_t(i)$, implied by $P_t(i) < P_t^*$, firms move along the concave production function to the right. In the case of a linear production function the increase in production is associated with no change in marginal costs. The presence of ϵ in the wedge $\phi_{n,t}$ and $\phi_{mc,t}$ represents the amplification that monopolistic competition adds to the inefficiency from price dispersion.

Furthermore, following from $Y_t^* = A_t \bar{K}^\theta N_t^{*1-\theta} = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t$, the ratio between output of a price re-setting firm and aggregate output is determined by the price adjustment gap which can be re-written using the aggregate price index as

$$Y_t^* = \phi_{o,t} Y_t, \qquad \text{where} \qquad \phi_{o,t} = \left[\frac{1 - \zeta \left(\Pi_t\right)^{\epsilon-1}}{1 - \zeta}\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{20}$$

It follows that,

$$Y_t \begin{cases} = Y_t^* & \text{for } \Pi_t = 1, \phi_{o,t} = 1, \\ > Y_t^* & \text{for } \Pi_t > 1, \phi_{o,t} < 1, \\ < Y_t^* & \text{for } \Pi_t < 1, \phi_{o,t} > 1 \end{cases} \tag{21}$$

In the economy with trend inflation where, on average, $\Pi_t > 1$, the average output is higher than optimal output, $Y_t^* < Y_t$. As prices are on average too low the economy produces on average too much. A negative productivity shock in this situation has therefore a much stronger impact on macroeconomic quantities. Firms, in the non-linear world with Calvo prices and trend inflation, need to meet the demand for goods and on average overproduce. If in this situation their production technology becomes, as a result of negative TFP shock, even less efficient without the possibility to adjust prices, the only option for accommodation of these adverse conditions is adjustment in quantities. It is this shock accommodation through quantities which leads to large model volatility. The next sub-section discusses the behavior of the subset of firms given the chance to adjust its price. Overproducing firms, given the chance to change their price, will need to adjust their, on average, low prices by more. This jump in adjustment leads to more volatile inflation in the economy.

5.2. TREND-INFLATION MARKUP CHANNEL

The presence of trend inflation leads firms to set their price at an additional markup over (current and future expected) marginal costs, which we denote the trend-inflation markup: a markup implied by sticky prices and elevated by trend inflation that occurs over and above the traditional markup from monopolistic competition. Trend inflation enters the firm's price decision problem, and therefore the first order condition for the optimal price represents another important channel. The price re-setting firm is forward-looking, it can foresee trend inflation and will therefore, on average, set its price above the aggregate price level (which includes non-resetting firms' prices from the past), $P_t^* > P_t$. It is because the optimal price has to equate to the present value of future marginal revenues with marginal costs,

$$\sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\epsilon-1} Y_{t+k} \left(\frac{P_t^*}{P_t}\right)^{1+\frac{\epsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\frac{\epsilon}{1-\theta}} Y_{t+k} MC_{t+k}(i). \quad (22)$$

The trend growth in prices increases both firms' costs of production and revenues from output sold. Nonetheless, nominal marginal costs (the expression in the infinite sum on the right hand side of equation (22)) grow at a faster rate than nominal revenues (the left hand side of equation (22)).[23] So, to keep the equality of marginal revenues with

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[23]As Π goes up, the right hand side of equation (22) grows faster – at rate $\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}$ – than the left hand side – which grows by $\Pi_{t+k}^{\epsilon-1}$.

marginal cost in present value terms, the price setting firm must set P_t^* above P_t.[24] The difference between the rate of growth in marginal cost and marginal revenue shapes the firm's markup over (present and future) marginal costs. Equation (23) defines the price adjustment gap, which depends on the weighted average of the firm's current and expected future real marginal costs:

$$\left(\frac{P_t^*}{P_t}\right)^{1+\frac{\epsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1}\sum_{k=0}^{\infty}\Upsilon_{t+k}MC_{t+k}(i), \quad \text{where} \quad \Upsilon_{t+k} = \frac{\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}Y_{t+k}}{\sum_{k=0}^{\infty}\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\epsilon-1}Y_{t+k}}.$$
(23)

Ascari and Sbordone (2014) show that the mark-up, Υ_{t+k}, increases with inflation and, thus, as trend inflation increases, the firm's trend-inflation markup amplifies the distortion implied by monopolistic competition. The rise in Υ_{t+k} means that firms put more weight on marginal costs far in the future compared to current marginal costs[25].

5.3. PRICE-INFLATION SPIRAL IN THE ECONOMY WITH TREND INFLATION

We shortly elaborate on the economic intuition behind the amplified effect of price-inflation spiral in the presence of trend inflation. The price-inflation spiral is best understood from the non-linear first-order difference equation governing the dynamics of price dispersion, which we presented in equation (14). High inflation, Π_t, at time t, leads to high dispersion of prices across product varieties, S_t, as only some firms can adjust their prices. The high dispersion of prices, S_t, in turn calls for a strong adjustment in prices and thus high inflation in the next period, Π_{t+1}. These reinforcing dynamics continue, and hence start the price-inflation spiral.

The price inflation-spiral shows that inflation depends on the already prevailing level of price dispersion. If prices are widely dispersed in the economy the dispersion will become self-reinforcing and lead to an inflation spiral. The higher the steady state of inflation the higher is the probability that exogenous inflationary shocks will trigger this price-inflation spiral. Trend inflation spreads out the distribution of prices as those firms which cannot change their low prices are left behind further and further from the optimal price as the price level grows. When these firms can finally change their price

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[24]Note that the trend-inflation markup channel is amplified through the parameter determining the strength of DRS, θ, which further widens the gap between costs and revenues.

[25]Ascari and Sbordone (2014) shows that overly forward looking agents de-anchor inflation expectations and decrease the determinacy region.

they create large inflation and, in fact, instead of decreasing the dispersion of prices they further contribute to the widening in the distribution of prices if their price adjustment is too big.

6. AN APPLICATION TO MACRO-FINANCE

6.1. GLOBALLY SOLVED MODELS

The inflation threshold determining the stability region is a global property of the model with Calvo pricing independent of the solution method applied to the model. There are several papers in the (optimal) monetary policy literature using the framework we describe solved by projection methods (i.e. Anderson, Kim, and Yun (2010), Leith and Liu (2014) and Miao and Ngo (2019)). None of these three papers report issues with stability. The model structure of these papers is very similar. Whereas Anderson, Kim, and Yun (2010) and Leith and Liu (2014) solve for optimal monetary policy Miao and Ngo (2019) use a calibrated Taylor rule.

