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Do output gap estimates improve inflation forecasts in Slovakia? *

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Abstract

The paper compares different output gap measures regarding their real-time reliability and usefulness for predicting inflation in Slovakia. The results indicate that estimated cycles from the Modified Hamilton filter, a Mixed-Frequency Bayesian Vector Autoregression and a Dynamic Factor Model are economically reasonable, similar in magnitude to the official central bank estimate and, more importantly, stable over time. Furthermore, among all the output gap estimates compared, the gap from the Mixed-Frequency Vector Autoregression can predict Slovak inflation better than other estimates of the cyclical position until the recent period of high inflation in 2021–2022.

Keywords: output gap; inflation; stability; forecasting;
JEL-Codes: C11, C32, E31, E32

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Non-technical summary

The output gap is an essential indicator of economic cycles, but it is unobserved, and therefore there is uncertainty in its estimation. Moreover, many output gap estimates are unreliable in real-time, whereas practitioners need to get precise estimates of the cyclical position when new data become available. Therefore, central bankers need to rely on stable output gap estimates. Unfortunately, primary methods for estimating the output gap are not satisfactory from different points of view: (1) the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997) is a univariate two-sided filter[1] that might produce spurious or unstable estimates (A.1), (2) Unobserved Component Models with additional information from surveys (European Central Bank, 2015; Benčik, 2019) might not filter the gap well when these surveys do not contain essential information for economic cycles (A.2), (3) proxies such as Principal Components from different economic indicators might incorporate measurement errors due to which it might be hard to estimate potential output (A.3). Many methods for estimating the gap are vulnerable to the revision problem (Orphanides and van Norden, 2002), and, as a consequence, there is tremendous uncertainty regarding the cyclical position of the economy.

Moreover, output gap estimates should also be informative regarding future inflation since the gap should indicate demand pressures in the economy. Recently questions on predicting inflation became popular again because of elevated inflation in many countries and its forecast uncertainty due to high energy prices, supply disruptions and geopolitical risks. Since Slovakia is a small open economy and not many studies have been conducted on the informativeness of various output gap measures, the paper addresses the question of which output gap measure precisely indicates inflation in Slovakia.

This paper uses several methods to estimate the output gap and compares estimated gaps in a Phillips curve type predicting exercise. The methods for estimating the output gap used in the study are the Hodrick-Prescott filter (Hodrick and Prescott, 1997), the Modified Hamilton filter (Hamilton, 2017; Quast and Wolters, 2020), the Beveridge-Nelson filter (Kamber, Morley, and Wong, 2018), the official National Bank of Slovakia estimate (NBS, 2022), the first principal component from Benčik (2019), and two multivariate filters.

Multivariate techniques for estimating the gap can exploit additional sources of information that might reduce estimation uncertainty, but their implementation is restricted because of data limitations (many economic indicators for Slovakia are available only

[1]There is also available a one-sided version of the Hodrick-Prescott filter. See, for instance, Wolf, Mokinski, and Schüler (2020).

from the 2000s), two recessions during the period studied (in 2008 and 2020), and structural (and possibly non-discrete) breaks in trends. These issues limit the scope of methods suitable for estimating the gap. Nevertheless, Bayesian methods are advantageous due to the possibility of taking prior information into account (for dealing with the problem of a small number of observations) and using probabilistic statements regarding the gap (for instance, it is possible to construct credible intervals instead of point estimates, density forecasts instead of point forecasts). Two main multivariate Bayesian techniques for estimating the gap are based on a Bayesian Vector Autoregression (Berger, Morley, and Wong, forthcoming) and a Dynamic Factor Model (Jarocinski and Lenza, 2018; D'Agostino, Giannone, Lenza, and Modugno, 2016).

The results show that the most stable output gap measures are obtained from the Modified Hamilton filter, the Beveridge-Nelson filter, a Mixed-Frequency Bayesian Vector Autoregression and a Dynamic Factor Model. Nevertheless, predicting inflation is challenging since almost all output gap estimates are not connected with inflation, except for the Mixed-Frequency Bayesian Vector Autoregression gap. Among possible reasons for the weak connection between output gaps and inflation might be the importance of global inflation (Ascari and Fosso, 2021) or the lower relative importance of demand shocks and cost pressures (Negro, Lenza, Primiceri, and Tambalotti, 2020).

1. Introduction

The output gap intends to indicate domestic demand pressures and there should be a positive link between the output gap and future inflation. Nevertheless, from an empirical point of view, the Phillips curve type relationship does not seem to hold. For example, Atkeson and Ohanian (2001) found that simple models that use previous inflation rates to predict future inflation produce more accurate inflation forecasts than Phillips curve models. Since Slovakia is an inflation-targeting country, finding a good measure of domestic demand pressures is extremely important. However, the questions on the prediction of inflation were studied mainly for the advanced economies or the euro area (Melolinna and Toth, 2019; Anderton, Aranki, Dieppe, Elding, Haroutunian, Jacquinot, Jarvis, Labhard, Rusinova, and Szorfi, 2014; Jarocinski and Lenza, 2018; Guérin, Maurin, and Mohr, 2011; Huber, Pfarrhofer, and Piribauer, 2020) and haven't been studied to the full extent yet for small open economies. Therefore, this paper aims to compare different output gap measures for predicting inflation in Slovakia.

Furthermore, the lack of observed Phillips curve connection in empirical exercises might be attributed to imprecise estimates of the economy's cyclical position. Moreover, Orphanides and van Norden (2002) and Kangur, Voigts, Natal, and Kirabaeva (2019) pointed out the instability problem of real-time inflation forecasts using output gap measures and, therefore, this study concentrates on output gap measures that are stable over time.

Contrary to previous studies, this paper implements an extensive evaluation of different output gap measures estimated on a small sample size for predicting inflation. The estimation is complicated for several reasons. First, comparable data in Slovakia are available for the last twenty years, which leads to the difficulty in obtaining stable estimates of the output gap. Second, the last twenty years contain volatile data due to the Global Financial Crisis, the COVID pandemic, and structural changes in the Slovak economy (such as joining the European Union in 2004, introducing inflation targeting in 2005 and joining the Monetary Union in 2009). Third, there is a disconnection between the economy's cyclical position and inflation after the Global Financial Crisis.

The study uses several methods to estimate the output gap and compares estimated gaps in a Phillips curve type predicting exercise. The methods for estimating the output gap implemented in the study are the Hodrick-Prescott filter (Hodrick and Prescott, 1997), the Modified Hamilton filter (Hamilton, 2017; Quast and Wolters, 2020), the Beveridge-Nelson filter (Kamber, Morley, and Wong, 2018), the official National Bank of Slovakia estimate (NBS, 2022), the first principal component from Benčik (2019), the Mixed-Frequency Bayesian Vector Autoregression (Berger, Morley, and Wong, forthcoming) and a Dynamic Factor Model (Jarocinski and Lenza, 2018; D'Agostino, Giannone, Lenza, and Modugno, 2016).

While previous studies concluded that none of the output gap estimates outperforms others in inflation forecasting (Barbarino, Berge, Chen, and Stella, 2020; Stock and Watson, 2009), this study finds that some output gap measures are better than others in predicting core inflation, namely the gap from the Mixed-Frequency Bayesian Vector Autoregression (Berger, Morley, and Wong, forthcoming) and the first principal component from survey data (Benčik, 2019). Moreover, the output gap obtained from the the Mixed-Frequency Bayesian Vector Autoregression is stable over time, making it reliable for policymakers. The findings support those of Ball and Mazumder (2020), who found that inflation is predicted by the output gap together with inflation expectations. The paper is related to the study of Huber, Pfarrhofer, and Piribauer (2020), who developed a multi-country output gap model and compared it in an out-of-sample predicting inflation exercise. Nevertheless, this study concentrates also on stability properties of the estimated output gaps and compares their real-time revision properties.

The paper is organised as follows. Section 2 reviews related literature and presents results from related studies, Section 3 describes various techniques used in the study for estimating the output gap, Section 4 presents estimation results and their stability properties, Section 5 discusses the usefulness of different output gap measures for predicting inflation in Slovakia, and Section 6 concludes.

2. Related Literature

This paper relates to several research areas in trend-cycle decompositions and Phillips curve relationships. Related studies mainly estimated the economy's cyclical position for major economies.

Usually, models for estimating the output gap require a relatively large sample size (Barigozzi and Luciani, forthcoming) to produce stable estimates, or studies try to find the most plausible model structures without looking at the stability properties of estimated gaps(Grant and Chan, 2017). Barbarino, Berge, Chen, and Stella (2020) investigated the stability of different output gap estimates and did not find a superior method. The authors found that the most crucial problem in revisions is the end-point problem, whereas instability of estimated parameters and data revisions play a minor role. Moreover, the authors found that including the unemployment rate in estimation improves the stability properties of the gap.

Nevertheless, some studies tried to combine stability with plausibility. Morley and Wong (2020) and Berger, Morley, and Wong (forthcoming) proposed using multivariate

Beveridge-Nelson decomposition in Vector Autoregressions and showed that the unemployment rate is a significant contributor to the cyclical position of the U.S. economy. In contrast, Jarocinski and Lenza (2018) and D'Agostino, Giannone, Lenza, and Modugno (2016) employed a Dynamic Factor Model for trend-cycle decomposition of U.S. GDP. However, Berger, Morley, and Wong (forthcoming) use a relatively large sample size and do not discuss the properties of the gap for shorter samples, while the gap estimated by Jarocinski and Lenza (2018) is somewhat unstable. Guérin, Maurin, and Mohr (2011) investigated various linear and non-linear output gap estimates for the euro area and found that model averaging improves the statistical properties of the estimated gaps. To achieve stability of estimates in real-time, Melolinna and Toth (2019) and Constantinescu and Nguyen (2017) modified an unobserved component model by adding a financial conditions index as an explanatory variable. Nevertheless, this study finds that financial variables are unimportant for Slovakia's output gap.

Huber, Pfarrhofer, and Piribauer (2020) estimated the euro area output gap from a cross-country dynamic factor model with a common cyclical component and stochastic country-specific trends and found that the gap helps predict inflation for the euro area. While Huber, Pfarrhofer, and Piribauer (2020) used output and inflation for a dynamic factor model, Berger and Kempa (2011) employed an addition equation for the exchange rate to calculate the output gap for Canada.

Several studies pointed out problems with the output gap estimates obtained. Kangur, Voigts, Natal, and Kirabaeva (2019) found that output gap estimates from the World Economic Outlook are biased due to judgements and forecast errors. Coibion, Gorodnichenko, and Ulate (2018) pointed out that many potential output measures obtained from statistical filters respond not only to supply but also to demand shocks.

Some studies used trend-cycle decompositions for answering various economic questions. For instance, Ascari and Fosso (2021) used a Vector Autoregression with stochastic trends, where several variables share common trends but different cycles, and found that domestic inflation might be detached from domestic labour markets and co-move with international prices. Berge (2020) found that innovations in the volatility of the Federal Reserve's output gap are connected with negative responses in macroeconomic indicators. Chen and Córnicka (2020) combined short-, long- and sign restrictions in a small open economy Structural Vector Autoregression to estimate the UK's output gap.

The second line of related research concerns Phillips curves. Numerous empirical studies (Orphanides and van Norden, 2002; Kangur, Voigts, Natal, and Kirabaeva, 2019) found that the output gap is a weak predictor of future inflation because of substantial revisions in its real-time estimation. Furthermore, Bańbura and Bobeica (2020) noted that the Phillips curve relationship is valid but difficult to estimate due to model instability. Stock and Watson (2007) found that the lack of Phillips curve relation might be attributed to the fact that inflation can be described through an unobserved component stochastic volatility model. Moreover, Stock and Watson (2009) noted that forecasts from the Phillips curve outperform other multivariate forecasts, but at the same time their performance is comparable to univariate benchmarks. Additionally, the authors found time variation in inflation forecasts. To improve inflation predictions, Lansing (2019) proposed including an interaction between inflation and the gap in Phillips curve type regressions, while Guérin, Maurin, and Mohr (2011) found that output gap measures improve inflation forecasts over the sample before the Great recession.

Moreover, some studies found the Phillips curve flatters over time. For instance, Negro, Lenza, Primiceri, and Tambalotti (2020) estimated both theoretical and empirical structural models and noted a disconnection between inflation and the business cycle, mainly due to the lower importance of cost pressures. Contrary, Eser, Karadi, Lane, Moretti, and Osbat (2020) evaluated the role of Phillips curve models at the European Central Bank and noted that slack is the main transmission channel of monetary policy to inflation.

3. Empirical Framework

3.1. Estimation of the output gap

Due to the importance of estimating the economy's cyclical position, many methods were proposed for estimating the output gap. Therefore, this study concentrates on ones that give empirically plausible results and are relatively stable in real time. The study also uses the standard Hodrick-Prescott filter (Hodrick and Prescott, 1997) as a benchmark. There are a few univariate filters that provide stable output gap estimates, among which are the Modified Hamilton filter (Hamilton, 2017; Quast and Wolters, 2020) and the Beveridge-Nelson filter (Kamber, Morley, and Wong, 2018). Moreover, the study also compares modern multivariate filters that should produce stable results, such as a Dynamic Factor model (Jarocinski and Lenza, 2018; D'Agostino, Giannone, Lenza, and Modugno, 2016), a Vector Autoregression Model (Berger, Morley, and Wong, forthcoming) and a dimensionality reduction technique (European Central Bank, 2015).

3.1.1 Hodrick-Prescott filter

The Hodrick-Prescott filter is the most popular filtering technique for estimating the output gap, despite its well-known problem of revisions in real time. The filter is based on the following minimisation problem (Hodrick and Prescott, 1997):

$$y_t^{GDP} = g_t + c_t \tag{3a}$$

$$\underset{\{g_t\}_{t=-1}^{T}}{Min} \left(\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} ((g_t - g_{t-1}) - (g_{t-1} - g_{t-2}))^2 \right) \tag{3b}$$

$$c_t = y_t^{GDP} - g_t \tag{3c}$$

where y_t^{GDP} is the log of real GDP multiplied by 100, g_t is a trend component, c_t is a cyclical component, λ is a penalty parameter.

3.1.2 Modified Hamilton filter

Hamilton (2017) criticised the Hodrick-Prescott filter and proposed a simple alternative based on eight quarters ahead forecast error from an autoregressive model with four lags. Quast and Wolters (2020) modified the filter so that instead of using eight quarters forward forecast errors, the authors proposed to use an average of four to twelve quarters ahead forecast errors. The stability properties of the proposed decomposition are better than the Hodrick-Prescott ones because the proposed filters are one-sided, unlike the Hodrick-Prescott filter.

3.1.3 Beveridge-Nelson filter

The other version of a stable and reliable one-sided filter employs a Bayesian version of an autoregressive model with twelve lags of quarterly GDP growth (the first difference of the log GDP multiplied by 100) with shrinkage priors for its coefficients and a special technique to find the signal-to-noise ratio (Kamber, Morley, and Wong, 2018).

3.1.4 National Bank of Slovakia estimate

The official National Bank of Slovakia model is similar to a multivariate Unobserved Component model that decomposes time series into trend and cyclical components with interactions between them and builds on price and wage Phillips curves, Okun's law and Cobb-Douglass production function, similar to the model described in Tóth (2021). The model is calibrated for the Slovak economy and is estimated using the Kalman filter and smoother. Therefore, the model is similar to a two-sided filter. Furthermore, the model also incorporates expert judgements, producing estimates which are not purely model-based.

3.1.5 Principal components

A popular and stable alternative to trend-cycle decomposition is to reduce the dimensionality of several variables, one of which is the Principal Component analysis. The method produces stable estimates since all variables should already be transformed into stationary form. Nevertheless, dimensionality reduction techniques cannot estimate the output gap precisely since it does not take into account the decomposition in a way $y_t^{GDP} = c_t + g_t$. Despite that, dimensionality reduction techniques can produce an estimate of the output gap that is highly correlated with the actual one.

3.1.6 Mixed-Frequency (U-MIDAS) Bayesian Vector Autoregression

The study follows the mixed frequency setup from Berger, Morley, and Wong (forthcoming) that implements the multivariate Beveridge-Nelson decomposition using a Mixed Frequency Vector Autoregression[2] (MF-BVAR) model. Namely,

$$Y_t = \begin{bmatrix} m_{month1} \\ m_{month2} \\ m_{month3} \\ \Delta y^{GDP}_{quarter} \end{bmatrix} \qquad X_t = [Y_t' \; Y_{t-1}' \ldots Y_{t-p}']'$$

$$X_t = FX_{t-1} + U_t \qquad u_{i,t} \sim N(0, \Sigma) \qquad i = 1, ..., N \tag{3d}$$

$$c_t = -s_f F(I - F)^{-1} X_t{}^3 \tag{3e}$$

, where m_{month1} is a vector of monthly variables for the first month within a quarter, $y^{GDP}_{quarter}$ is real GDP, p is a number of lags, N is the total number of variables at a quarterly frequency, c_t is a cyclical component associated with the s_f^{th} variable, the equation (3d) casts a stacked mixed frequency model in companion form, and the equation (3e) shows how to extract a cycle from the estimated parameters where s_f is a selection row vector that selects a corresponding variable. The Beveridge-Nelson decomposition assumes that a trend component is a random walk with constant drift, which might not be valid for real GDP, and, therefore, the study implements dynamic de-meaning of GDP growth using a backwards-looking rolling 10-quarter window[4]. Appendix B discusses the specification of priors for this model.

[2] This Mixed-Frequency model is a U-MIDAS type of model and not a Mixed-Frequency type of model as in Schorfheide and Song (2015).
[3] The proof for this equation can be found in Morley (2002).
[4] Similarly to Berger, Morley, and Wong (forthcoming), who used a 40-quarter window.

3.1.7 Dynamic Factor Model

The study employs the general setup of Jarocinski and Lenza (2018) and D'Agostino, Giannone, Lenza, and Modugno (2016) for a small dynamic factor model. The model can be summarised as follows:

$$y_t^n = \lambda^a c_{t-1} + \lambda^b c_t + \lambda^c c_{t+1} + g_t^n + \varepsilon_t^n \tag{3f}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \eta_t^c \tag{3g}$$

$$\Delta g_t^n = \Delta g_{t-1}^n + \eta_t^n \tag{3h}$$

, where ε_t^n, η_t^c, η_t^n are independent Gaussian errors, $n = 1, ..., N$, where N is the total number of observables, Λ and ϕ are coefficients. The first equation (3f) is an observation equation, where the first variable is the log of real GDP multiplied by 100. The coefficients of the first equations are restricted as $y_t^1 = c_t + g_t^1$, and, therefore, c_t can be interpreted as the output gap, g_t^1 is the trend.

The model is specified so that the selected observables share a common cyclical component and have individual trends. The first state equation (3g)[5] determines the evolution of the gap, and the second state equation (3h) specifies the evolution of the trends, where trend growth rates are modelled as random walks. The technical details on estimation and priors are discussed in Appendix C.

3.1.8 Mixed-Frequency Dynamic Factor Model

The study also uses a Mixed-Frequency approach with a Dynamic Factor model. The model described in the previous subsection was recalculated at a monthly frequency using a precision-based approach described in Chan, Poon, and Zhu (2021). For this, additional equations were added to sample GDP at a monthly frequency at each sampling step. The details are presented in Appendix D. The rest of the model is as described in (3f)-(3h).

3.2. Data

GDP growth in Slovakia is quite volatile (Figure 1) and changes rapidly by 5–10% in just one quarter during some episodes. The reasons for these sharp GDP changes might be that Slovakia is a small open economy with several political changes and recessions

[5] The study also checked the financial cycle in the specification for the gap as in Melolinna and Toth (2019), but the latter was not found to be relevant for the business cycle based on posterior estimates. Similarly, Odor and Kucserova (2014) estimated a state-space model with financial variables affecting the cyclical component in Slovakia and found that only household credit out of seven financial variables is statistically significant.

during the period studied. Slovakia became an inflation targeter in the early 2000s and joined the European Union in 2004 and the Monetary Union in 2009. In 2008 Slovakia was hit by the Global Financial Crisis, and in 2020 by the COVID pandemic. The kinks in GDP growth complicate the output gap estimation and tend to make those estimates unstable over time[6].

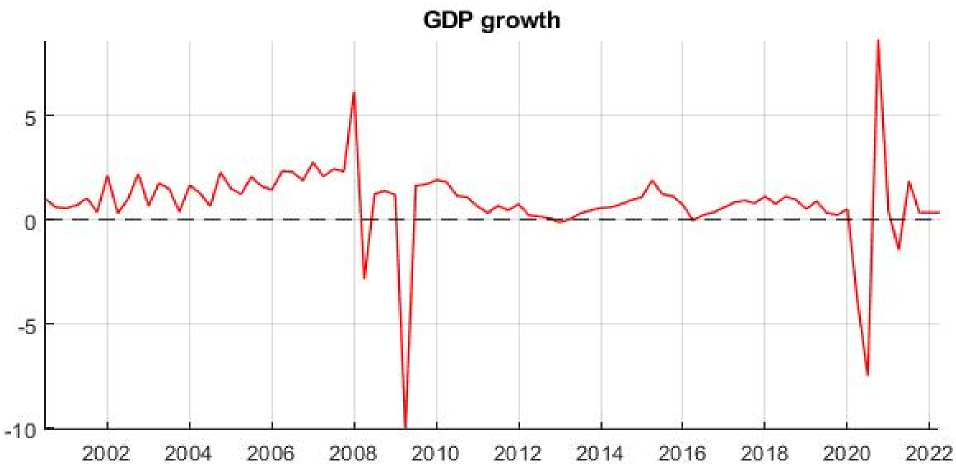

Figure 1: GDP growth in Slovakia

Additionally, the GDP data are subject to real-time revisions, which in turn might also make estimations of the output gap unstable over time. Figure 2 shows real GDP revisions in Slovakia using different GDP vintages from 2019Q1 to 2022Q1. One can see that the level of GDP was somewhat revised from 2019Q3 compared to the previous estimates. Moreover, the drop during the COVID pandemic was revised downwards according to the most recent estimates. While univariate filters use only one variable, namely real GDP, to estimate the gap, multivariate models use additional variables that might improve the stability properties of the estimated gaps in real time. Therefore, the question of variable selection becomes crucial for the results.

Multivariate filters usually use a small set of variables that should be informative regarding the economy's cyclical position. The official model uses the following time series: real GDP, the unemployment rate, capital stock, working age population, average hours worked, the participation rate, world demand, HICP excluding energy, compensation per employee, the long term unemployment rate and an inflation target trend. All data are taken from the projection exercises (NBS, 2022). The inflation target trend is detrended inflation with the Hodrick-Prescott filter ($\lambda = 1,600$) before 2008 and is set to 0.0256 after 2008.

[6]All models used here have constant coefficients.