Figure 6: Determinacy vs. Stability region in Miao and Ngo (2019)
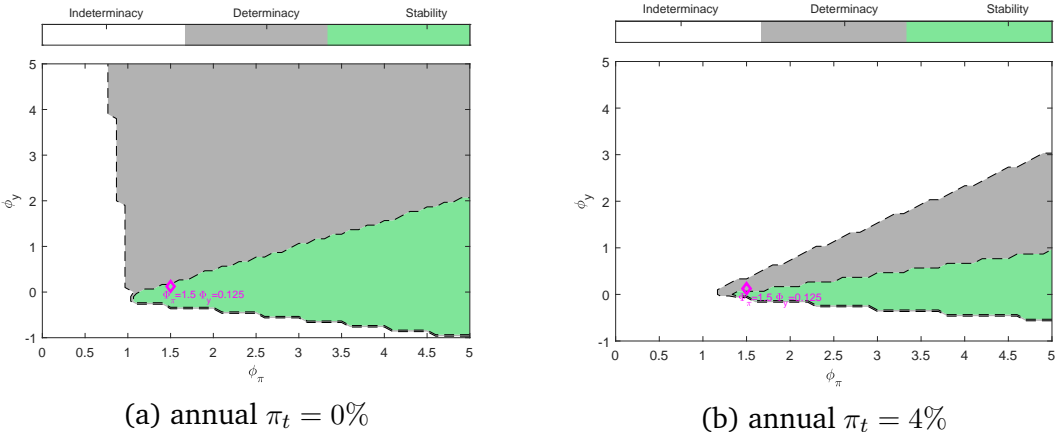
(a) annual $\pi_t = 0\%$ (b) annual $\pi_t = 4\%$

Note: The left panel shows stability region based on zero trend inflation. The stability region denotes the region where $S_{t-1} < S_t$ over the 1st to 99th percentile of the simulated bivariate distribution state variables, denoted $P_{1,99}(S)$ and $P_{1,99}(A)$. The region 95 % Conf. does the same for $P_{5,95}(S)$ and $P_{5,95}(A)$. The diamond indicates the model calibration by Miao and Ngo (2021). The right panel plots stability region with 4% annual trend inflation.

We replicate these papers and demonstrate on the example of Miao and Ngo (2019) in Figure 6a that the reason for the stability of global solution is that these models are calibrated to be in the stability region. Nevertheless, setting the coefficient on output gap only slightly higher would set the model in Figure 6a already to the explosive path.

By assuming non-zero trend inflation in figure 6b the stability region[26] narrows and the calibration of Miao and Ngo (2019) does fall outside the stability region despite the linear solution being still determined.

6.2. MACROFINANCE

We now present an application that illustrates that the mechanism described in the previous sections is relevant and quantitatively pronounced in widely used modeling framework of term structure of interest rates. We adopt as our example application the model of Rudebusch and Swanson (2012)[27] (RS), which has become the state-of-the-art model in the macro asset pricing literature. We choose this model because it represents a central model in the literature which is doomed to rely on non-linear solution methods and Calvo pricing. In particular, the RS model explains the economics of risk premia and precautionary saving effects which are inherently non-linear phenomena. In addition, price dispersion implied by Calvo pricing has been stressed in the macro asset pricing literature as an important driver explaining risk premia. In a New Keynesian model with Calvo prices, Swanson (2015) uses the price dispersion index as a source of conditional volatility in consumption to generate sizable term premia in bond prices. Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018) argue, in a mid-scale model with Calvo contracts, that their model can match the volatility of term premia by using trend inflation to amplify the non-linearities in the price dispersion index.

Price dispersion therefore plays an important role in the macro-asset pricing literature to explain the behavior of risk. A negative TFP shock received at the time with high dispersion in prices might lead to a price-inflation spiral which in turn leads to high conditional volatility of the stochastic discount factor and, thus, a high risk premium.

The literature on asset pricing in New Keynesian models with Calvo contracts resorts dominantly to third-order approximate solutions to the policy functions. The third-order approximation is justified by the fact that only third and higher-order approximations can capture positive and volatile risk premia (i.e. van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2012) and Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018).

Figure 7 shows the stability and determinacy region for RS model. The left panel 7a

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[26]Ascari and Ropele (2009) shows that trend inflation narrows the determinacy region.

[27]In the interest of space we do not review here the model of Rudebusch and Swanson (2012) and refer the reader to Appendix C for the summary of equilibrium conditions and to the original article. The online appendix on authors web-page provides details on the model derivation.

Figure 7: Determinacy vs. Stability region in Rudebusch and Swanson (2012)
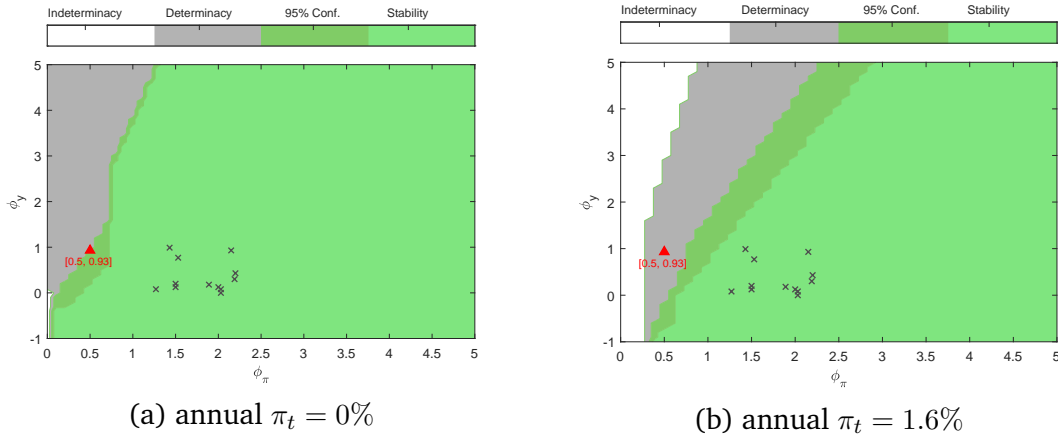
(a) annual $\pi_t = 0\%$ (b) annual $\pi_t = 1.6\%$

Note: The left panel shows stability region based on zero trend inflation. The stability region denotes the region where $S_{t-1} < S_t$ over the 1st to 99th percentile of the simulated bivariate distribution state variables., denoted $P_{1,99}(S)$ and $P_{1,99}(A)$. The region 95 % Conf. does the same for $P_{5,95}(S)$ and $P_{5,95}(A)$. The diamond indicates the calibration by Rudebusch and Swanson (2012). The right panel plots stability region with 1.6% annual trend inflation.

show the original calibration in Table 3 of RS. It shows that the model is calibrated on the boarder of the stability region and that in the simulation 5 percent of observations feature $S_{t-1} > S_t$. The right panel 7b demonstrates that already for small inflation of 1.6% annually the RS benchmark calibration features unanchored inflation expectations and the model feature explosive moments despite the fact that it is lying in the determinacy region.

Table 4 follows RS and reports the moments of macro and finance variables, where $NTP^{(40)}$ denotes the 10-year nominal term premium, and $i^{(40)}$ denotes the 10-year bond yield. In addition to RS we also report, consistent with Table 2, the statistics describing the loss due to price dispersion, Δ_t, and the statistics on the violations of bounds.

The first column of Table 4 provides targeted empirical moments. The subsequent columns are model-based unconditional moments, calculated from third-order approximated and pruned model simulations of various RS model versions, where we highlight modeling features and channels discussed previously. Column RS gives simulated moments from the original baseline RS model (with zero trend inflation), using the RS best fit calibration (which we summarize in Table 7) from Table 3 of their paper. All the remaining columns assume positive annual trend inflation, $\bar{\pi} = 1.6\%$.

Column 'RS with trend inflation' shows that as soon we add even moderate trend inflation into the RS model the simulation is becomes unstable; both lower (51.5% of periods) and upper bound (7.46% of periods) on S_t are violated and for instance, the standard deviation of inflation reaches an enormous 32.92%. Column 'RS with linear-

in-labor' production function highlights the quantitative impact of the marginal cost channel driven by the degree of real rigidity in the model. Both macro and finance moments show that the absence of the real rigidity (by using a linear-in-labor production function) significantly tames the model dynamics. For instance, the volatility of consumption drops from 10.59 to 0.54, and the model also rarely violates the bounds on S_t.