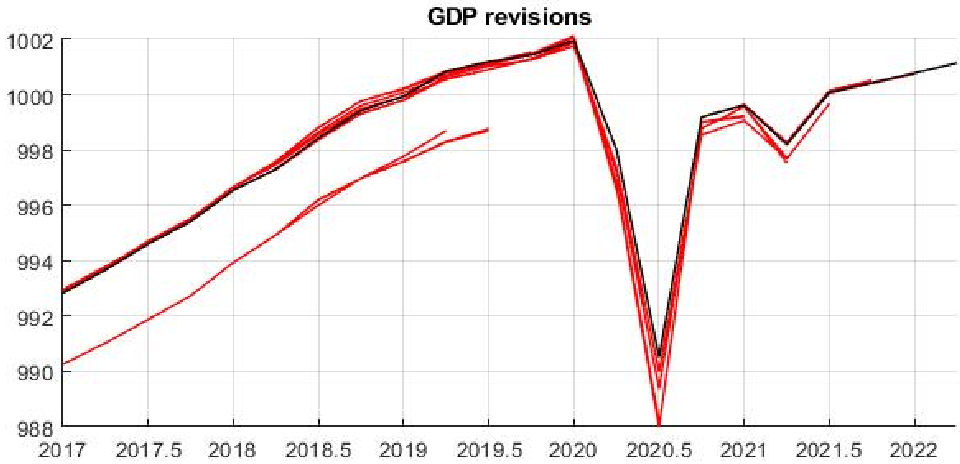

Figure 2: Real-time GDP revisions in Slovakia

Furthermore, the study uses the Principal component version of Benčik (2019), who employed 37 survey-based indicators to construct the output gap. These indicators can be broken down into the following categories: construction indicators (15 variables), manufacturing indicators (12 variables), services/retail indicators (6 variables) and labour market indicators (4 variables)[7]. The study uses these variables because they are already in stationary form, and there is no need to transform them since an output from principal component analysis depends heavily on data transformation, and different de-trending techniques might distort results and provide different outcomes.

There is a need to select variables for other multivariate models. Standard variable selection techniques, such as minimising predictive Mean Squared Error (MSE), Least Absolute Shrinkage and Selection Operator (LASSO), Stochastic Search Variable Selection (Korobilis, 2011) or expert judgements (Hucek, Karsay, and Vavra, 2015), select variables important for GDP growth, which might or might not be contributors to the gap. On the other hand, the Morley and Wong (2020) approach selects mainly stationary variables from surveys as significant contributors to the gap. However, output gap estimates from chosen variables are not stable over time. Therefore, the study selects variables from surveys and statistical indicators based on stability criteria.

In the case of the mixed-frequency models, relevant monthly variables should be selected. The study selects monthly variables that give a stable estimate of the gap and predict inflation better than other variables. The following domestic variables are employed in the estimation: order books, export order books, major purchases at present (European Commission, 2022), industrial production of intermediate goods, nominal import of goods, the number of registered unemployed (National Bank of Slovakia,

[7]The detailed list of variables can be found in Benčik (2019).

2022). Moreover, the following foreign variables are included: industry expectations in the European Union (European Commission, 2022), Germany's share price index (Organization for Economic Co-operation and Development, 2022b), world trade (CPB Netherlands Bureau for Economic Policy Analysis, 2022). All variables, except for Germany's share price index, are seasonally adjusted.

The real-time data availability is presented in Table 1. The following variables are available for the previous three months in the middle of May: order books, export order books, major purchases at present, industry expectations in the European Union and Germany's share price index (DAX). These are the data on expectations and stock prices. Hard time series data are available for February and March only: industrial production of intermediate goods, nominal import of goods, and the number of registered unemployed. World trade is available only for February since it takes time to aggregate this variable. Moreover, a flash estimate of real GDP for the first quarter is already released in the middle of May. The final estimate is released at the beginning of June.

Table 1: Real-time data availability in the middle of May

Variable	February	March	April
Industry confidence indicators: order books	v	v	v
Industry confidence indicators: export order books	v	v	v
Major purchases at present	v	v	v
Industry expectations in the European Union	v	v	v
Industrial production of intermediate goods	v	v	x
Nominal import of goods	v	v	x
Number of registered unemployed	v	v	x
Germany's Share price index	v	v	v
World trade	v	x	x
Real GDP	Q1		

Variables for the Dynamic Factor model are selected similarly to Jarocinski and Lenza (2018) and based on stability criteria. The observed variables are real GDP, the rate of capacity utilisation from the Business Tendency survey (Organization for Economic Co-operation and Development, 2022a), order books (European Commission, 2022), the registered unemployment rate, total gross capital formation, the total export of goods and services (National Bank of Slovakia, 2022). The data frequency is quarterly. All variables are seasonally adjusted. Stationary variables are left untransformed, while non-stationary variables are transformed similarly to real GDP (100 multiplied by the log of a variable).

One remark on the set of variables should be made. When different multivariate filters use different variables, the question arises of whether differences from the models come from different variables. Because of that, it might not be possible to compare the performance of the models directly. Therefore, the study also compares the models using the same variables to be sure that the differences come from models and not from variables.

4. Estimation Results and Stability

Orphanides and van Norden (2002) pointed out the revision problem of estimated output gaps, meaning that estimates are unreliable in real time. Massive revisions complicate the estimation of the gap and reduce the forecasting abilities of models when new data become available. Therefore, estimated gaps need to be stable over time. The COVID pandemic complicates the issue since it is exogenous to the economy, huge in magnitude, and different economic indicators reacted differently to it.

The estimation results are presented in Figure 3 and Figure 4. Figure 3 shows the stability results for the Hodrick-Prescott filter, the Modified Hamilton filter, the Beveridge-Nelson filter and the first principal component from Benčik (2019). The black lines show the final estimates as of 2022Q1, while the red lines show the real-time estimates from 2019Q2.

All filters indicate economic expansions in 2007–2008 and 2018–2020 and economic downturns after 2008, 2009 and 2020. But the magnitudes of these expansions and downturns are filter specific. The gap from the MH filter shows about twelve basis points expansion in 2008, the HP filter shows about eight basis points expansion, the BN filter indicates about five basis points expansion, while the first principal component from surveys shows about four basis points expansion. The drops in 2009 and 2020 are similar in magnitude among all three filters.

Moreover, all three filters show revisions in 2020 similar to the revisions in GDP (Figure 2). Nevertheless, the first principal component from the survey data does not exhibit these revisions in 2020, but in fact it does not even use GDP data in estimation, so it cannot be considered as the gap per se. Regarding stability, the results from the HP filter are quite unstable, with sizeable revisions during 2018–2020, while all the other methods show relatively stable estimates in real time.

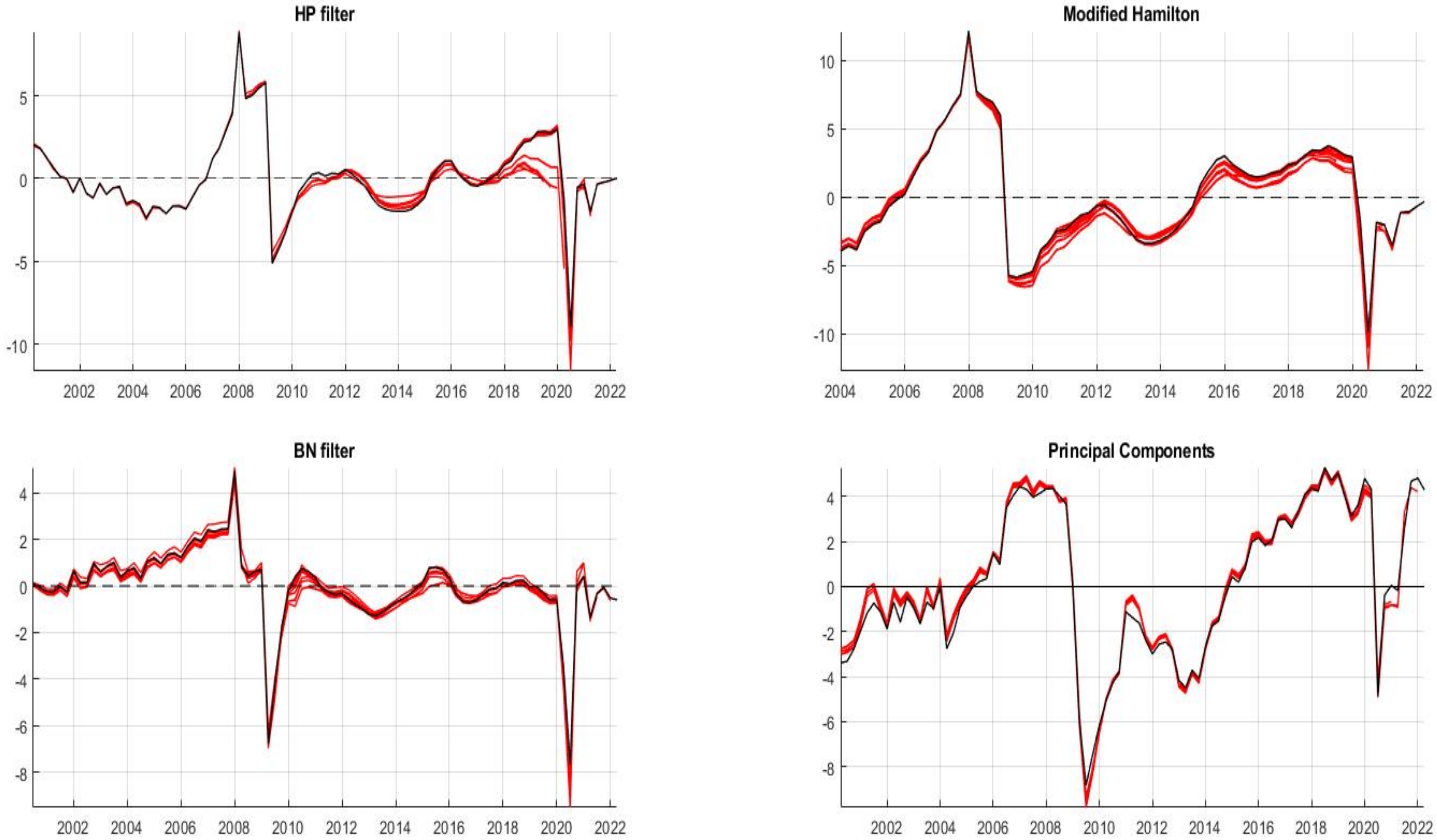

Figure 3: Estimated output gaps from the selected univariate models and the principal component analysis
the red lines in the figure present different data vintages starting from 2019Q2, the black line – final estimates; posterior means; real-time data

Figure 4 presents the stability of the output gaps from the multivariate models. A few things might be noted from the comparison of the estimates. Firstly, the quarterly DFM model results are quantitatively similar to the estimates of the official model after 2009, whereas it estimated a higher expansion during 2006–2008. Secondly, the MF-BVAR results are also quantitatively similar to the official estimate, but the expansion in 2008 is lower in magnitude. Thirdly, the estimates of DFM at a monthly frequency with the MF-BVAR variables show higher magnitudes than all other models, but the business cycle turning points and relative magnitudes are similar to other multivariate methods.

The upper left figure shows the real-time estimates of the official model with expert judgements. The figure looks similar to the HP filter results, except for 2000–2003. It is more stable than the HP filter during 2018–2020 and less stable after 2020. The differences in 2000–2003 might depend on the starting date of estimation. The gap fell in 2008 from 7 to 5 basis points and continued falling in 2009 from 5 to -7 basis points. The second significant fall was during the COVID-pandemic period, when the gap fell from 1.5 to -10 basis points. Therefore, the pandemic had a higher negative effect on cyclical fluctuations in Slovakia than the Great Recession.

The top right figure presents the results of MF-BVAR estimation[8]. There are not many visible revisions in the recursive re-estimation of the MF-BVAR gap. According to the results, the MF-BVAR gap fell from 3.55 to -5.72 basis points in 2009 but moved back to a positive value in 2011. During the COVID crisis, the gap fell from around 0.78 to -10.12 basis points several quarters after the start of the pandemic. That might reflect the consequences of lockdowns on the economy during the first pandemic wave. Nevertheless, the gap increased from -10.12 to -2.7 basis points in the third quarter of 2020, reflecting positive economic development after all measures were lifted. It declined again in the first quarter of 2021, which might likewise be due to anti-COVID containment measures during the second pandemic wave.

The potential output from this decomposition is more volatile than, for example, the one from the HP filter. In comparison, the latter fixes the signal-to-noise ratio in trend-cycle decomposition, while the former pins down the signal-to-noise ratio from the time series included in the model. The HP filter might over-smooth a trend component, or the time series might not be informative enough for the gap in the case of the multivariate BN decomposition. This excess volatility of the trend might be due to the effects of real shocks on the potential output since from a theoretical perspective the potential output is output under fully flexible prices, so it should be more volatile than the one from the HP filter (Neiss and Nelson, 2005).

[8]The study uses the code provided by Berger, Morley, and Wong (forthcoming).

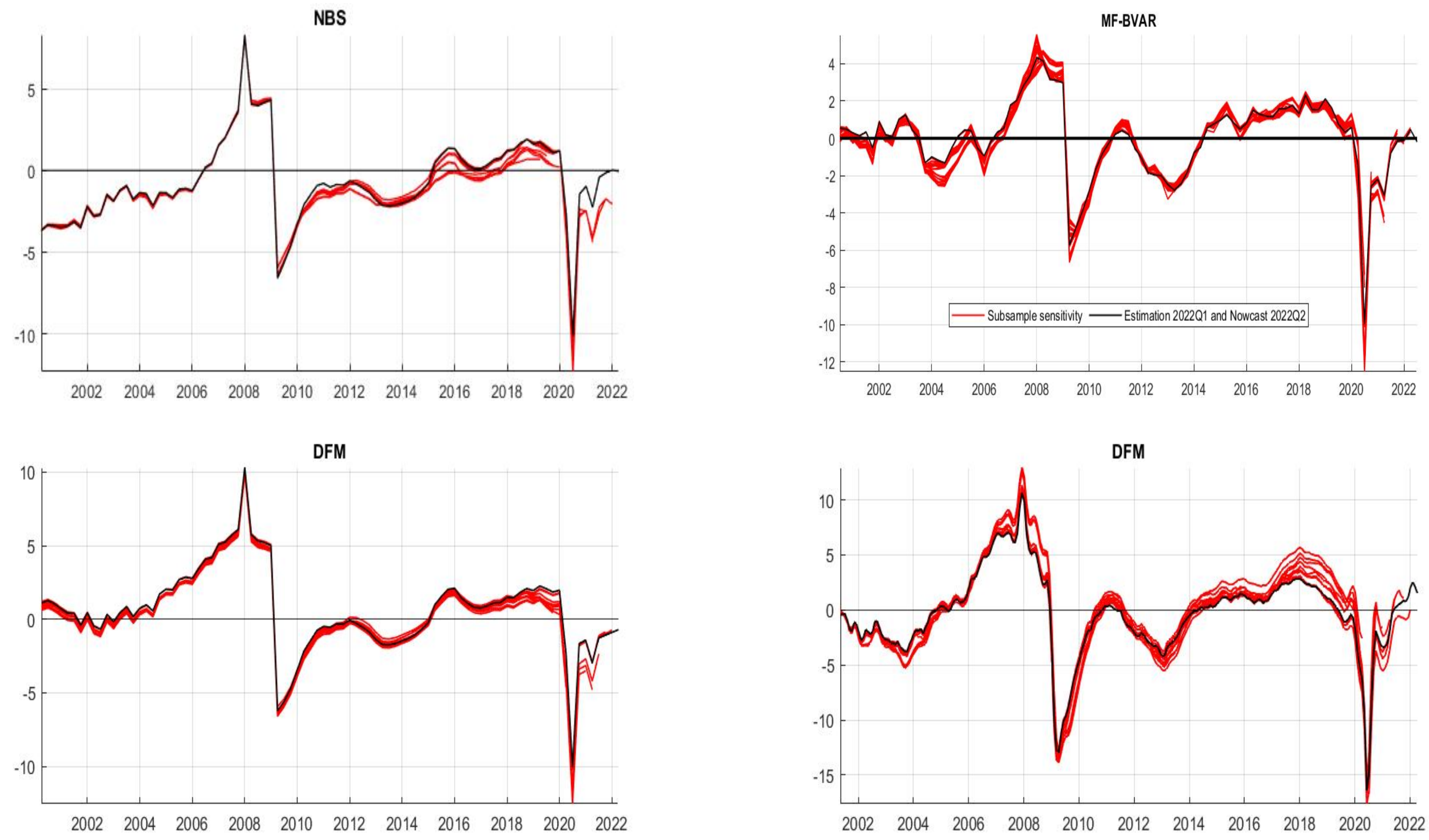

Figure 4: Estimated output gaps from the selected multivariate models
the red lines in the figure present different data vintages starting from 2019Q2, the black line – final estimates; posterior means; real-time data

The bottom left figure presents the estimated gap from the DFM model, which evolves as an AR(1) process (Table C.1), and is similar in shape and magnitude to the official bank estimate and the estimate from MF-BVAR except for 2000–2006, where assessments are slightly different. Moreover, the magnitudes in expansion periods (2006–2008) are different: the dynamic factor model estimated the output gap twice more positively in 2007–2008 compared to the MF-BVAR gap. However, the effects of both recessions are almost identical in all quarterly models. Appendix E presents all decomposed trends from the selected time series.

Appendix F discusses additional estimation details of the quarterly DFM model with the same variables as in the MF-BVAR and official models. The estimates with different variables are quantitatively similar except for 2021–2022. But the stability properties of the estimated gaps with different variables differ and depend on selected variables. The DFM model with the variables specified in this study has the best stability properties among all DFM models with different variables. Moreover, the DFM model that uses the variables from the official UCM model[9] produces quantitatively similar results to the official model (Figure F.4). Furthermore, Appendix G presents the comparison of magnitudes of different output gap estimates.

Table 2 shows standard deviations of the estimated gaps in real time for Slovakia from 2018Q1 to 2021Q1. The BN filter produces the most stable output gap estimates on the full sample and also on the sample excluding the pandemic period. The gap from the MF-BVAR model has the second best stability properties on the sample excluding the COVID pandemic period and the third best on the full sample, not taking into account the first principal component from surveys. Nevertheless, the gap from the MF-BVAR has the worst stability properties in 2020Q2. This instability of the estimates might be attributed to large outliers, such as the COVID shock and the small sample size. Nevertheless, it was one of the most stable filters before the COVID period.

Interestingly, the gap from the HP filter is less stable than the other filters that use quarterly data on both samples, but it is one of the most stable filters during the last three quarters. Overall, the gap from the HP filter is the least stable because of the two-sided properties of the filter. Whereas the gap from the quarterly DFM model also uses leads and lags in the estimation, it is more stable because it is a multivariate filter and utilises information from different variables.

The DFM model at a monthly frequency produces estimates that are the least stable among all estimated output gaps in real time. Nevertheless, this gap is relatively stable during 2020Q2. Interestingly, the DFM gap at a quarterly frequency has similar stability

[9]This DFM model uses only historical values of variables without projections.

properties to the official estimate that uses an unobserved component model.

Table 2: Standard deviations of estimates. Real-time data

	Univariate filters			Multivariate filters				
Standard deviations for	HP	MH	BN	NBS	PCA[1]	MF-BVAR	DFM	DFM_m
2018Q1	0.42	0.25	0.09	0.32	0.10	0.18	0.24	1.00
2018Q2	0.53	0.22	0.08	0.35	0.09	0.12	0.25	1.04
2018Q3	0.73	0.23	0.10	0.39	0.10	0.14	0.26	1.10
2018Q4	0.87	0.27	0.09	0.33	0.08	.015	0.28	1.17
2019Q1	1.13	0.40	0.14	0.38	0.08	0.21	0.35	1.08
2019Q2	1.31	0.45	0.12	0.35	0.11	0.16	0.43	1.01
2019Q3	1.36	0.44	0.09	0.34	0.16	0.28	0.49	1.04
2019Q4	1.30	0.39	0.08	0.32	0.23	0.28	0.55	1.25
2020Q1	1.28	0.84	0.37	0.50	0.15	0.98	0.91	1.47
2020Q2	1.22	1.30	0.76	0.75	0.28	1.40	1.07	0.81
2020Q3	0.1	0.22	0.27	0.47	0.22	0.45	0.96	1.54
2020Q4	0.17	0.23	0.30	0.63	0.35	0.42	0.95	1.40
2021Q1	0.08	0.16	0.06	0.89	0.32	0.61	0.87	1.06
2021Q2	0.03	0.05	0.07	1.06	0.37	0.19	0.57	0.94
2021Q3	0.02	0.07	0.02	0.92	0.18	0.30	0.11	1.03
2021Q4	0.01	0.01	0.07	1.47	0.42	0.09	0.11	1.28
Mean	0.66	0.34	0.17	0.59	0.20	0.37	0.53	1.13
Mean before 2020	0.96	0.33	0.10	0.35	0.12	0.19	0.36	1.08

[a] The first principal component from Benčik (2019).

5. Forecasting inflation

5.1. Full sample results

A popular setup for comparing different output gap estimates is based on the Phillips curve relationship since a positive output gap should lead to higher inflation in the future and vice versa. Therefore, the study uses the general setup of Kamber, Morley, and Wong (2018) and Quast and Wolters (2020) with one lag of each right-hand-side variable to estimate the coefficient $\hat{\beta}_1$ (5a), which should be positive.

$$\pi^h_{t+h} - \pi_t = \beta_0 + \beta_1 \hat{c}_t + \beta_2 \Delta\pi_t + \varepsilon_{t+h|t} \tag{5a}$$

$$\pi_t^h = \frac{400}{h}(log(HICP_t) - log(HICP_{t-h}))$$

where $HICP_t$ is the overall inflation index excluding energy, food, alcohol and tobacco (National Bank of Slovakia, 2022). The data are seasonally adjusted.

Table 3[10] discusses the obtained $\hat{\beta}$. The only regressor that gives a positive and significant coefficient for inflation one and two quarters ahead is the survey indicator extracted with the first principal component. It is because it is the only indicator that is highly positive after 2021. Nevertheless, the gaps from the DFM model that uses monthly variables from the MF-BVAR and from the MF-BVAR model are positively connected with inflation during the studied period.

Table 3: $\hat{\beta}_1$

	Univariate filters			Multivariate filters				
Quarters ahead	HP	MH	BN	NBS	PCA[1]	MF-BVAR	DFM	DFM_m
Q1	0.04	0.02	0.04	0.03	0.06*	0.06	-0.00	0.04
Q2	0.06	0.03	0.05	0.03	0.08*	0.09	-0.00	0.04
Q3	0.05	0.03	0.06	-0.00	0.07	0.08	-0.01	0.05
Q4	0.04	0.03	0.07	0.00	0.06	0.08	-0.03	0.04
Q5	0.03	0.02	0.07	0.01	0.06	0.06	-0.05	0.06
Q6	-0.02	0.01	0.00	-0.02	0.06	0.04	-0.06	0.05
Q7	-0.04	-0.00	-0.05	-0.04	0.06	0.03	-0.07	0.04
Q8	-0.00	0.02	-0.03	-0.03	-	0.13	-0.07	0.05

Note: *, **, and *** denote significance on the 10, 5, and 1% significance level based on Newey and West (1987) standard errors.