Column 'RS with $2 \times \phi_\pi$' shows that doubling the weight on inflation in the Taylor rule helps to anchor inflation expectations which lowers the trend inflation markup. The increased reaction of the central bank to inflation leads to lower price changes of optimizing firms. As price-resetting firms adjust their prices by smaller magnitudes the trend inflation markup drops. We see from this column that, although the dynamics of macro variables are much improved and the model is able to generate surprisingly large nominal term premia ($NTP^{(40)} = 1.66$) without compromising the macro moments[28], the numerical properties related to price dispersion remain compromised (45.68% of periods below the bound on S_t) and the match of the dynamics of output loss implied by price dispersion remains poor (in some periods up to 26.14% loss of quarterly output).

Column 'RS with indexation' substantially decreases the occurrence of the price-inflation spiral, by introducing full inflation indexation into the model with trend inflation. In this case the upper bound on inflation is violated only in 0.11% of periods as opposed to 7.46% and the model dynamics is comparable to original RS model. We identify indexation therefore as one of the modeling remedies attenuating the amplification effects of trend inflation in non-linear Calvo settings. In the last two columns we report the most common alternative pricing mechanisms found in the literature: Rotemberg quadratic price adjustment costs and the staggered Taylor pricing mechanisms. Results in these columns show that these alternative methods of modeling nominal rigidities match equally well both the macro and finance stylized facts, but do not generate the counterfactual size of the cost of inflation.

We therefore confirm the results from the analysis conducted in the standard New Keynesian DSGE model. Price dispersion drives both the finance and macroeconomic fluctuation. Anchoring inflation expectations, destabilized by the price dispersion inflation spiral, is a necessary condition for accuracy of perturbation solution and existence of global solution.

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[28]This has been shown to be challenging in the literature, see, e.g., Rudenbusch and Swanson (2008).

7. CONCLUDING REMARKS

Applications in modern macroeconomics increasingly require non-linear solution methods, among them models with asset pricing implications. Our paper studies the consequences of the widely used Calvo pricing mechanism in a non-linear world. We show, in a standard New Keynesian and later in a workhorse macro-finance model, that in a non-linear setting a price dispersion driven inflation spiral governs business cycle fluctuations. This provides a new challenge for the central bank to anchor inflation expectations. We introduce the concept of the stability region as a non-linear counterpart to the determinacy region. The price-inflation spiral strongly compromises model dynamics of all other variables and the non-linear model solution outside of the stability region implies counterfactual second moments. In addition, the explosive character of price dispersion implies that no global solution can be computed outside of the stability region.

Perturbation methods with pruning allow us to solve the model even outside of the stability region. This is, however, at the cost of large approximation errors. Should this finding imply that macroeconomic models abstract from models with Calvo pricing altogether? In our opinion not necessarily. One can understand the Calvo pricing mechanism purely as a modeling device of nominal rigidities without having a solid microeconomic foundation, which serves its purpose well in a linearized setting. As long as the model delivers realistic predictions (matches moments and impulse response functions) about the economy under scrutiny, it can be useful to explain observable data. A vast amount of literature proved that in linearized models Calvo pricing works well to explain second moments and match impulse responses to those found in VAR estimates. For these reasons, we believe that Calvo pricing, even if subject to substantial approximation errors and not having support in microeconomic evidence, is useful in linear models. Using Calvo pricing in non-linear setting, however, delivers, in many model setups, a counterfactual cost of inflation and extremely large dispersion of prices in the economy which are not in line with observed evidence. In non-linear Calvo pricing models with trend inflation empirically estimated Taylor rule coefficients are not sufficient to stabilize the economy.

Table 4: Empirical and Model-Based Unconditional Moments

Unconditional Moment	US data 1961-2007	RS replication	RS with trend inflation	RS with linear $N_t(i)$ in PF	RS with $2 \times \phi_\pi$ on π_t	RS indexation	RS Rotemberg pricing	RS Taylor pricing
		$\bar{\pi} = 0$	$\bar{\pi} > 0$	$\bar{\pi} > 0$	$\bar{\pi} > 0$	$\bar{\pi} > 0$	$\bar{\pi} > 0$	$\bar{\pi} > 0$
Moments of macroeconomic variables								
std(C)	0.83	0.88	10.59	0.54	1.20	0.89	0.50	0.70
std(N)	1.71	2.51	30.01	1.42	2.91	2.50	1.50	1.82
std(i)	2.71	3.41	38.69	2.49	2.97	3.43	2.13	3.07
std(π)	2.52	3.01	32.92	2.35	1.98	2.98	2.14	2.78
Moments of finance variables								
std($i^{(40)}$)	2.41	3.94	4.79	5.42	4.16	5.24	4.03	4.34
mean($NTP^{(40)}$)	1.06	0.91	3.68	0.65	1.66	1.08	0.84	1.47
std($NTP^{(40)}$)	0.54	0.42	7.04	0.11	0.98	0.55	0.36	0.13
Behavior of price dispersion and implied output loss, $\Delta_t = 100\left(1 - S^{-1}\right)$								
violation of $\pi_t \leq \pi^{upper}$	-	0.05	7.46	0.00	0.06	0.11	0.00	0.00
violation of $S \geq 1$	-	8.77	51.50	0.08	45.68	12.95	0.00	19.10
mean(Δ)	0.4	0.55	-1.24	0.29	0.45	0.52	0.76	0.10
std(Δ)	-	0.99	61.87	0.35	1.73	1.03	0.88	0.15
max(Δ)	-	17.45	98.55	4.94	26.14	41.00	0.00	4.25
min(Δ)	-	-7.48	-6227.45	-0.42	-14.80	-7.17	0.00	-0.12

Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. All models feature trend inflation, $\bar{\pi} = 1.6\%$, but the RS replication. The column with firm i's linear-in-labor production function shows the quantitative effect of the marginal cost channel. The column with $2 \times \phi_\pi$ shows the effect of the reduced trend markup channel. The column with indexation shows the effects of decreased occurrences of the price-inflation spiral. The last two columns show that alternative pricing mechanism do a better job at matching stylized facts.

A. GLOBAL APPROXIMATION METHOD

The obtain a global solution of our benchmark model we adopt a time iteration method as developed by Coleman (1990, 1991), where we solve for the approximate non-linear policy functions in an iterative algorithm. The non-linear model equations are summarized in Table 5. Below we briefly outline the steps of the algorithm used:

- In the following, denote $t+1$ variables and non-predetermined state variables at time t with a prime, e.g. $C = C_t$, $C' = C_{t+1}$, $S' = S_t$, $S = S_{t-1}$, etc. we construct a two-dimensional grid over the model's state variables at time t, that is, over $S = S_{t-1}$, $A = A_t$ consisting of $n_s n_A$ grid points. The grid in dimension of S ranges from 1 to $S_{\max}$, where we obtain $S_{\max}$ from the relevant region of price dispersion that is traveled in a simulation of the third-order perturbation solution. The gridpoints in dimension of A are obtained by discretizing the continuous AR process with the Rouwenhorst (1995) method, which, for persistent processes, has been shown to yield more accurate approximations than, e.g., the conventional discretization by Tauchen and Hussey (1991) (c.f. Kopecky and Suen (2010)). The number of gridpoints is chosen to be $n_s = 61$ and $n_A = 11$.

- Set counter equal to 1. Using the perturbation solution make initial guesses on the consumption policy, inflation policy, and the policy functions of the two auxiliary variables in the Calvo pricing block.