[a]The first principal component from Benčik (2019).

It is also possible to compare models in the out-of-sample predictive exercise (5b), where lags (p and q) are selected based on the Bayesian Information Criterion (BIC). Predictions are calculated using an expanding window from 2010Q1. The results are presented in Table 4, where all forecasts are estimated relative to predictions (5b) using the HP filter.

$$\pi_{t+h}^h - \pi_t = \alpha + \sum_{i=0}^{p} \beta_i \hat{c}_{t-i} + \sum_{j=0}^{q} \gamma_j \Delta\pi_{t-j} + \varepsilon_{t+h|t} \tag{5b}$$

$$\pi_t^h = \frac{400}{h}(log(HICP_t) - log(HICP_{t-h}))$$

[10]The study does not take into account the generated regressor problem since all estimates are shown to compare different gap measures.

Table 4[11] presents the improvement of predictions with the selected output gap measures compared to the predictions with the HP output gap. The survey indicator shows improvement in the prediction from five to seven quarters ahead, while using the MF-BVAR output gap improves eight quarters ahead inflation forecasts. There are also occasional improvements in forecasts while using the BN gap, the official gap and the monthly DFM gap.

Table 4: Out-of-sample prediction relative to the HP filter. Real-time data

	Univariate filters		Multivariate filters				
Quarters ahead	MH	BN	NBS	PCA[1]	MF-BVAR	DFM	DFM_m
Q1	0.92	0.99	1.00	0.97	1.03	1.00	0.98
Q2	0.88	0.99	1.02	0.96	1.04	1.01	1.00
Q3	0.92	0.99	0.98	1.03	1.12	1.04	1.00
Q4	1.15	0.96	1.02*	0.98	0.97	0.99	0.97
Q5	1.41	0.90**	0.96	0.87*	1.02	0.95	0.95**
Q6	1.55	0.97	0.94**	0.86*	1.05	0.94	1.24
Q7	1.65	0.99	0.97	0.85*	1.03	0.95	1.23
Q8	1.78	1.02	0.99	-	0.90***	1.02	1.10

Note: *, **, and *** denote significance on the 10, 5, and 1% significance level based on a two-sided Diebold and Mariano (1995) test with the small sample size correction of Harvey, Leybourne, and Newbold (1997).

[a]The first principal component from Benčik (2019).

Furthermore, it is also interesting to compare predictions from models with the gap relative to models without the gap. That will allow us to investigate whether information from the cyclical component helps to predict inflation. In this setting, the study uses only one lag of each right-had-side variable (5c). Therefore, the first model will include $\beta\hat{c}_t$ in the equation while the second will exclude it.

$$\pi^h_{t+h} - \pi_t = \alpha + \beta\hat{c}_t + \gamma\Delta\pi_t + \varepsilon_{t+h|t} \tag{5c}$$

$$\pi^h_t = \frac{400}{h}(log(HICP_t) - log(HICP_{t-h}))$$

Table 5[12] shows the relative improvements in inflation predictions with the output gap measures in the predictive model. The only two gap measures that statistically sig-

[11]The values are $\frac{RMSE_{cycle}}{RMSE_{HP\_cycle}}$

[12]The values are $\frac{RMSE_{cycle}}{RMSE_{no\_cycle}}$

nificantly improve forecasts of inflation eight quarters ahead are the gaps from the MF-BVAR and monthly DFM models.

Table 5: Out-of-sample prediction relative to no gap included. Real-time data

	Univariate filters			Multivariate filters				
Quarters ahead	HP	MH	BN	NBS	PCA[1]	MF-BVAR	DFM	DFM$_m$
Q1	1.00	1.01	1.00	1.00	1.00	1.01	1.01*	1.00
Q2	1.01	1.14*	1.01	1.01	0.98	1.01	1.01*	1.01
Q3	1.01	1.04*	1.02	1.01	1.00	1.03	1.01*	1.00
Q4	1.02	1.07**	1.03	1.01	1.01	1.04	1.01	1.02
Q5	1.01	1.06*	1.04	1.01	1.01	1.05	1.00	1.01
Q6	1.01	1.06*	1.05	1.01	1.01	1.05	0.99	1.01
Q7	1.01	1.05*	1.03	1.00	1.00	1.04	0.98	1.02
Q8	1.00	1.03	1.01	1.01	-	0.97**	0.99	0.97***

Note: *, **, and *** denote significance on the 10, 5, and 1% significance level based on a two-sided Diebold and Mariano (1995) test with the small sample size correction of Harvey, Leybourne, and Newbold (1997).

[a]The first principal component from Benčik (2019).

5.2. Results for 2000–2019

Since the COVID pandemic is quite a turbulent period and all models used in the estimation are linear with constant coefficients, it is important to compare the performance of the selected output gap measures excluding the pandemic period. Table 6 discusses $\hat{\beta}$ coefficients similarly to Table 3, excluding the pandemic period from the sample. The gap from the MF-BVAR model had the strongest connection with inflation from one to five quarters ahead. Interestingly, the other gap that has a positive and statistically significant coefficient is the DFM gap calculated at a monthly frequency that uses the same variables as MF-BVAR. This gap has a strong positive connection with inflation from two to six quarters ahead. All the other output gap measures, including the one from principal components, do not exhibit a strong connection with inflation.

According to the results on relative predictive accuracy, the gap estimated from the MF-BVAR model has the best predictive properties among all models: predictions with this gap are statistically better than those based on the HP filter from three to eight quarters ahead (Table 7). The DFM gap calculated at a monthly frequency also predicts core inflation better than the HP filtered gap four and five quarters ahead. Moreover, the official NBS gap and the gap from the BN filter are also occasionally statistically better than the HP gap in predicting inflation from five quarters ahead. Finally, the gap estimated from the principal component predicts inflation better than the HP gap from five quarters ahead, but the differences between the two are not statistically significant according to the Diebold and Mariano (1995) test.

Table 6: $\hat{\beta}_1$

	Univariate filters			Multivariate filters				
Quarters ahead	HP	MH	BN	NBS	PCA[1]	MF-BVAR	DFM	DFM$_m$
Q1	0.06	0.03	0.06	0.03	0.05	0.10*	0.01	0.05
Q2	0.09	0.05	0.10	0.04	0.07	0.16**	0.03	0.07*
Q3	0.10	0.06	0.13	0.05	0.07	0.18**	0.03	0.08*
Q4	0.08	0.06	0.15	0.05	0.07	0.19*	0.01	0.09*
Q5	0.05	0.05	0.15	0.04	0.06	0.18*	-0.01	0.09*
Q6	0.03	0.04	0.13	0.02	0.06	0.15	-0.02	0.08*
Q7	0.00	0.02	0.10	-0.00	0.06	0.13	-0.04	0.08
Q8	-0.02	0.01	0.03	-0.03	-	0.12	-0.08	0.04

Note: *, **, and *** denote significance on the 10, 5, and 1% significance level based on Newey and West (1987) standard errors.

[a]The first principal component from Benčik (2019).

Table 7: Out-of-sample prediction relative to the HP filter. Real-time data

	Univariate filters		Multivariate filters				
Quarters ahead	MH	BN	NBS	PCA[1]	MF-BVAR	DFM	DFM$_m$
Q1	0.92	1.03	1.02	1.06	1.00	1.05	0.98
Q2	0.86	1.03	1.03*	1.04	0.94	1.07**	0.99
Q3	0.90	1.03	0.99	1.00	0.87***	1.10***	0.99
Q4	1.16	0.97	1.02	1.02	0.87***	1.02	0.92**
Q5	1.45	0.92**	0.95	0.88	0.86***	0.98	0.89***
Q6	1.60	0.96	0.92**	0.89	0.90***	0.98	1.33
Q7	1.71	0.99	0.97	0.90	0.90***	1.00	1.28
Q8	1.85	1.00	0.97*	-	0.90***	1.00	1.16

Note: *, **, and *** denote significance on the 10, 5, and 1% significance level based on a two-sided Diebold and Mariano (1995) test with the small sample size correction of Harvey, Leybourne, and Newbold (1997).

[a]The first principal component from Benčik (2019).

Table 8 discusses improvements in the predictive model while including the output gaps. The gap from the MF-BVAR model improves forecasts for three, six, seven and eight quarters ahead. At the same time, the monthly DFM gap improves all predictions from three to eight quarters ahead. However, the results differ from the results in Table 5. The differences in predictive abilities of the output gap measures might be explained by the nature of the COVID period, which can be characterised by a large exogenous shock and relatively fast recovery after the pandemic.

Table 8: Out-of-sample prediction relative to no gap included. Real-time data

	Univariate filters			Multivariate filters				
Quarters ahead	HP	MH	BN	NBS	PCA[1]	MF-BVAR	DFM	DFM_m
Q1	1.00	1.02	1.00	1.00	1.02	0.99	1.00	0.99
Q2	0.99	1.03	1.00	1.00	1.00	0.95	1.01	0.97
Q3	0.99	1.05	1.00	0.99	1.00	0.93*	1.01	0.96*
Q4	1.01	1.07	1.01	1.00	1.01	0.94	1.02	0.96**
Q5	1.01	1.07*	1.01	1.00	1.01	0.95	1.02	0.95**
Q6	1.01	1.06*	1.01	1.00	1.01	0.96*	1.01	0.96*
Q7	1.00	1.04	1.00	1.00	1.00	0.96**	1.02	0.97*
Q8	1.00	1.03	1.00	1.00	-	0.97**	0.99	0.99*

Note: *, **, and *** denote significance on the 10, 5, and 1% significance level based on a two-sided Diebold and Mariano (1995) test with the small sample size correction of Harvey, Leybourne, and Newbold (1997).

[a]The first principal component from Benčik (2019).

6. Conclusions

This paper compares different univariate and multivariate output gap estimates for the Slovak Republic. However, estimation using multivariate models is complicated due to the small sample size (many economic indicators are available from the 2000s), two recessions during the studied period (the Great Resection and the COVID pandemic) that are different in nature, many extreme changes, and possible multiple structural breaks in GDP growth (some of which might not be discrete).

Nevertheless, output gap estimates from the multivariate models presented in this paper have good properties in terms of stability, they are similar in magnitude to the official NBS estimate and can predict inflation better than the gap from the Hodrick-Prescott filter. Moreover, the MF-BVAR gap is among the best-performing gap measures in price Phillips curves specifications before the recent period of high inflation in 2021–2022. Whereas all output gap measures struggled to predict high inflation in 2021–2022, alternative indicators, such as principal components from various survey indicators, might be used for these purposes.

A. Issues with methods for estimating the gap

A.1. Issues with the Hodrick-Prescott filter

The Hodrick-Prescott filter is a univariate two-sided filter that is often revised in real-time. The following graph shows recursive estimates of the gap from 2019Q2.