- Having guesses $C(S, A)$, $\Pi(S, A)$, $K(S, A)$, $F(S, A)$ in hand, we can write the model's conditional expectations, $CE_{EE} = \left\{ \frac{C_{t+1}^{-\varphi}}{\Pi_{t+1}} \right\}$, $CE_K = \left\{ C_{t+1}^{-\varphi} \Pi_{t+1}^{\frac{\epsilon}{1-\theta}} K_{t+1} \right\}$ and $CE_F = \left\{ C_{t+1}^{-\varphi} \Pi_{t+1}^{\epsilon-1} F_{t+1} \right\}$, as

$$CE_{EE}(S', A) = \sum_{A'} \pi(A'|A) \frac{[C'(S', A')]^{-\varphi}}{\Pi'(S', A')}, \tag{24}$$

$$CE_K(S', A) = \sum_{A'} \pi(A'|A) [C'(S', A')]^{-\varphi} [\Pi'(S', A')]^{\frac{\epsilon}{1-\theta}} K'(S', A'), \tag{25}$$

$$CE_F(S', A) = \sum_{A'} \pi(A'|A) [C'(S', A')]^{-\varphi} [\Pi'(S', A')]^{\epsilon-1} F'(S', A'), \tag{26}$$

where we integrate out the expectations over future A' using the Markov transition matrix $\pi(A'|A)$.

- Using the guesses for the conditional expectations and given the time t values of grid variables, (S, A), the model's endogenous variables S', C, N, $(1 + i_t)$, Π, p_t^*, K, F, W, MC, Y can be computed by solving the system given in Table 5 with a

Table 5: System of model equations, baseline NK model

(NK1):	$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\varphi} \frac{(1+i_t)}{\Pi_{t+1}} \right\}$
(NK2):	$K_t = \frac{\epsilon}{\epsilon-1} MC_t Y_t + \beta\zeta \left(\frac{C_{t+1}}{C_t} \right)^{-\varphi} \Pi_{t+1}^{\frac{\epsilon}{1-\theta}} K_{t+1}$
(NK3):	$F_t = Y_t + \beta\zeta \left(\frac{C_{t+1}}{C_t} \right)^{-\varphi} \Pi_{t+1}^{\epsilon-1} F_{t+1}$
(NK4):	$\chi_0 (1-N_t)^{-\chi} C_t^{\varphi} = W_t$
(NK5):	$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{K_t}{F_t}$
(NK6):	$MC_t = \frac{1}{1-\theta} \overline{K}^{\frac{\theta}{1-\theta}} \frac{W_t}{A_t} \left(\frac{Y_t}{A_t} \right)^{\frac{\theta}{1-\theta}}$
(NK7):	$S_t Y_t = A_t \bar{K}^{\theta} (N_t)^{1-\theta}$
(NK8):	$S_t^{\frac{1}{1-\theta}} = (1-\zeta)(p_t^*)^{\frac{-\epsilon}{1-\theta}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}$
(NK9):	$\Pi_t^{1-\epsilon} = (1-\zeta)(p_t^* \Pi_t)^{1-\epsilon} + \zeta$
(NK10):	$Y_t = C_t + \bar{I} + G$
(NK11):	$\left(\frac{1+i_t}{1+i} \right) = \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_Y}$
(NK12):	$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}$

non-linear equations solver, at each gridpoint (S, A). Price dispersion next period, S', is part of the variables that need to be solved for, and is also the argument in evaluating expressions 24 at value S'. We use linear interpolation (extrapolation) when evaluating expressions $CE_{EE}(S', A)$, $CE_K(S', A)$, and $CE_F(S', A)$ at value S' off of (outside of) the exogenously specified S-grid.

- Having in hand solutions for $C(S, A)$, $\Pi(S, A)$, $K(S, A)$, $F(S, A)$ provides us with new policy function guesses to compute an updated guess of the conditional expectation terms, 24. We set counter = counter+1 and return to step 3. The above steps are repeated until convergence is achieved. The convergence criterium is specified as the maximum absolute value of the difference from policy function guesses in two consecutive iterations to be smaller than 1e-6.

A.1. NO GLOBAL SOLUTION OUTSIDE THE STABILITY REGION

The reason why no global solution can be obtained whenever the model falls outside of the stability region is that for some values of the exogenous state A_t, the relevant state space in the S_{t-1} dimension becomes unbounded, so that no specified grid over S_{t-1} is large enough. In particular, this is the case for low values of the exogenous state A_t which imply (high) inflation rates that exceed the level of threshold inflation,

i.e., $\Pi_t > \Pi_t^{threshold}$, so that (as explained in section 3.3) $S_t > S_{t-1}$. This means that whatever maximum value of the S_{t-1}-grid one takes, the value of S_t falls outside of the specified grid. Section 3.2 discussed the existence of an upper bound, so that one may be tempted to think that it could be of guidance in specifying a maximum grid in the S_{t-1} dimension. However, unfortunately, the upper bound is solely an upper bound on inflation, Π^{upper}, and the fact that $\Pi^{upper} > \Pi_t^{threshold}$ continues to imply that because $S_t > S_{t-1}$ no 'upper bound' on price dispersion can be specified. In the practise of the time iteration algorithm described above, this problem implies that the algorithm is unable to converge. This is because policy function guesses $CE_{EE}(S', A)$, $CE_K(S', A)$ and $CE_F(S', A)$ need to be evaluated at S', when solving the exact non-linear system of model equations; and S' falls beyond the maximum value of S and thus outside the prespecified (S-A) grid. As a result, the very nature that price dispersion is explosive in some regions ($S_t > S_{t-1}$), implies that a current guess for the policy function needs to be evaluated outside of the grid over which it is computed, i.e. one needs to extrapolate. Because extrapolation is likely to be inexact (particularly for this region of high non-linearity), convergence cannot be achieved. As explained, extending the grid is not an option to resolve these issues.

We have experimented with a number of alternatives to obtain a globally approximated model solution. This includes turning to a version of our time iteration/ policy function iteration algorithm that adopts an endogenous grid, specifying the grid over S_t and A_t and solving for S_{t-1} as part of the variables in the non-linear equations solver step. Alternatively we have also explored other, alternative global solution methods. As a robustness check we have explored using a projection method (Chebyshev collocation) as in, e.g., Anderson, Kim and Yun (2010). Similarly, we explored the 'extended perturbation method' of Andreasen and Kronborg (2017). Whenever the solution falls into the stability region, all these methods imply almost identical policy functions and simulated model moments are virtually identical; a global solution for parameterization cases falling outside of the stability region can, however, not be obtained.

B.   POLICY FUNCTIONS OF THE SIMPLE NK MODEL

This section deepens the intuition for unstable price dispersion dynamics by looking at the policy function plots for the parameterization cases A1-C2 from section 4. Policy functions in our simple NK example model depend on two state variables, on endogenous state S_{t-1}, and on exogenous state A_t. Before doing so, Figure 8, however, plots

price dispersion, S_t, as a function of both arguments it depends on according to its true non-linear law of motion, equation (12), which is past price dispersion, $S_{t-1} \geq 1$, and on inflation, Π_t. In a model solution Π_t is itself a function of state variables S_{t-1}, A_t, so that there is a correspondence between the plot of price dispersion as a function of S_{t-1}, Π_t and as a function of S_{t-1}, A_t. The advantage of the former is that it is not dependent on a model solution; Figure 8 here does not need to rely on any approximation method at all: what is plotted is the true non-linear dynamics, as given by the non-linear law of motion of price dispersion.[29] It can clearly be seen that, for a large region of Π_t price dispersion is an increasing function of past price dispersion, so that dynamics are explosive, confirming that unstable dynamics are a true feature of the model. We know from section 3 that this is the case whenever $\Pi_t > \Pi^{threshold}$, then $S_t > S_{t-1}$. For easier visibility this region of S_t is highlighted in red.