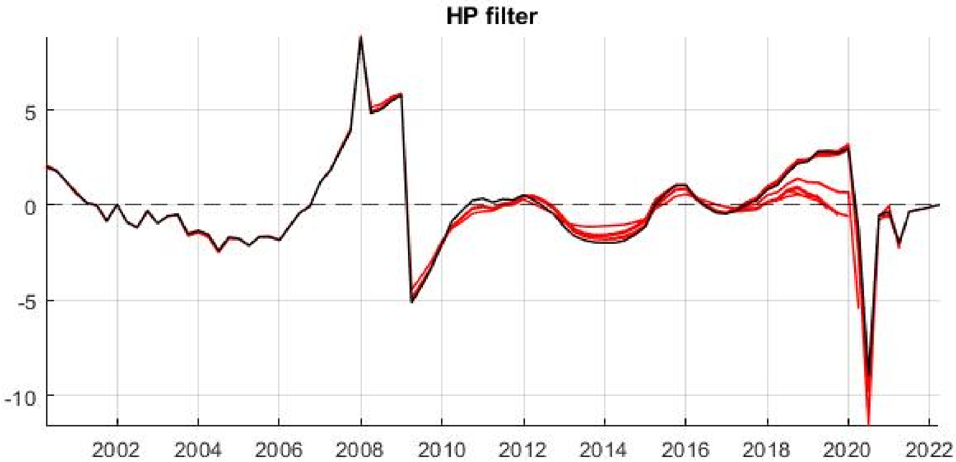

Figure A.1: Output gap from the Hodrick-Prescott filter, recursive re-estimations from 2019Q2, real-time data
the red lines at the top figure present intermediate recursive re-estimations, the black line – final estimates

A.2. Issues with Unobserved Component Models

Unobserved Component Models are often vulnerable to the revision problem in a similar way as the Hodrick-Prescott filter. To deal with this problem, researchers use additional information to make estimates of the gap more stable over time (Melolinna and Toth, 2019). A popular source of additional information comes from surveys. For example, European Central Bank (2015) aggregated information from surveys by a weighing scheme, while Benčik (2019) used the first principal component from surveys. Later this aggregated information from surveys is used in a state equation for the cycle in an unobserved component model (European Central Bank, 2015).

The problem with this approach is that over time there might be changes (breaks or time variation) in the coefficient for a survey-based indicator in a state equation, the coefficient might become non-informative, or different indicators might be important in different periods. In this case, this additional information will not help with the revision properties of an estimated gap. Moreover, non-informative data might not pin down the signal-to-noise ratio properly in a model, and filtering might produce spurious results.

A.3. Issues with Principal Components as a proxy for the gap

Alternatively, it is possible to use principal components from several economic activity variables as a proxy for the output gap. The proxy might be a good indicator of an economy's cyclical position, but it is challenging to estimate potential output with a proxy. In this case, a principal component measures the gap with an error, i.e. $pca_t = gap_t + \varepsilon_t^{ec}$, where ε_t^{ec} is a measurement error. GDP can be decomposed into the trend and cyclical components $gdp_t = gap_t + trend_t$ and the problem is that $gdp_t \neq pca_t + trend_t$ since pca_t also includes a measurement error and $gdp_t - pca_t = gdp_t - gap_t - \varepsilon_t^{ec} = trend_t - \varepsilon_t^{ec}$.

B. Monthly time series

B.1. The priors for MF-BVAR

The equations (B1) and (B2) describe hyperparameters for a natural conjugate prior (a Normal Inverse-Wishart prior). $\phi^{j,k}$ is the j,k element of the matrix F. σ_j^2 is the variance of the j^{th} variable and it is estimated from data, l is a lag of the k^{th} variable in the j^{th} equation, α is a hyperparameter that controls overall prior tightness. The decomposition requires the eigenvalues of the matrix F to lie inside a unit circle and, therefore, non-stationary variables are usually transformed into stationary series. The estimation is done via adding dummy observations to observables and using a closed-form solution as in Berger, Morley, and Wong (forthcoming).

$$E(\phi^{j,k}) = 0 \tag{B1}$$

$$Var(\phi^{j,k}) = \begin{cases} \frac{\alpha^2}{l^2} & if \quad j = k \\ \frac{\alpha^2\sigma_j^2}{l^2\sigma_k^2} & otherwise \end{cases} \tag{B2}$$

MF-BVAR models depend heavily on model specification (such as selected variables and lags) and priors. A higher α puts more weight on data and filters GDP better (produces a smoother trend). Nevertheless, it also overfits more and delivers less reliable nowcast due to over-parametrisation. At the same time, a smaller α produces more accurate forecasts but filters GDP worse. α is optimised based on minimising pseudo-out-of-sample mean squared errors (Berger, Morley, and Wong, forthcoming) for a variable of interest (GDP growth in our case).

B.2. Monthly time series

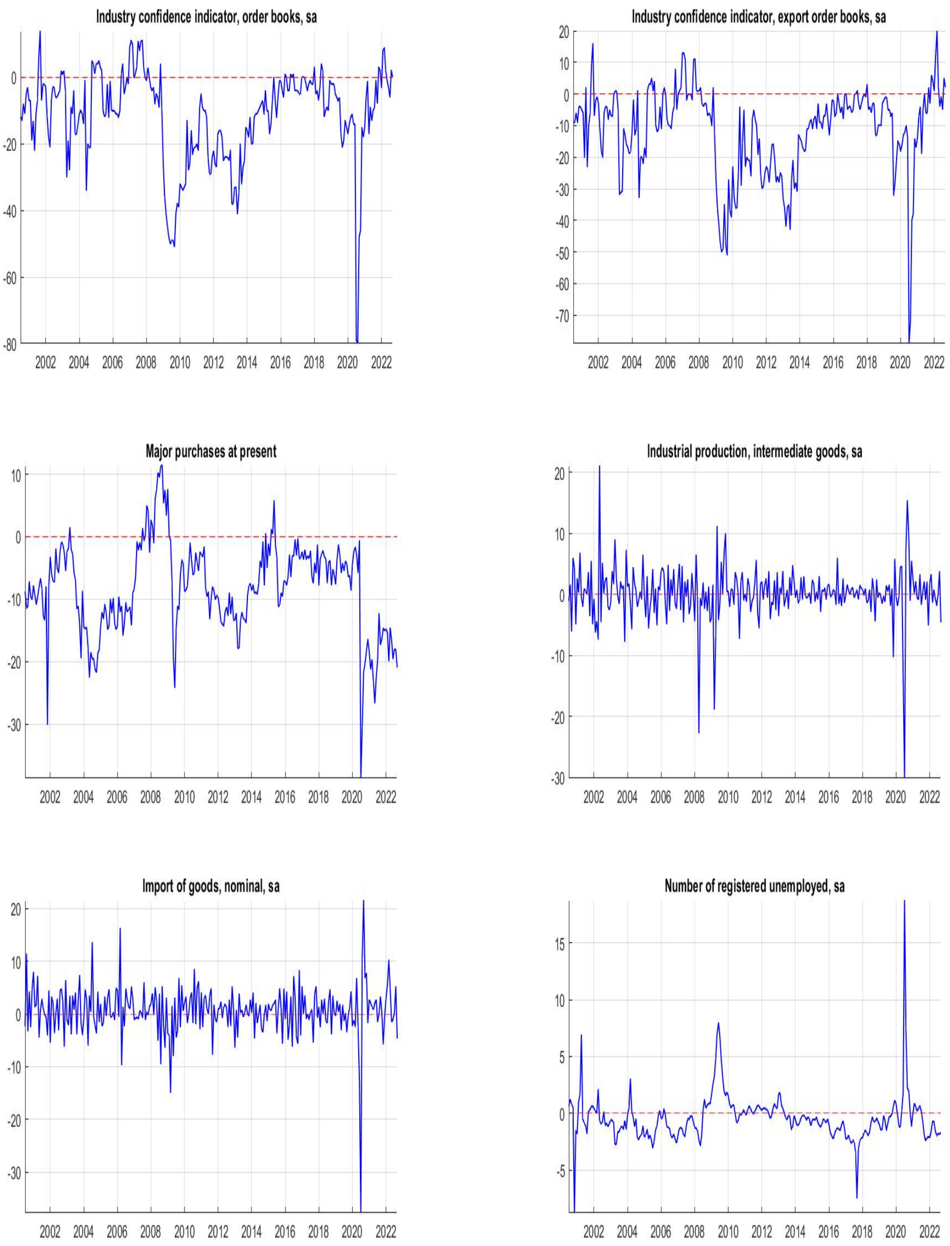

Figure B.1: Input data for the MF-BVAR

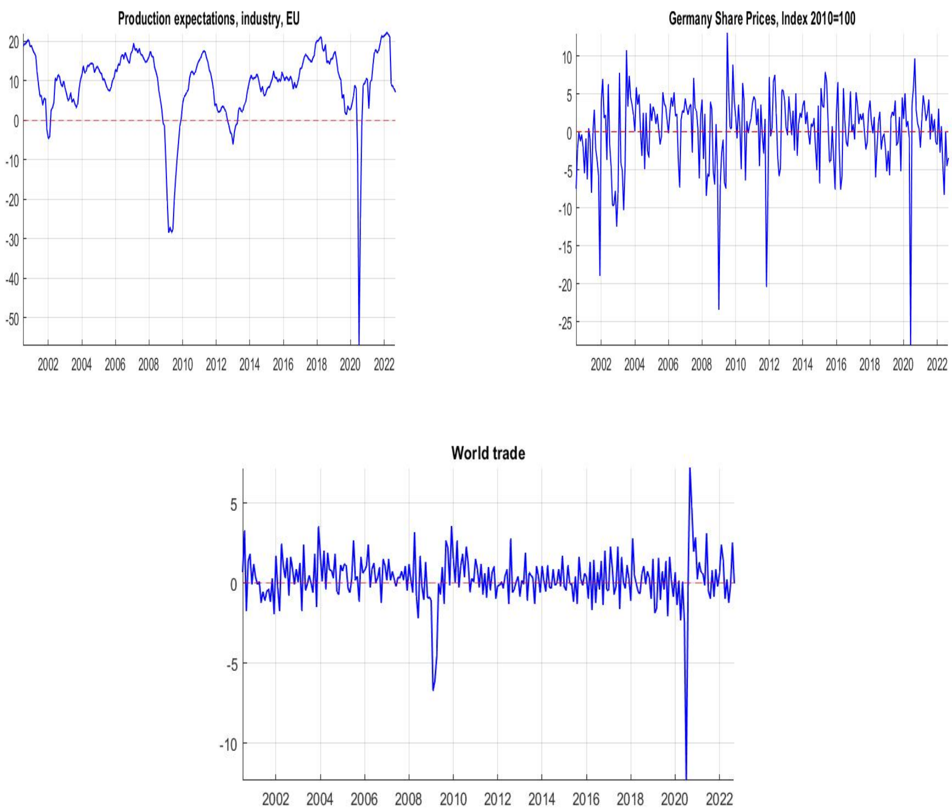

Figure B.2: Input data for the MF-BVAR

C. Estimation of the Dynamic Factor Model

Priors

The study follows Jarocinski and Lenza (2018) in setting priors. The priors for Λ_t (D1) are independent Gaussian with means equal to zeros, and prior variances are set in a Minnesota style, i.e. variances are equal to $\alpha\frac{\sigma^2_{y^n}}{\sigma^2_{y^1}}$, where α is 0.25. $\sigma^2_{y^n}$ are set based on the training sample (from 1995 till 2000)[13]. The prior for the output gap parameters (ϕ) is multivariate Normal (D2) with means and variances are set based on the periodicity of business cycles in the euro area (Jarocinski and Lenza, 2018).