Figure 8: Price Dispersion Dynamics According to the True Non-linear Law of Motion
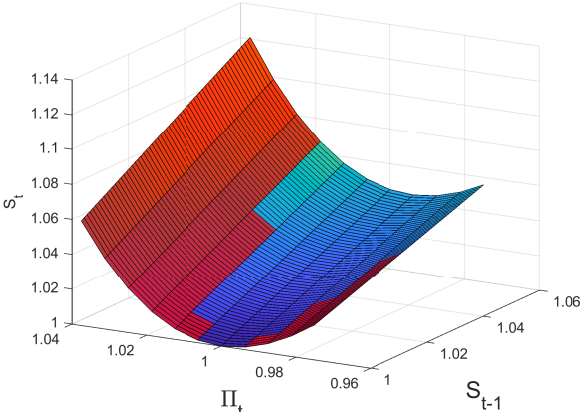
Note: Price dispersion S_t plotted as implied by the exact non-linear law of motion (equation (14)) in dependance of S_{t-1} and Π_t. The red part of the surface indicates the region where price dispersion dynamics are unstable, i.e. where $\Pi_t > \Pi^{threshold}$ and $S_t > S_{t-1}$.

Figure 9 presents policy function plots for price dispersion, S_t, for parameterization cases A1-A3 (benchmark scenario), for cases B1-B2 (scenarios with positive trend inflation) and cases C1-C2 (scenarios with zero trend inflation and no decreasing returns to scale), respectively. Policy functions for S_t, are now plotted as a function of the model's fundamental state variables, which are past price dispersion, S_{t-1}, and total factor technology, A_t. Also included in the panels is a transparent surface which plots the S-Grid on itself, i.e. providing a surface with slope equal to one in the S_{t-1}-dimension. Similar to Figure 8 regions with increasing price dispersion dynamics, $S_t > S_{t-1}$, are colored in red. As discussed in the main text, scenarios A1, B1, and C1, are all parameteriza-

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[29]The true non-linear law of motion depends on three model parameters, θ, ϵ, and ζ. These are chosen as in the benchmark parameterization (case A).

tion examples that fall well into the stability region. The policy function plots deliver the same insight: policy function S_t falls below the transparent surface indicating the instability threshold, thus price dispersion dynamics are stable over the entire relevant state space to which price dispersion travels. Also, we observe that the policy function obtained from the third order solution is virtually identical to the one obtained from the global solution, mirroring the simulation results in the main text (section 4). This drastically changes when moving to parameterization scenarios A2-A3, B2, or C2, for which we know that they lie outside of the stability region. Here, we observe that – highlighted in red– price dispersion dynamics are unstable, $S_t > S_{t-1}$, for the region of the state space where A_t is low (e.g., resulting from a negative supply shock), which is associated with high levels of inflation, leading to high price dispersion, and vice versa (the price-inflation spiral). The dynamic behavior visualized in the policy function plots is Figure reminiscent of the unstable region highlighted in 8, the difference being that the policy function plots recognize that inflation is itself a function of the model's fundamental state variables, S_{t-1} and A_t. For parameterization cases A2-A3, B2, and C2, because of the wildly unstable price dispersion dynamics no global solution can be obtained, as explained in section A.

C.   RUDEBUSCH AND SWANSON (RS) MODEL

This appendix gives a summary of the equilibrium conditions of Rudebusch and Swanson (2012). Table 6 summarizes the system of equations of the Rudebusch Swanson model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables) defined as $c_t = \frac{C_t}{Z_t}$, $y_t = \frac{Y_t}{Z_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t Z_t}$, $p_t^* = \frac{P_t^*(i)}{P_t}$, $mc_t(i) = \frac{MC_t(i)}{P_t}$, $y_t = \frac{Y_t}{Z_t}$, $\mu_t = \frac{Z_t}{Z_{t-1}}$. The best fit calibration of the RS model based on their Table 3 is summarized in Table 7. In this setting, model dynamics are driven by three types of shocks, stationary technology shocks, government spending shocks, and inflation target shocks (in particular, there are no trend productivity shocks, so that $\mu_t = \frac{Z_t}{Z_{t-1}} = \mu$ is constant).
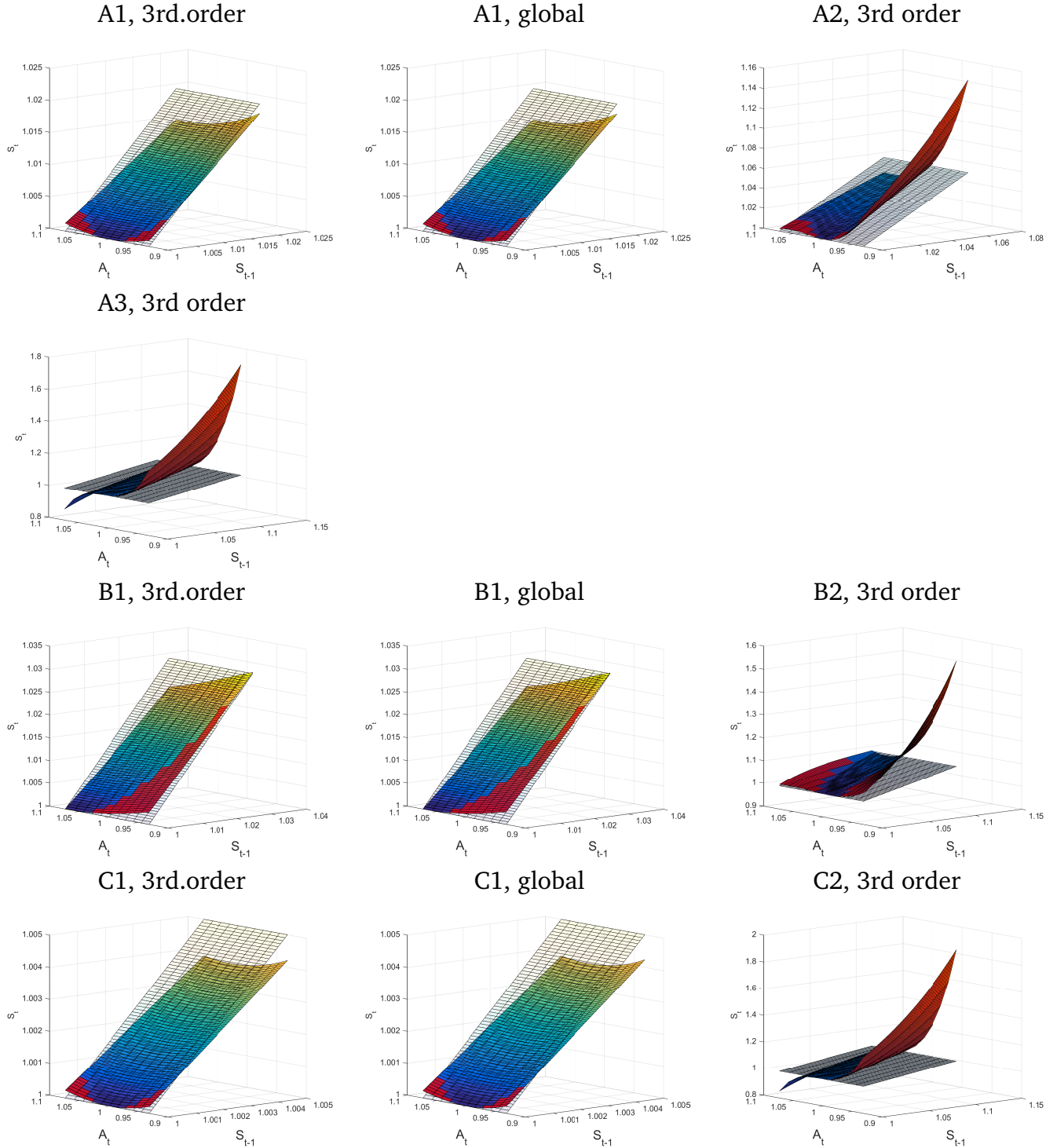
C.1.   BOND PRICING

The price of a default-free n-period zero coupon bond that pays \$1 at maturity can be described recursively as:

$$p_t^{(n)} = E_t\{Q_{t,t+1} p_{t+1}^{(n-1)}\}$$

where $Q_{t,t+1}$ is the stochastic discount factor; $p_t^{(n)}$ denotes the price of the bond at time

Figure 9: Policy functions of price dispersion for cases A1-A3, B1-B2, and C1-C2

Note: Policy functions for price dispersion S_t for scenarios A1-A3, B1-B2, and C1-C2. Policy function S_t is plotted in dependance of state variables S_{t-1} and A_t. The transparent surface included plots the S_{t-1} grid on itself for each given value of A_t, i.e. having slope 1 in S_{t-1} dimension. The red part of the policy function surface indicates the region where price dispersion dynamics are unstable, i.e. where $\Pi_t > \Pi^{threshold}$ and $S_t > S_{t-1}$.

Table 6: System of model equations, Rudebusch Swanson model

(RS1):	$V_t = \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}}$
(RS2):	$Q_{t-1,t} = \mu_t^{-\gamma} \left(\frac{(V_t\mu_t^{1-\gamma})}{[E_{t-1}(V_t\mu_t^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \left(\frac{c_t}{c_{t-1}} \right)^{-\varphi}$
(RS3):	$\chi_0(1-N_t)^{-\chi}c_t^{\varphi} = w_t$
(RS4):	$1 = \beta E_t \left\{ Q_{t,t+1} \frac{(1+i_t)}{\Pi_{t+1}} \right\}$
(RS5):	$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{aux_{1t}}{aux_{2t}}$
(RS6):	$aux_{1t} = \frac{\epsilon}{\epsilon-1}mc_t y_t + \beta\zeta Q_{t,t+1}\Pi_{t+1}^{\frac{\epsilon}{1-\theta}} aux_{1t+1}$
(RS7):	$aux_{2t} = y_t + \beta\zeta Q_{t,t+1}\Pi_{t+1}^{\epsilon-1} aux_{2t+1}$
(RS8):	$S_t Y_t = A_t \bar{K}^\theta (N_t)^{1-\theta}$
(RS9):	$S_t^{\frac{1}{1-\theta}} = (1-\zeta)(p_t^*)^{\frac{-\epsilon}{1-\theta}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}$
(RS10):	$\Pi_t^{1-\epsilon} = (1-\zeta)(p_t^*\Pi_t)^{1-\epsilon} + \zeta$
(RS11):	$MC_t = \frac{1}{1-\theta}\overline{K}^{\frac{\theta}{1-\theta}} \frac{W_t}{A_t} \left(\frac{y_t}{A_t} \right)^{\frac{\theta}{1-\theta}}$
(RS12):	$y_t = c_t + \overline{I} + g_t$
(RS13):	$4i_t = 4\rho_i i_{t-1} + (1-\rho_i) \left[4(\bar{i}-\bar{\pi}) + (\pi_t^{avg}) + \phi_\pi(4(\pi_t^{avg}) - (\pi_t^*)) + \phi_Y \left(\frac{\mu_t Y_t}{\bar{\mu}\bar{Y}} - 1 \right) \right]$
(RS14):	$\pi_t^* = (1-\rho_{\pi^*}) 4\pi_t^{avg} + \rho_{\pi^*}\pi_{t-1}^* + \zeta_{\pi^*}(4\pi_t^{avg} - \pi_t^*) + \sigma_{\pi^*}\varepsilon_{\pi^*,t}$
(RS15):	$\pi_t^{avg} = \theta_{\pi^{avg}}\pi_{t-1}^{avg} + (1-\theta_{\pi^{avg}})\pi_t$
(RS16):	$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}$
(RS17):	$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G$

t with maturity n, and $p_t^{(0)} \equiv 1$, i.e. the time-t price of \$1 delivered at time t is \$1.

The price of a bond can be decomposed into the risk-neutral price and a term premium. The risk-neutral bond price, $\hat{p}_t^{(n)}$, is defined through the expectations hypothesis of the term structure:

$$\hat{p}_t^{(n)} = e^{-i_t} E_t \hat{p}_{t+1}^{(n-1)} \qquad NTP_{n,t} = i_t^{(n)} - \frac{1}{n} \sum_{j=0}^{n-1} E_t[i_{t+j}] \tag{27}$$

where the bond price is discounted by one period rate, i_t. The price of bond reflects, in this case, expectations about inflation and economic activity but abstracts from the uncertainty surrounding the expectations[30]. The continuously compounded yield to maturity of the n-period zero-coupon bond can be written as $i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}$, (see for instance Cochrane (2001)). The term premium, $NTP_{n,t}$ is defined as the difference between the yield expected by the risk-averse investor ($i_t^{(n)}$) minus the yield awaited by the risk-neutral investor ($\hat{i}_t^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} E_t[i_{t+j}]$).

C.2. AGGREGATION

Here we describe in detail the aggregation across the i-firms in case of decreasing return to scale (i.e., the model version with fixed capital, as in the original model specification of RS), and constant return to scale production function (i.e., with variable capital).

C.2.1 Aggregate Price Index

The aggregate price index $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i) di \right]^{\frac{1}{1-\epsilon}}$ can be written using the Calvo result as,

$$\frac{P_t^*}{P_t} = \left[\frac{1 - \zeta (\Pi_t)^{\epsilon-1}}{1 - \zeta} \right]^{\frac{1}{1-\epsilon}}, \tag{28}$$

C.2.2 Aggregation for DRS

The production function of intermediate firm i is given by $Y_t(i) = A_t K^{\theta} N_t^{1-\theta}(i)$. Using this, plug in for $Y_t(i)$ into the demand for variety i, equation 3, solve for $N_t(i)$ and

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[30]This can be understood as the bond price of a 10-year bond expected by the so-called risk-neutral investor who is rolling over a one-period investment for 10 years.

integrate over all varieties i. Since workers are all the same the aggregation of hours worked is $N_t = \int_0^1 N_t(i)di$. Aggregation thus delivers,

$$N_t = \left(\frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} di, \tag{29}$$

which can be re-written as

$$Y_t = S_t^{-1} A_t K_t^\theta N_t^{1-\theta}, \tag{30}$$

where variable $S_t^{\frac{1}{1-\theta}} = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$ defines price dispersion.