The priors for the variances of the shocks to all observables (D3), except for GDP ($\varepsilon_t^{n(n\neq 1)}$), are independent inverse Gamma with scale parameter o and degrees of freedom v. v is set to 5 and o is $(v-2)\frac{\sigma^2_{y^n}}{4}$. The study uses the Uniform prior $U(0, 0.01)$ with the upper bound 0.01 for the first variable (real GDP) and also for the shock variances to trends to make trends more smooth than otherwise implied by a model with unconstrained inverse Gamma priors. The prior for the shock variance in the gap equation (3d) is inverse Gamma with degrees of freedom v and scale o_g equal to $(v-2)0.2\sigma^2_{y^1}$.

$$\Lambda_t^n \sim N(0, \alpha^2\frac{\sigma^2_{y^n}}{\sigma^2_{y^1}}) \tag{D1}$$

$$p(\phi_{1,2}) \sim N\left(\begin{array}{c} 1.352 \\ -0.508 \end{array}\right), \left(\begin{array}{cc} 0.0806 & -0.0578 \\ -0.0578 & 0.0806 \end{array}\right)_{I(\phi_{1,2}\in R)} \tag{D2}$$

$$var(\varepsilon_t^{n(n\neq 1)}) \sim IG(v, o) \tag{D3}$$

$$var(\varepsilon_t^1) \sim U(0, 0.01) \tag{D4}$$

$$var(\eta_t^n) \sim U(0, 0.01) \tag{D5}$$

$$var(\eta_t^g) \sim IG(v, o_g) \tag{D6}$$

Initial values

Starting values of observables are used as prior means for initial values, except for the gap, where hyperparameters for initial values are set to zero since it is a zero mean stationary process. Prior variances are set to 100.

[13]Since variables are trending, first differences of observables are used for setting the priors.

Estimation

The model (3f)-(3h) together with the priors (D1)-(D6) is a linear state-space model. Therefore, it is possible to use the Gibbs sampling to draw from the posterior in this case. The precision-based sampler of Chan and Jeliazkov (2009) is employed instead of one of more standard forward filtering backward sampling algorithms based on the Kalman filter (Carter and Kohn, 1994; Frühwirth-Schnatter, 1994; Durbin and Koopman, 2002), because the precision-based sampler is more efficient. A greedy Gibbs algorithm (described in Chan, Koop, Poirier, and Tobias (2019), for instance) is used to draw posteriors for $var(\varepsilon_t^1)$ and $var(\eta_t^n)$ because full conditional densities of $var(\varepsilon_t^1)$ and $var(\eta_t^n)$ are non-standard. I generate 8,000,000 draws, discard the first 4,000,000, and keep every 20th draw of 4,000,000 remaining (200,000 retained draws in total). Trace plots for the autoregressive parameters of the gap are shown in Figure C.1.

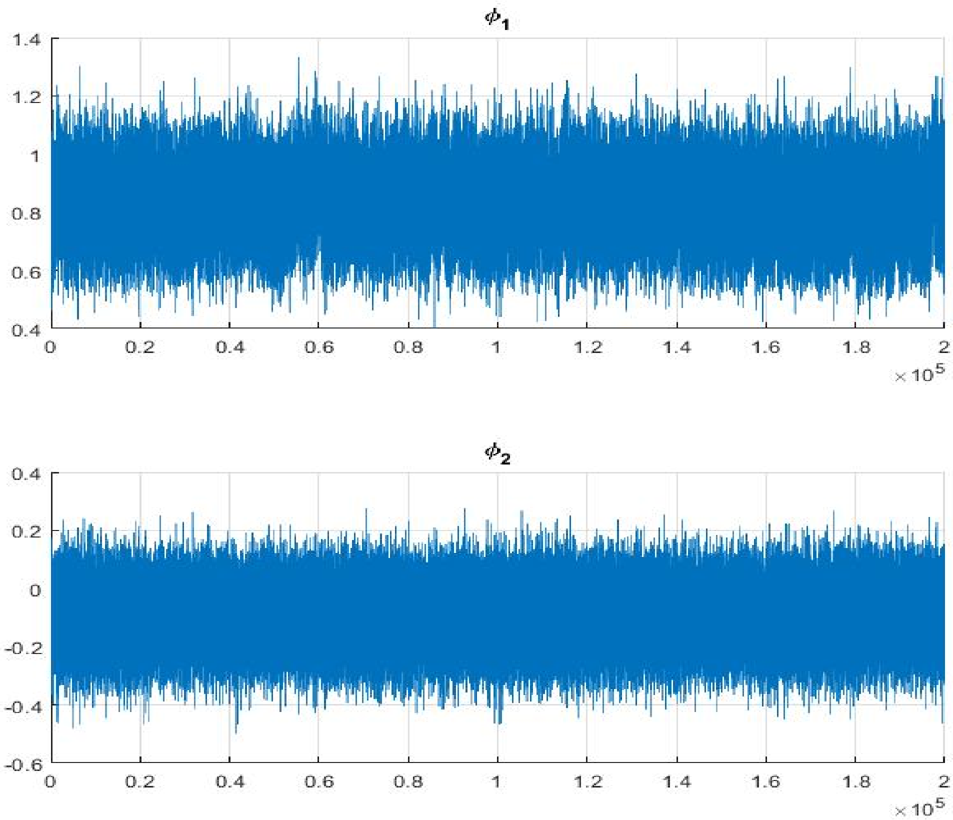

Figure C.1: Trace plots for the autoregressive coefficients in the state equation (3d)

Table C.1: Estimated parameters

Parameter	Estimate
ϕ_1	0.85 (0.11)
ϕ_2	-0.11 (0.09)

Note: posterior means and numerical standard errors.

Table C.2 presents the estimated parameters[14]. According to the results, industry confidence and exports are leading indicators with respect to the gap, unemployment and investment are lagging, while capacity utilisation is both a leading and lagging indicator. Survey indicators are leading with respect to the gap, which is a well-known forward-looking property of surveys. Moreover, all variables are important for the gap contemporaneously. All coefficients are positive except for unemployment, which is negative.

Table C.2: Estimated parameters

Parameter	$\lambda^a(t-1)$	$\lambda^b(t)$	$\lambda^c(t+1)$
Capacity utilisation	**0.45**	**0.73**	**0.34**
	(0.14)	(0.16)	(0.13)
Industry confidence	0.33	**2.20**	**1.36**
	(0.43)	(0.50)	(0.42)
Unemployment	**-0.16**	**-0.09**	0.02
	(0.03)	(0.03)	(0.03)
Investment	**1.89**	**2.01**	-0.06
	(0.43)	(0.51)	(0.42)
Export	0.16	**2.18**	**0.46**
	(0.22)	(0.26)	(0.22)

Note: coefficients with t-stat higher than two are marked in bold. Posterior means and numerical standard errors.

[14] Posterior means and numerical standard errors.

D. Mixed-Frequency Dynamic Factor Model

The derivations are based on Chan, Poon, and Zhu (2021).

$$Y = M_o Y^o + M_u Y^u \tag{E1}$$

where Y is a $NT_m \times 1$ vector of all variables, Y^o are $N_o T_m \times 1$ variables that are available at a monthly frequency, Y^u are $N_u T_m \times 1$ variables that are unobserved at a monthly frequency, M_o and M_u are full column rank matrices and constructed as described in Chan, Poon, and Zhu (2021).

$$\tilde{Y}^u = M_a Y^u + u \qquad u \sim N(0, O) \tag{E2}$$

$\tilde{Y}^u$ is a $N_o T_q \times 1$ vector of variables at a quarterly frequency, which are linked to unobserved variables at a monthly frequency, M_a is a $N_o T_q \times N_o T_m$ matrix of restrictions and has a full column rank and constructed as described in Chan, Poon, and Zhu (2021). O can be very small meaning that errors are close to zeros. Combining equations (3f) and (E1) and writing an expression for Y^u one can get:

$$\mu_Y = K_Y^{-1}(M_u' \Omega^{-1}(vec((g\Lambda)') + vec(w') - M_o Y^o))) \tag{E3}$$

where $K_Y = M_u' \Omega^{-1} M_u$ and $\Omega^{-1} = (T_m N \otimes \mathcal{E})^{-1}$. Λ is a $N_{leads/lags} \times N$ matrix of λ^s, g is a $T_m \times N_{leads/lags}$ matrix of the lead/lagged output gap, w is a $T_m \times N$ matrix of individual trends from (3f), T_m is a monthly number of observations, N is a total number of variables (both monthly and quarterly), $\mathcal{E}$ a $T_m \times N$ matrix of errors from (3f).

Then, deriving Y^u from (E2) it is possible to obtain:

$$\mu_{Y_u} = (\Omega^{Y^u})^{-1}(M_a' O^{-1} \tilde{Y}^u) \tag{E4}$$

where $\Omega^{Y^u} = M_a' O^{-1} M_a$. Finally, from (E3) and (E4) we obtain:

$$\mu_{Y^u} = K_{Y^u}^{-1}(M_a' * O^{-1} * \tilde{Y}^u + M_u' \Omega^{-1}(vec((g\Lambda)') + vec(w') - M_o Y^o)) \tag{E5}$$

where $K_{Y^u} = M_a' O^{-1} M_a + M_u' \Omega^{-1} M_u$. The rest of the model is as described in (3f)-(3h).

E. TRENDS FROM THE DYNAMIC FACTOR MODEL

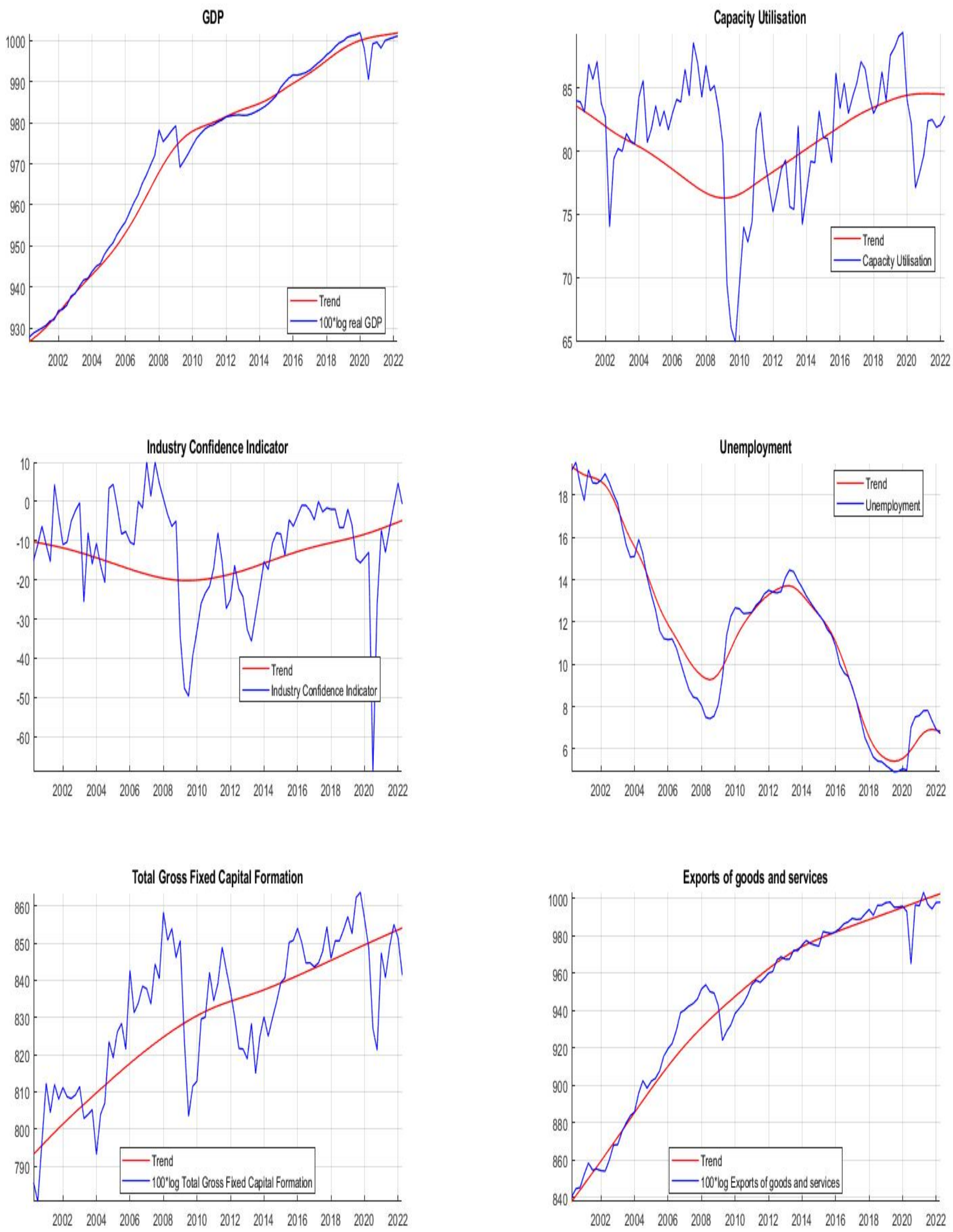

Figure E.1: Trends from the Dynamic Factor Model and the observed series; posterior means