C.2.3 Re-setting firm vs. aggregate quantities for DRS

The demand function at time $t + k$ for the firm re-setting its price at time t is given by,

$$Y_{t+k}^* = A_{t+k}\bar{K}^\theta N_{t+k}^{*(1-\theta)} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{31}$$

where P_t^* is the optimal price of firm resetting its price at time t for the horizon k. Factor demand of the price re-setting firm, N_t^* is,

$$N_{t+k}^* = \left(\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^\theta}\right)^{\frac{1}{1-\theta}}. \tag{32}$$

The ratio of the price re-setting (equation (32)) and the aggregate firm's factor demands (29)), expressed in terms of time t quantities, is given by

$$\frac{N_t^*}{N_t} = \frac{\left(\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} \frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}}}{\left[\frac{Y_t S_t}{A_t K^\theta}\right]^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]}. \tag{33}$$

An analogous ratio can be derived for aggregate marginal cost and marginal costs of the price resetting firm. Marginal costs for the price resetting firm are,

$$MC_t^* = \frac{W_t}{(1-\theta)A_t K^\theta N_t^{-\theta}} \frac{N_t^{-\theta}}{N_t^{*-\theta}}, \tag{34}$$

Aggregate marginal cost come from $\frac{\partial W_t N_t}{\partial Y_t}$ and, using $N_t = \left[\frac{Y_t}{A_t K^\theta}\right]^{\frac{1}{1-\theta}} S_t^{\frac{1}{1-\theta}}$, delivers,

$$\frac{MC_t}{S_t} = \frac{W_t}{(1-\theta)\left(A_t K^\theta N_t^{-\theta}\right)}. \tag{35}$$

Plugging equation (33) into (34) and rearranging delivers,

$$MC_t^* = MC_t \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} \tag{36}$$

C.2.4 Aggregation for CRS

For the case of a constant returns to scale production function, where capital is variable, the cost minimization problem is given by

$$\min_{N_t(i)} W_t N_t(i) + R_t^k K_t + MC_t^r(i)\left[Y_t(i) - A_t K_t(i)^\theta N_t^{1-\theta}(i)\right], \tag{37}$$

subject to production function, $Y_t(i) = A_t K_t(i)^\theta N_t^{1-\theta}(i)$, and where $MC_t(i)$ is the multiplier associated with the constraint.

The firm's demands for labor and capital are, respectively,

$$W_t = MC_t^r(i)(1-\theta)A_t K_t(i)^\theta N_t^{-\theta}, \tag{38}$$

$$R_t^k = MC_t^r(i)A_t \theta K_t(i)^{\theta-1} N_t^{1-\theta}(i), \tag{39}$$

Plugging the factor demands into the definition of total costs, $TC_t(i) = W_t N_t(i) + R_t^k K_t(i)$ delivers,

$$TC_t(i) = [MC_t^r(i)]\,Y_t(i). \tag{40}$$

Marginal costs are defined as a change in total cost when output changes, $\frac{dTC_t(i)}{dY_t(i)} = MC_t^r(i)$, which shows that the Lagrange multiplier equals real marginal costs. From the ratio of equation (38) and equation (39) we get that,

$$\frac{1-\theta}{\theta} = \frac{W_t N_t(i)}{R_t^k K_t(i)}. \tag{41}$$

Since factor prices are common for all the firms, the ratio of $\frac{1-\theta}{\theta}\frac{R_t}{W_t} = \frac{N_t(i)}{K_t(i)}$ is the same

for all firms. Plugging factor demands from equation (38) and equation (39) into production function of firm i we get $Y_t(i) = A_t \left(\frac{MC_t^r(i)\theta Y_t(i)}{R_t^k} \right)^\theta \left(\frac{MC_t^r(i)(1-\theta)Y_t(i)}{W_t} \right)^{1-\theta}$ which after expressing for MC_t^r delivers,

$$MC_t^r = \int_0^1 MC_t^r(i)di = \frac{\left(R_t^k\right)^\theta W_t^{1-\theta}}{A_t \theta^\theta (1-\theta)^{1-\theta}}, \tag{42}$$

Marginal costs are therefore the same for all firms, both of price setters and firms with staggered prices.

C.2.5 Re-setting firm vs. aggregate quantities for CRS

From the relationship $Y_t^* = A_t K_t^{*\theta} N_t^{*1-\theta} = \left(\frac{P_t^*}{P_t} \right)^{-\epsilon} Y_t$ we can express for amount of labor input hired by the price re-setting firm as

$$N_t^* = \frac{\left(\frac{P_t^*}{P_t} \right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} di \right]} \left(\frac{K_t}{K_t^*} \right)^{\frac{\theta}{1-\theta}} N_t. \tag{43}$$

The ratio of capital demand equations for the price resetting firm and the aggregate firm delivers,

$$\frac{K_t}{K_t^*} = \frac{Y_t}{Y_t^*}, \tag{44}$$

We have shown that $Y_t^* \leq Y_t$ in the presence of trend inflation. Therefore, as $\frac{K_t}{K_t^*} = \frac{Y}{Y_t^*}$, then $K_t^* \leq K_t$ and $N_t^* \leq N_t$.

C.3. PROOFS AND PROPOSITIONS

Proposition 1 Price dispersion is bounded by one, $S_t \geq 1$.

Proof. The aggregate price index is $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i) \right]^{\frac{1}{1-\epsilon}}$. Dividing by P_t gives $1 = \left[\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$. Defining $v_{i,t} = \left(\frac{P_t(i)}{P_t} \right)^{1-\epsilon}$ we get that $\left[\int_0^1 v_{i,t} \right]^{\frac{1}{1-\epsilon}} = 1$. Writing price dispersion, $S_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} di \right]^{1-\theta}$, in terms of $v_{i,t}$ yields $v_{i,}^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}} = \left[\left(\frac{P_t(i)}{P_t} \right)^{1-\epsilon} \right]^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}}$ Thus, price dispersion can be written in terms of variable v as, $S_t^{\frac{1}{1-\theta}} = \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}}$ And

as $\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta} > 1$, Jensen's inequality implies that

$$1 = \left[\int_0^1 v_{i,t}\right]^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} \leq \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} = S_t^{\frac{1}{1-\theta}}. \tag{45}$$

$\square$

Proposition 2 The ratio of price indexes, $\phi_n = \dfrac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]} \gtreqless 1$, for

$$\phi_n \begin{cases} < 1 & \text{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t > 0, \\ & \text{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t > -\bar{\pi}, \\ = 1 & \text{for } \bar{P} = P_t^* = P_t(i) = P_t, \\ > 1 & \text{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t < 0, \\ & \text{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t < -\bar{\pi}, \end{cases} \tag{46}$$

where $\bar{P}$ is the deterministic steady state of price and $\hat{\pi}_t$ is deviation of inflation from its steady state.

Proof. The ratio $\phi_n < 1$ if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} < \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$. From the Proposition 1, $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di \geq 1$. Thus, it must be true that if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} \leq 1$ then $\phi_n \leq 1$. This will hold for all cases when $P_t^* \geq P_t$. Because $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}$ (equation (28)) for $\Pi_t \geq 1$ it holds that $P_t^* \geq P_t$ In case of positive steady state inflation, $\bar{\pi}_t > 0$ the inflation deviation from its steady state can reach $\hat{\pi}_t > -\bar{\pi}$ for $\phi_n \leq 1$. $\square$

C.4. CALIBRATION

Table 7: Calibration of the RS table 3 (best fit) model

Symbol	Variable	Value
β	Discount factor	0.99
$CRRA$	Risk aversion	110
IES	Intertemporal elasticity	0.09
ϵ	Elasticity of substitution	6
$Frisch$	Frisch elasticity	0.28
ϕ_π	Response to inflation	0.53
ϕ_y	Response to output	0.93
ρ_i	i_t smoothing	0.73
ζ	Price adjustment	0.76
$\bar{G}/\bar{Y}$	Government spending on output	0.17
ρ_G	Autocorrelation Government spending shock	0.95
σ_G	Volatility of Government spending shock	0.004
ρ_A	Autocorrelation of TFP shock	0.95
σ_A	Volatility of TFP shock	0.005
$\theta_{\rho_{pi*}}$	Inflation target shock persistence	0.995
σ_{π^*}	Volatility of inflation target shock	0.0007
ζ_{π^*}	Inflation target adjustment	0.003
θ	Capital share of output	1/3
$\bar{\Pi}$	Steady state inflation	1.004
δ	Capital depreciation	0.02

C.5. SENSITIVITY ANALYSIS TO RS MODEL

In what follows, we focus our analysis further on the dispersion of prices in the economy. As the model parameters of RS were calibrated to match moments for the case of $\bar{\pi} = 0$, it may be argued that the model might be not well calibrated, when simply allowing for $\bar{\pi} > 0$ but leaving other model parameters unchanged. We first confirm that the patterns documented in Table 2 hold across a wide range of parameter values.