F. Additional Results

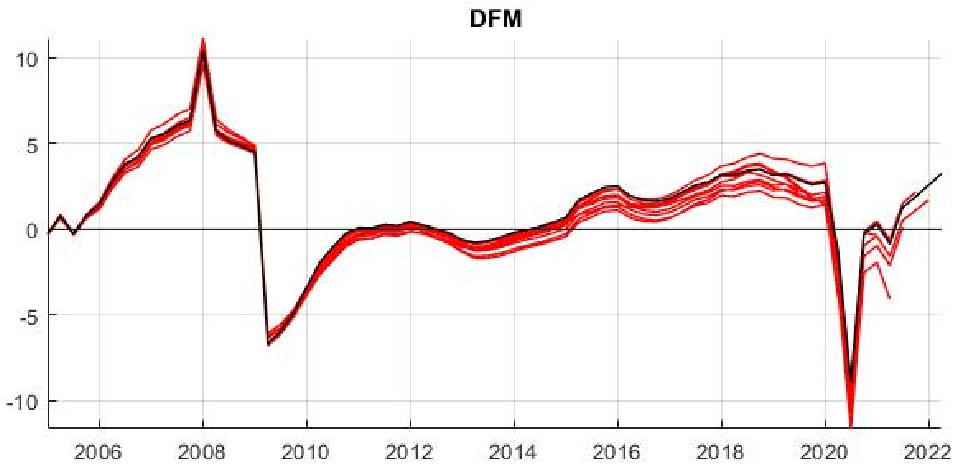

Figure F.1: Estimated Dynamic Factor Model with the MF-BVAR data
the red lines in the figure present different data vintages starting from 2019Q2, the black line – final estimates; posterior means; real-time data

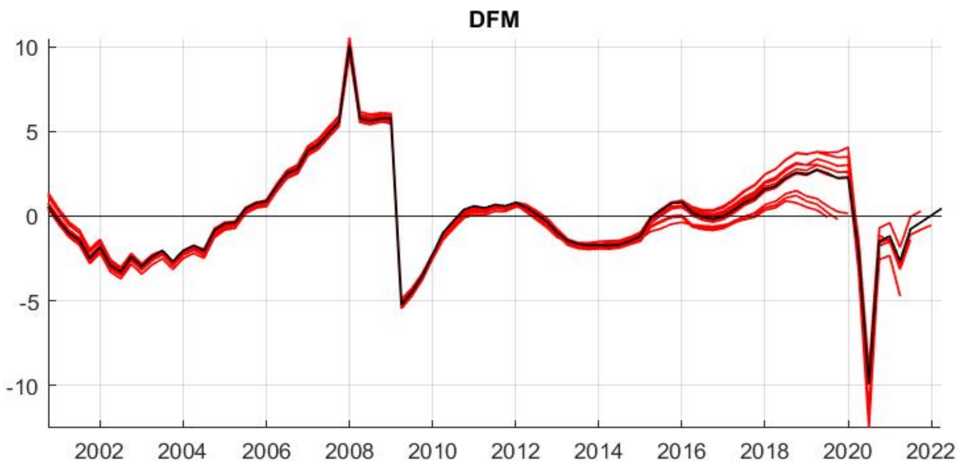

Figure F.2: Estimated Dynamic Factor Model with the UCM data
the red lines in the figure present different data vintages starting from 2019Q2, the black line – final estimates; posterior means; real-time data

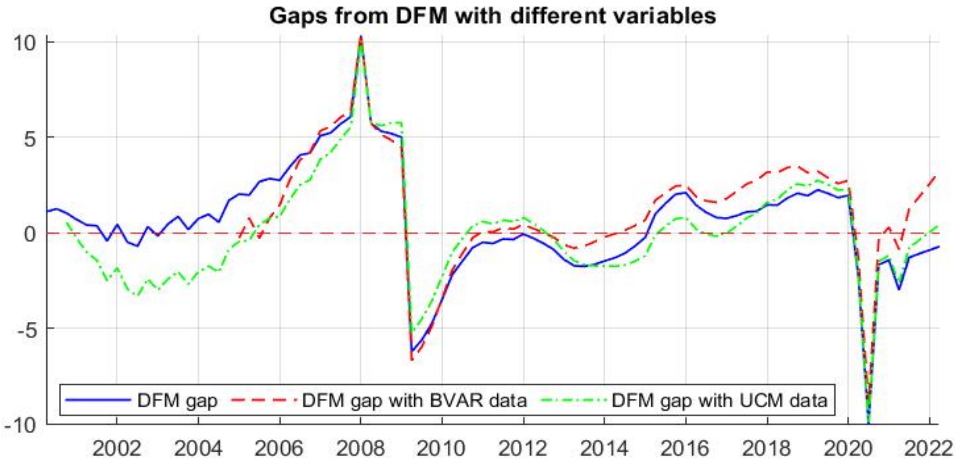

Figure F.3: Estimated output gaps from the quarterly Dynamic factor model with different variables

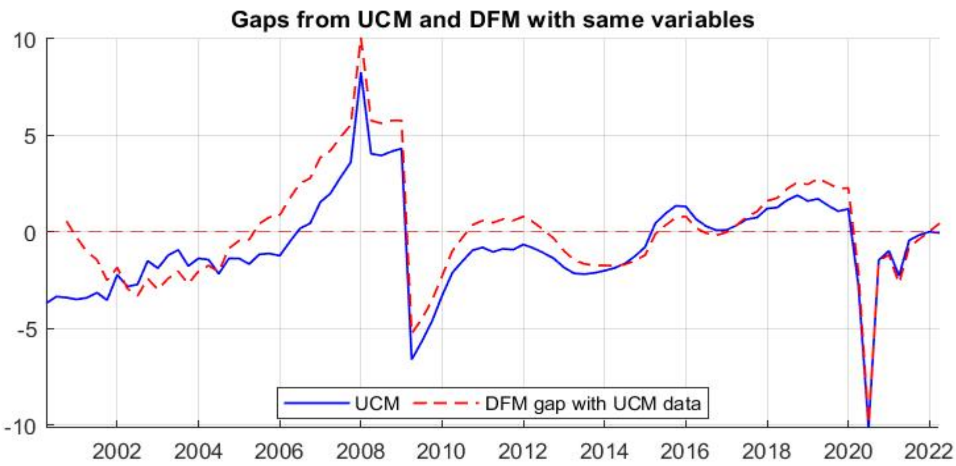

Figure F.4: Estimated output gaps from the UCM model and the quarterly Dynamic factor model with the UCM variables

G. Comparison of the output gaps

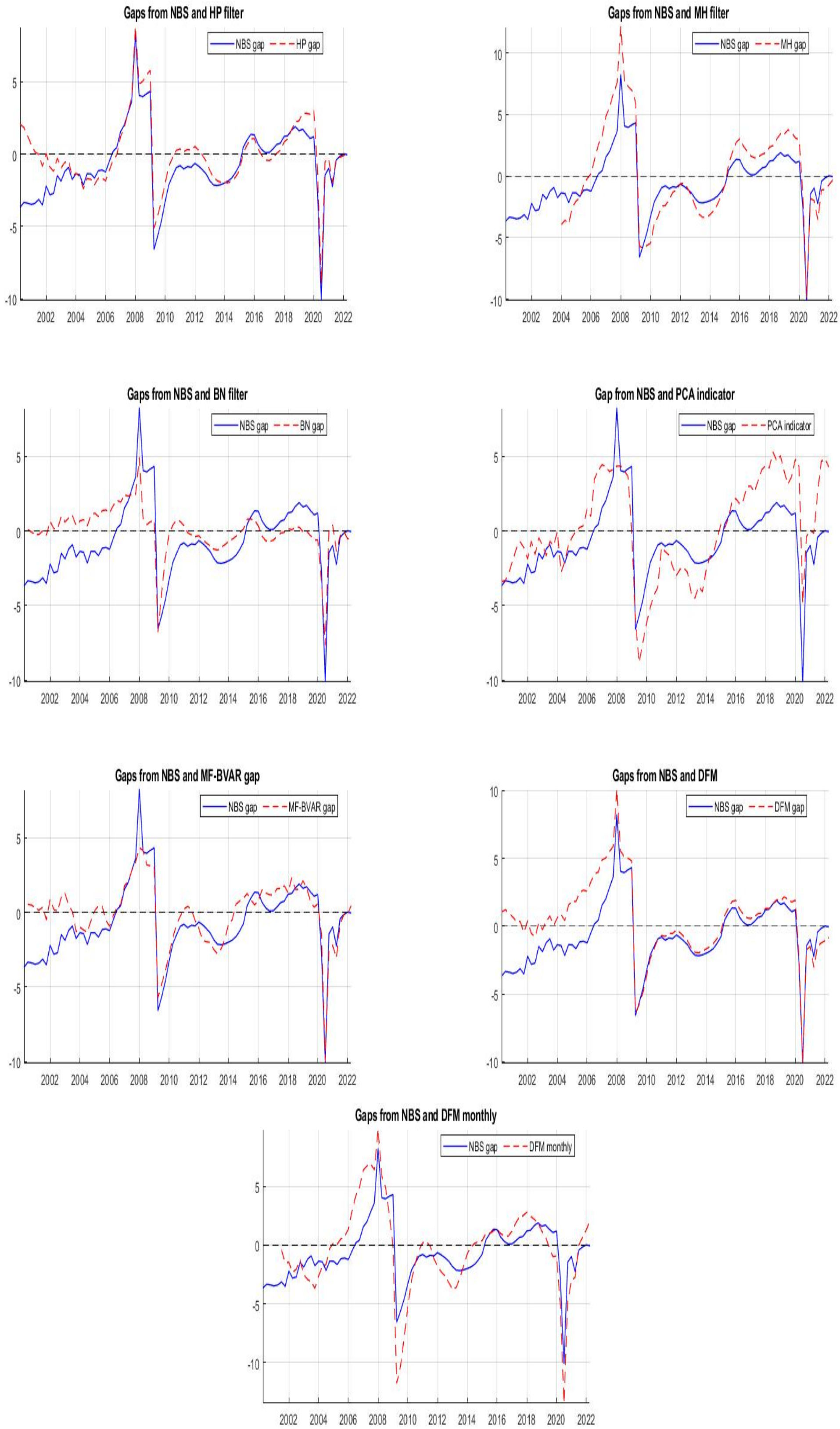

Figure G.1: Comparison of the output gaps

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