Figure 10 shows how the mean of the inverse price dispersion changes over different ranges of other model parameter values and orders of approximation. The first set of panels shows the sensitivity of mean simulated price dispersion to changes in key

model parameters for the case of zero trend inflation, for different orders of approximation (first, second, third-order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. Whereas the mean simulated price dispersion is affected strongly by varying trend inflation (panel 1), including pushing S^{-1} to the infeasible region bigger than one[31], varying other model parameters does not affect the simulated mean price dispersion drastically (and never pushes S^{-1} to an infeasible region). Other than variations in trend inflation, only regions of relatively high elasticities of substitution or high price rigidities lead to large costs from price dispersion (of, e.g. more than 1%, reflected in S^{-1} falling below 0.99). The second set of panels presents comparable figures for the case of positive trend inflation. Pink diamonds reflect the 'RS Table 3'-baseline parameterization, apart for steady-state inflation, which now is $\bar{\pi} = 1\%$. Since the accuracy of the mean price dispersion is already somewhat compromised at $\bar{\pi} = 1\%$, regions of relatively high elasticities of substitution or high price rigidities quickly lead to problems (flat lines in the last two reported panels represent cases with indeterminate solutions). Variations in other key parameters continue to leave mean price dispersion mostly unaffected.

D.   RESULTS

Table 8: Taylor rule estimates for US

| Study	Period	ϕ_{pi}	ϕ_y
Taylor (1999)	1987 - 1997	1.53	0.77
Judd and Rudebusch (1998)	1987 - 1997	1.54	0.99
Clarida, Gali, and Gertler (2000)	1979 - 1996	2.15	0.93
Orphanides (2003)	1979 - 1995	1.89	0.18
Coibion and Gorodnichenko (2011)	1983 - 2002	2.2	0.43
Smets and Wouters (2007)	1966 - 2004	2.03	0.08
Jermann and Quadrini (2012)	1984 - 2010	2.41	0.121
Rotemberg and Woodford (1997)	1980 - 1995	1.27	0.08
Lubik and Schorfheide (2004)	1982 - 1997	2.19	0.3
Boivin and Giannoni	1979 - 2002	2.03	0
Calibrated models			
Galí (2015)	-	1.50	0.125
Ascari and Sbordone (2014)	-	2.00	0.125
Guerrieri and Iacoviello (2015)	-	2.50	0.50
Basu and Bundick (2017)	-	1.5	0.2
Levin et.al. (2000)	-	1.27	0.08

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[31]S^{-1} is bounded from above by one. See Proposition 1

Figure 10: Parameters sensitivity in the RS (2012) model

Parameter sensitivity of RS (2012) model, case of zero trend inflation, $\bar{\pi} = 0\%$
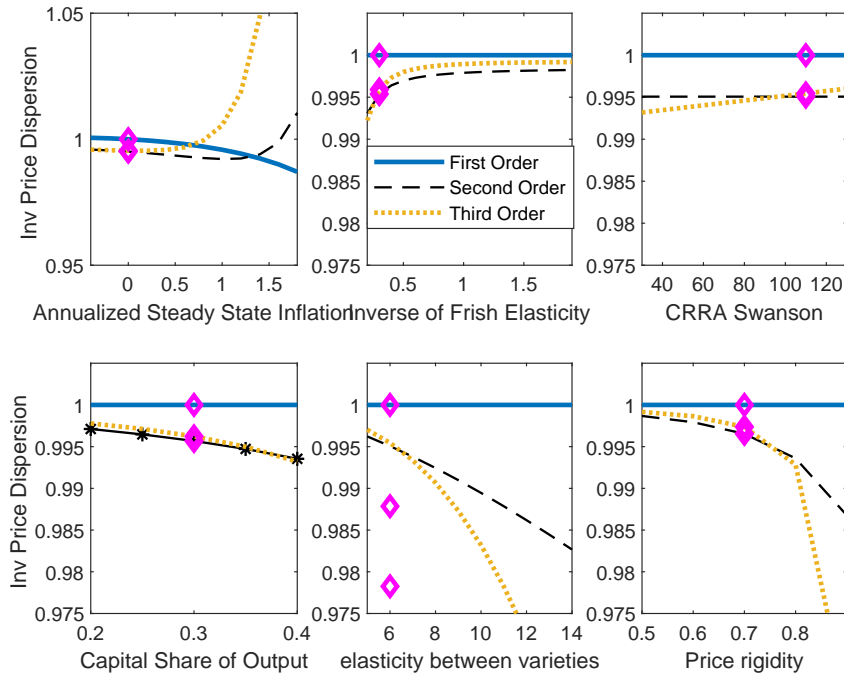
Parameter sensitivity of RS (2012) model, case of positive trend inflation, $\bar{\pi} = 1\%$
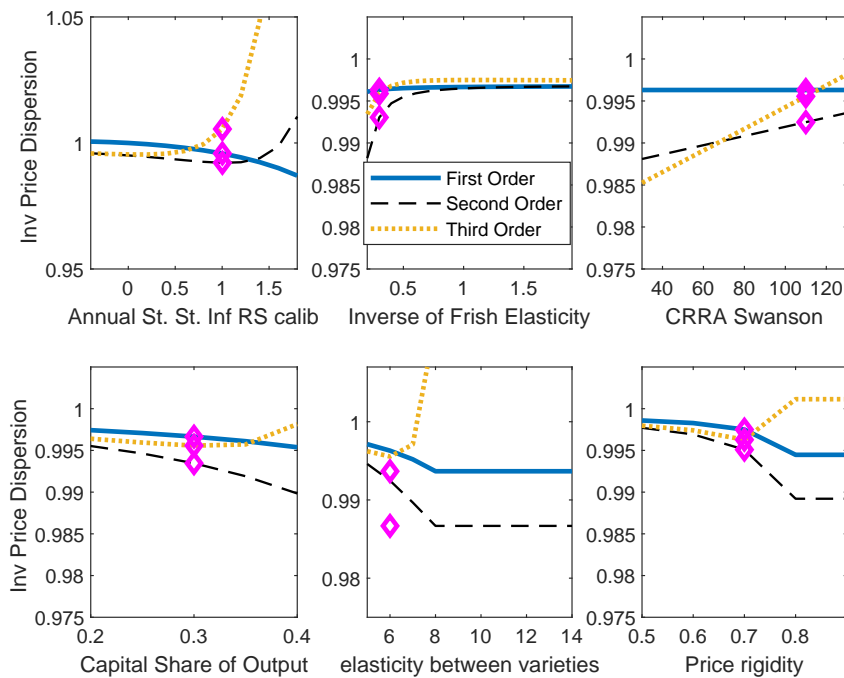
Note: The first set of panels shows the sensitivity of the mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second and third order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. The second set of panels presents analogous figures for the case of positive trend inflation. Flat lines in the last two reported panels represent cases with indeterminate solutions.

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